

A CRITICAL REAPPRAISAL OF CARL — GÔRAM MALER'S ENVIRONMENTAL MODEL

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1. INTRODUCTION

Karl-Gôram Måler (KGM), in his well known monograph on Environmental Economics ¹, constructs a simple model to investigate the optimal time path that maximizes the welfare function (1) under the restrictions (2-7).

$$W = \int_0^T U(C, Y) \exp(-rt) dt \quad (1)$$

$$K(T) = K_s, \quad Y(T) = Y_s \quad (2)$$

$$s \int_0^T f[K(t)] dt = S \quad (3)$$

$$X + C - f_m K - f(K) = 0 \quad (4)$$

$$C + m K - Z = 0 \quad (5)$$

$$dY/dt = L' - L'Y - GZ, \quad U = a/A, \quad G = 1/A \quad (6)$$

$$X + mK = 0$$

where, U : an instantaneous utility function

C : rate of consumption

Y : environmental quality

r : discount factor that represents the time preference of the government

t : time

T' : length of the planning period

K(t) : capital stock, at time t

K_s : the minimum level of capital stock required at the horizon

Y_s : the minimum quality of the environment required at the horizon

f[K(t)]: production function (with the assumption that labour input remains constant), which equals the rate of natural resources exploitation.

S : the maximum exploitation allowed during the planning period

X : rate of net capital accumulation = dK/dt

m : rate factor, at which the capital stock is assumed to depreciate owing to physical wear and tear.

Z : rate of residuals generation (tons of pollutant per unit of time)

a: lake outflow (m^3 per unit of time)

A : water volume of the lake (m^3).

These equations can be checked by means of dimensional analysis². For this purpose we determine the dimensions of the above quantities (variables or parameters of the system), as a function of the primary quantities R, L, T :

$$f [K(t)], Z, C, X=[RT^{-1}]^*, \quad r, m = t[T^{-1}], \quad t, T' = [T], \quad K, S = [R]$$

$$Y=[RL^{-3}]^{**}, \quad a=[L^3T^{-1}], \quad A=[L^3],$$

where R = Mass, L = Length, T = Time.

* KGM considers a composite commodity that can be used for production, consumption and capital accumulation, in physical units. This commodity exists as a nonreproducible natural resource in a certain limited quantity.

** KGM uses the ambient concentration of discharged pollutant η (tons/ m^3) to formulate a measure of environmental quality.

Since Eq. (6) must be dimensionally homogeneous, the exponents of the individual primary units on the left hand side must equal those of each term on the right hand side :

Term	of the left hand side :	$dY/dt = [RL^{-3}] [T^{-i}]$	$= [R^1L^{-3}T^{-1}]$
1st»	» right »	$: L' = a/A = [L^3T^{-i}][L^{-3}]$	$= [R^0L^0T^{-i}]$
2nd»	» » »	$: L'Y = [L^3T^{-i}][L^{-3}][RL^{-3}]$	$= [R^1L^{-3}T^{-1}]$
3rd»	» » »	$: GZ = [L^{-3}][RT^{-i}]$	$= [R^1L^{-3}T^{-1}]$

As the exponents of the individual primary units of the 1st term do not equal those of the rest terms, we may suggest that this equation is not correct.

2. CORRECTION OF THE UNSTEADY - STATE MATERIAL BALANCE EQUATION

Eq. (6) is derived straight-forward from KGM's assumption of a lake-like environmental system where Z tons of pollutants per unit of time are discharged in. According to his notation,

nA is the total amount of pollutant in the lake

na is the amount of pollutant that is transported away from the lake per unit of time

Z is the transportation to the lake of the pollutant and

$-an + Z$ is the change of the total amount of pollutant in the lake per unit of time ; thus

$$A \frac{dn}{dt} = -na + Z \quad \text{or} \quad \frac{dn}{dt} = -\frac{an}{A} + \frac{Z}{A}, \quad A = \text{const.} \tag{8}$$

Instead of studying the ambient concentration n , he is introducing the variable $Y = 1 - n$. By substituting Y into Eq. (8), one has Eq. (6), which has the following dimensionally non homogeneous solution :

$$Y = Y(0) \exp(-L't) + (J - GZ/L') [1 - \exp(-L't)], \quad Z = \text{const.} \tag{9}$$

We may observe that giving to 1, in the expression $Y = 1 - n$, the dimensions of n , we avoid the dimensional inconsistency of Eqs (6) and (9). Nevertheless it remains a question about the dimensions of η and a in the first term on the right-hand side of Eq. (8). If $n = [RL^{-3}]$ and $a = [L^3T^{-1}]$ then this term implies that there is an outflow of water from the lake carrying within polluting material na and non polluting material $n'a$, where n' is the concentration of the rest materials including the carrier (water). If $n' = 0$ then either η is meaningless or the system has collapsed, because it contains nothing but pollutant. If $n' > 0$ then there is an amount of material $n'a$ which outflows from the lake without being replenished, as pollutant is by definition the only substance discharged in the lake (with a rate Z); in this case, A is not a constant and the term Adn/dt must be replaced by $d(An)/dt$.

KGM claims that his «first-order linear differential equation (6) is formally identical to the Streeter-Phelps equation, relating the dissolved oxygen deficit to biochemical degradable wastes» with a difference in interpretation. Most probably this is the main source of his error in Eq. (6) or in its equivalent unsteady - state materials balance Eq. (8).

The Streeter - Phelps equation* in its simple form $dD/dt = K_1S' - K_2D$, where D the dissolved oxygen deficit, S' the concentration of organic pollutant expressed in Biochemical Oxygen Demand (BOD) and K_1, K_2 rate constants, is based on a closed system mechanism with inflow (oxygen absorption from the atmosphere) and a single sink (utilization by microorganisms consuming organic pollutant, thus reducing BOD). This equation is used in stream analysis and its derivation is based on ideal streams simulated by a plug flow reactor. The main assumption is that no mixing of fluid elements occur longitudinally along the flow path; i.e. each fluid element in this type reactor is analogous to a completely mixed batch reactor moving along a time axis. Therefore, no outflow occurs from the elementary system examined for derivation of the Streeter - Phelps equation.

The best way to find out a correct form for Eq. (8) is to investigate the spe-

* The two parts of this equation were borrowed in 1925 from chemical kinetics by Streeter and Phelps to describe the course of BOD of polluted waters. However, the use of a first order kinetics equation for this purpose was foretold very earlier in Phelps's analysis of the results of methylene blue stability tests^{3, 4}.

cial cases which result from a general unsteady-state material balance equation for a homogeneous lake* simulated by a continuous stirred tank reactor.

We proceed to formulate this equation, first in word and then in mathematical terms, by the aid of the schematic diagram of Fig. 1.

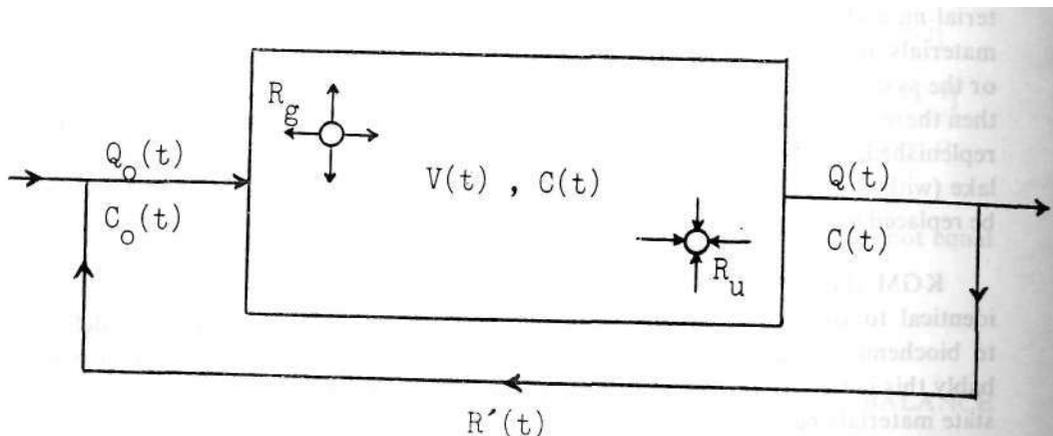


Fig. 1. Schematic diagram of a lake-like homogeneous environmental system simulated by a continuous stirred tank reactor.

a. General word statement :

Net rate of accumulation of pollutant within B = rate at which the pollutant enters B - rate at which the pollutant leaves B + rate of generation of pollutant within B - rate of utilization of pollutant within B,

where

* A closed lake-like environmental system with homogeneous dispersion of sinks and sources (both existing as particulates) has been also used by the MIT project team, which worked on «The Limits to Growth», in their World 3 Model⁵. An interdisciplinary team at the Science Policy Research Unit at Sussex Univ., including economists, engineers, physicists, biologists, political scientists and social psychologists, published «A Critique of the Limits to Growth». P. K. Marstrand and T. C. Sinclair⁶, representing this team in the topic of pollution, consider the concept of homogeneously dispersed particulates-like pollution sinks and sources as a highly unrealistic device implying a mechanism «more characteristic of the spread of an insidious and largely unidentified menace». Nevertheless, they are not in a position to give even a slight idea about an alternative model.

B = the system boundary*.

b. Simplified word statement :

Accumulation = input - output + generation - utilization

c. Quantitative representation :

$$d(VC)/dt = Q_0(t)C_0(t) - Q(t)C + R_g V - R_u V \quad (10)$$

where $V = V[V(0), Q_0(t), Q(t)]$, $C = C[C_0(t), V, Q_0(t), Q(t), R_g, R_u(C)]$,

R' = Control variable, $V(0)$ = volume of the lake at $t = 0$,

V = volume of the lake, $[L^3]$

C = concentration of pollutant in the lake and the effluent, $[RL^{-3}]$

Q_0 = volumetric rate of flow into the lake, $[L^3T^{-1}]$

Q = volumetric rate of flow out of the lake, $[L^3T^{-1}]$

C_0 = concentration of pollutant in the influent, $[RL^{-3}]$

R_g = rate of pollutant generation within the lake, $[RL^{-3}T^{-1}]$

R_u = rate of pollutant utilization within the lake, $[RL^{-3}T^{-1}]$

R = volumetric rate of recycle flow, $[L^3T^{-1}]$

In order to obtain a form of the unsteady-state material balance Eq. 10 comparable with Eq. (8), we assume $V = A = \text{const.}$, $Q_0(t) = Q(t) = Q = \text{const.}$, $R_g A = Z' = \text{const.}$, $R_u = K_1 C$ (first order kinetics). Eq. (10) is now reduced to $A dC/dt = Q C_0(t) - Q C + Z' - K_1 C A$ (11) or $dC/dt = -bC + (Q/A)C_0(t) + Z'/A$ (11a) where $b = (Q/A) + K_1$, which has the solution

* When the boundaries act as sources, e.g. release of phosphorus from the sediments of the lake, their product is considered as an input for the improvement of the predictability of the environmental model⁷.

$C = C(0) \exp(-bt) + \exp(-bt) [Z' (\exp(bt) - 1) + bQ \int_0^t S_{in}(t) \exp(bt) dt] / bA$ (12)
 where

$$C = C(0) \quad \text{at} \quad t = 0.$$

If $C_0(t) = C_0 = \text{const.}$, Eq. (11a) is reduced to

$dC/dt = -bC(Q/A)C_0 + Z'/A$ (11b), which has the solution

$$C = C(0) \exp(-bt) + Z'[1 - \exp(-bt)]/bA + QC_0 [1 - \exp(-bt)]/bA \quad (13).$$

The first term of Eq. (13), and of the general solution (12), evaluates the initial condition of the system, the second term evaluates the concentration change brought about by a source generating pollutant with a rate R_g while the third term evaluates the concentration change brought about by the input QC_0 .

Eq. (11b) is similar to Eq. (8) for $QC_0 + Z' = Z$ and $Q + K_x A = a$, but in the course of the investigation this resemblance will be proved impossible to occur, unless an open system concept may be adopted, under special assumptions about the inflow carrier.

We are now in a position to investigate special cases of Eq. (11b)*, which may represent possible situations described by Eq. (8).

1st case : When $Q = 0$, i.e. no inflow - outflow takes place, Eq. (11b) is reduced to $dC/dt = Z'/A - K_x C$, which has the solution

$$C = C(0) \exp(-K_1 t) + (Z'/AK_1) [1 - \exp(-K_1 t)] \quad \text{and is similar to Eq. (8)}$$

for $Z' = R_g A = Z$ and $a/A = K_x$.

2nd case: When $R_u = 0$, $R_g = 0$, i.e. there are neither sinks nor sources within the system, Eq. (11b) is reduced to $dC/dt = QC_0/A - QC/A$, which has the solution $C = C(0) \exp(-Qt/A) + C_0 [1 - \exp(-Qt/A)]$, and is similar to Eq.(8) for $QC_0 = Z$.

* which equation in its complete form we may call case 0.

3rd case : When $R_u = 0$, $R_g > 0$, i.e. there are only pollutant sources within the system, Eq. (1b) is reduced to $dC/dt = QC_0/A - QC/A + Z'/A$, which has the solution $C = C(0)\exp(-Qt/A) + (C_0 + Z'/Q) [1 - \exp(-Qt/A)]$, and is similar to Eq. (8) for $QC_0 + Z' = Z$.

4th case: When $R_u > 0$, $R_g = 0$, i.e. there are only pollutant sinks within the system, Eq. (1b) is reduced to $dC/dt = QC_0/A - bC$, which has the solution $C = C(0)\exp(-bt) + QC_0[1 - \exp(-bt)]/bA$, and is similar to Eq. (8) for $Q + K_1A = a$ and $QC_0 = Z$.

In all cases where $Q > 0$, we must have $Q = R$, so that the system can be considered as a closed one. This implies that the pollutant cannot enter the system within an inflow from outside, without a change of A . But even if we might accept an inflow, implying an open system, then its volumetric rate must be a , so that $A = \text{const.}$; under these conditions Eq. (8) lacks the self-purification term, the lake acting now as a dilution tank. Moreover, as the pollutant concentration in R is C , there must exist a wastewater treatment installation within the system if $R_u = 0$. But KGM states clearly that such an investment has no place in his paradigm. So, $C = C_0$ if $Q > 0$ and finally cases 2, 3, 4 are rejected while case 0 is reduced to case 1, which is the only possible if the concept of the closed system is to be kept. On the contrary, if we accept an open system we have to assume a pollutant quasi-outflow because of self-purification ($K_x A$) and volumetric outflow (Q), as it is shown in cases 0 and 4, and $Q > R = 0$, $A = \text{const.}$; but remains the question about the inflow carrier (water or inert material) which can be answered only by changing Eq. (8) to $Adn/dt = -na + wZ$ ($1 > w > 0$), where $1 - w$ is the portion of Z which is inert material, simulated by water in the lake system. This is an assumption closer to reality which determines also C_0 and Q as $wZ / (Z/d)$ and Z/d , respectively, where d stands for apparent density ($d = \text{const.}$).

3. EXTENSION OF THE UNSTEADY-STATE MATERIAL BALANCE EQUATION**

In real world, C_0 as well as Q_0 are not constants. The unsteady-state mate-

* The apparent density may vary with C but in simulation of real aquatic environmental systems it is frequently assumed to be independent of C .

** An analysis of pollution accumulation phenomena in closed systems, based on this part of the article, has been presented by the Author at the Annual Seminar on «Quantitative Methods

rial balance Eq. (11a) gives the following solutions for the most common forms of input concentration in polluting substances. In the same way, we may obtain solutions from Eq. (10) for $Q_0(t) = Q(t)$, by solving first the differential equation $dV/dt = Q_0(t) - Q(t)$.

- a. Linear change of pollutant concentration, $C_0(t) = C_0 + ct$
(increase when $c > 0$, decrease when $c < 0$, Eq. (13) when $c = 0$).

$$C = C(0)\exp(-bt) + (Z' + QC_0) [1 - \exp(-bt)] / bA + cQ [bt - 1 + \exp(-bt)] / b^2A$$

- b. Exponential change of pollutant concentration, $C_0(t) = C_0 \exp(ct)$ (increase when $c > 0$, decrease when $c < 0$, Eq. (13) when $c = 0$).

$$C = C(0)\exp(-bt) + Z' [1 - \exp(-bt)] / bA - fQC_0 [\exp(ct) - \exp(-bt)] / (b - fc)A$$

- c. Periodic change of pollutant concentration, $C_0(t) = C_0 + c \sin(vt)$ (the simplest case after a Fourier analysis).

$$C = C(0)\exp(-bt) + Z' [1 - \exp(-bt)] / bA + Q [C_0(1 - \exp(-bt)) / b + c(b \sin vt - v \cos vt + v \exp(-bt)) / (b^2 - f v^2)] / A$$

The last case can be extended to involve influent flow rate changes, which are of special importance. In practice, if the changes observed in the influent wastewater flow rate and strength are expected to cause a severe shock load a flow equalization technique is applied. The damping of flow rate changes, through an external basin, favours the sinks of the system to respond in a natural way, so that the self-purification characteristics can be protected.

Of extreme importance is the effect of concentration increase on the rate constant of the sink. This point has been widely misunderstood. E.g. the MIT project team of «The Limits to Growth» considered two opposing mechanisms influencing the rate at which pollution is eliminated. In the first mechanism, the higher the level of pollution, the greater is the rate at which it is absorbed, *ceteris paribus*. According to the second mechanism, rising of pollution levels leng-

in Economics and Operational Research», running at the Pireaus Graduate School of Industrial Studies under the direction of Prof. A. Panayotopoulos, and has been briefly discussed with Prof. G. Drakos. I take the opportunity to thank both of them.

thens the pollution elimination time, which in turn depresses the sink rate. The net result is in favour of pollution increase. It is worth noting that neither T. C. Sinclair⁸ nor P.K. Marstrand⁶ of the Sussex Univ. project team were able to fight this argument, in their immediate critique of «The Limits to Growth». Supporting the view that «neither the upper bound for pollution absorption capacity nor the level of pollution at which it would be reached is known for any pollutant», they conclude that there is no means at all of checking the suggested by the MIT project team limits against the real state of affairs.

Actually, there is not any upper bound capacity, in a static meaning, as environmental sinks are renewable resources ; i.e. they usually consume or deteriorate or weaken the polluting substances instead of accumulating them in an internal store, which will be filled up in a definite time. The latter is true in certain cases of organisms that accumulate heavy metals or toxic substances and become dangerous for human or animal consumers (e.g. mercury poisoning in the region of Minamata Bay by soda producing industrial units⁹)*. In most cases the polluting substances are finally transformed to inert or useful material stored in Nature to be used in the rings of the food chain. E.g. nutrient loading of wastewater discharged in the lake indicates pollution but it is possible to serve, under special management, as a driving force for lake restoration " .

Instead of an upper bound capacity concept we may introduce the function: sink capacity parameters = $f(C, T'_c, t)$ where T'_c is the time period during which the sink is exposed to pollutant concentration C (actually a distribution) or, for simplicity, $K = f(C, T'_c, t)$ as the rate constant K is the only parameter in first-order kinetics of pollution absorption by renewable sinks. At low ambient concentration C , K is unaffected, at medium concentration K probably increases (especially if defined as specific rate constant, reduced to mass per mass or volume unit of the lake) but at higher concentration begins to decrease. This decrease is initially a linear reversible change that becomes accelerated fall leading suddenly (as a critical ring in the internal sink subsystem breaks down) to catastro-

* We can classify sinks into two categories : Sinks of permanent neutralization (e.g. biodégradation of organic waste) and sinks of transient accumulation (e.g. heavy metals or nutrient elements accumulation by living aquatic organisms). Sinks of the last category behave also as sources, at the same time (e.g. phosphorus uptake by the roots of macrophytes and release through the stems and leaves) or with a time lag (e.g. concentration of phosphorus upon the sediments of the lake by settling of suspended inorganic and organic particles and chemical release when the hypolimnetic water becomes anaerobic 1°).

phes of the sink. This process can be incorporated into the unsteady-state material balance Eq. (11) by setting $K = K(C, T_c, t)$.

Another point that deserves attention is that a polluting substance L' is not usually transformed directly to the inert product N' , by an acting sink ; instead, it happens very frequently to be transformed to an intermediate substance M' , which in turn will be transformed by the same or another sink to N' . If M' is also a pollutant, its generation (by the subsystem which acts as a sink for L' but as a source for M') and utilization sources must be incorporated into the unsteady-state mass balance Eq. (11) or a modified form of Eq. (8), otherwise $Y = 1 - n$ will not represent environmental quality any more. For this reason we have to determine dM/dt as well as dL/dt (where L, M, N , stand for the concentration of the substances L', M', N' , respectively) and put their functions in Eq. (11).

If K_3, K_4 are the first - order kinetics rate constants for the transformations $L' \rightarrow M'$ and $M' \rightarrow N'$, respectively, the rate equations, for the simplest case where $Q = 0$ and no other sinks or sources exist within the system, are :

$$dL/dt = -K_3L \quad (14)$$

$$dM/dt = K_3L - K_4M \quad (15)$$

$$dN/dt = K_4M \quad (16), [L=L(0), M = 0, N = 0 \text{ at } t = 0]$$

The integral form of Eq. (14) is

$$L = L(0)\exp(-K_3t)$$

Through substitution, Eq. (15) becomes

$$dM/dt = K_3L(0) \exp(-K_3t) - K_4M$$

This is a first order linear differential equation, which has the solution

$$M = K_3[\exp(-K_3t) - \exp(-K_4t)] L(0) / (K_4 - K_3) \quad (17)$$

The time path of pollutant and inert product concentrations L, M, N is shown in Fig. 2. The concentration of the intermediate pollutant M passes through a maximum, which can be determined by differentiating Eq. (17) with respect to time and setting the differential equal to zero. The result is $\text{max}t = [\ln(K_4/K_3)] / (K_4 - K_3)$ which gives, by substitution in Eq. (17) $\text{max}M = L(0) (K_3/K_4) K_4 / (K_4 - K_3)$.

This analysis must be extended to involve continuous production of L so that a substitution of η or C in Eqs (8) or (11) by L +M may take place.

Last, if the sink is another substance D, with concentration C_j , which reacts with the pollutant to form an inert product, with concentration X' at time t, the rate of pollution elimination becomes :

$$-R_u = K_2 C C_1 \quad (18) \quad \text{or} \quad -R_u = K_2 [C(0) - X'] [Q(0) - X'] \quad (19),$$

where K_2 is the rate constant and $C(0)-X', C_1(0) -X'$ are the concentrations of the two substances at time t, assuming the simplest case of second - order kinetics* and $Q = 0, R_g = 0$.

Since $R_u = d[C(0) - X']dt = -dX'/dt$, Eq. (19) becomes

$$dX'/dt = K_2 [C(0) - X'] [C_x(P) - X'] \quad (20)$$

Eqs (18), (19) and (20) give finally the solution

$$K_2 t = [C(0) - d(0)] - \ln [C C_1(0) / C(0) C_j] \quad \text{or}$$

$$C = [C(0) Q / Q t 0] \exp [K_2 t [C(0) - C_1(0)]]$$

When D is present in large excess its concentration is hardly changed during the course of pollution elimination, i.e. $C_1 \sim C_1(0) = \text{const}$, and Eq. (18) simplifies to the pseudo-first-order kinetic equation $-R_u = K_2' C$, where $K_2' = K_2 C_1(0)$. When D is present in low concentration, its rate of generation determines the rate of pollutant elimination. At any case a substitution of the second order kinetic equation (18) into Eq. (11) is necessary.

* This is the case, where equal parts of the two substances contribute to inert product generation.

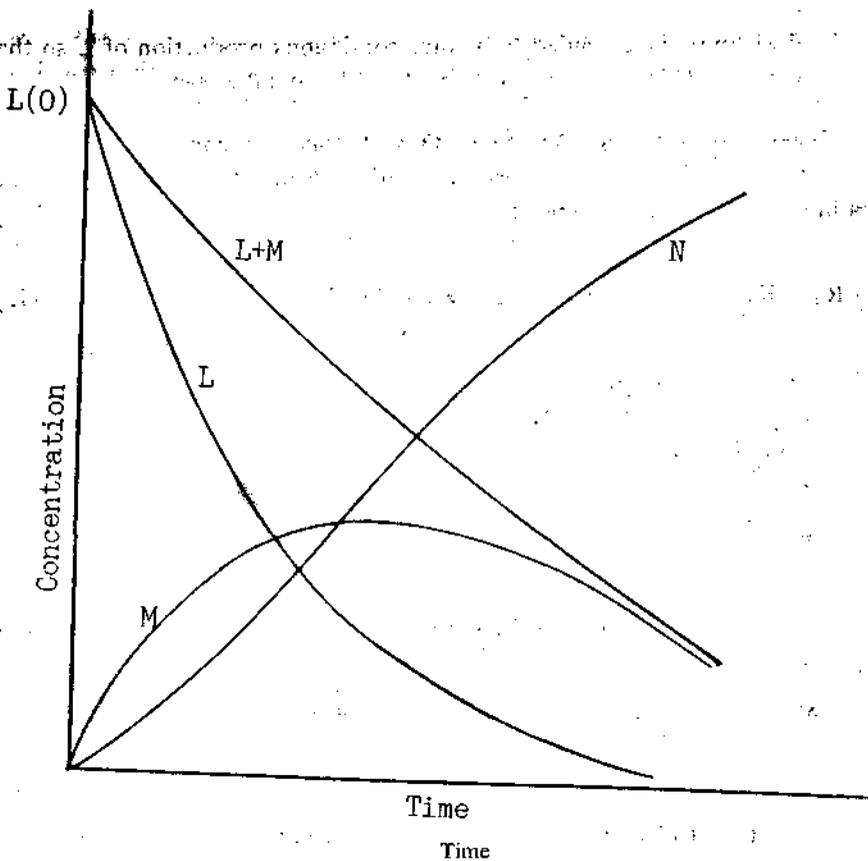


Fig. 2. Time path of pollutant L transformation to inert product N through an intermediate polluting substance M . Curve $L+M$ represents total polluting material extinction.

4. SOME FURTHER REMARKS AND A CASE STUDY IN BRIEF

So far, all criticism and restoration attempt has been focused on the unsteady-state material balance Eq. (8), as the examination of the economic grounds of the model is beyond the scope of this article. Nevertheless, we cannot bypass two main points of technological interest.

- a. The environmental damage depends not only on $\eta = 1 - Y$ but also on the time period T_c or T_n (independent of t), during which the system is exposed to pollutant concentration C or n , continuously. This cumulative effect is widely neglected in the determination of the optimum pollution level, too.

An indicator of special importance for the accumulation of a pollutant is its

half-life, depending on its elimination process which is different in its sink subcategory. Half-life is the time T_m it takes for the concentration of a pollutant to fall to half its initial value. For a first-order kinetic equation, which is the case closest to KGM's environmental model, we have :

$-R_u = -dC/dt = K'C$, [$K' = KV$, $Q = 0$, $R_g = 0$], or $\ln[C/C(0)] = -K't$ (21). Setting in Eq. (21), $C = C(0)/2$ and $t = T_m$, we have $\ln(1/2) = -K'T_m$ or $T_m = (1/K')\ln 2$, i.e. half life T_m of the first-order kinetic equation of pollutant absorption by a sink is independent of the initial concentration of the pollutant $C(0)$. This means that if the concentration at some arbitrary stage of the process is C , then the concentration will have fallen to $C/2$ after a further time interval of $(1/K')\ln 2$. But this is not the case in higher than one or in zero-order pollution absorption processes, where half-life depends on initial concentration as it is shown below.

For a second-order kinetic equation, of the simplest form, $-R_u = -dC/dt = K'C^2$ we have $1/C - 1/C(0) = K't$ or $T_m = 1/[K'C(0)]$ and for the general n -order kinetic equation of the simple form $-R_u = -dC/dt = K'C^n$, we have $[C^{1-n} - C(0)^{1-n}] / (n-1) = K't$, which gives $T_m = (2^{1/n} - 1) / [(n-1) [K'C(0)^{n-1}]$. Some examples of $\max C(0)$ dependence on the kind of certain special subcategories of sinks are given in the following case study of DDT.

- b. There is a portion of polluting residuals wZ (e.g. pesticides or fertilizers) that is coming straightforward from applications with a specified purpose which leads directly to productivity increase. So, there is a dependence of the technological function parameters upon the product (multiplication effect), though a process of diminishing returns seems to be valid ; e.g. the MIT project team estimates a function of the form $Y' = aP^b$, where Y' = average agricultural yield (kg per ha year), P = pesticides or fertilizers input (kg per ha year), $a, b = \text{const.} > 0$, $b < 1$. This process of diminishing returns becomes more intensive in the case of the above mentioned environmental damage because of pollutants accumulation with a high half-life period, as it is the case of DDT which we study below in brief.

DDT was prepared by O. Zeidler in 1874, as part of his Ph.D. thesis, but he had not suspected its insecticidal properties. In 1939, a potato crop in Switzerland was threatened by Colorado potato beetle. P. Muller, a chemist working for J. R. Geigy Co. and searching for an effective insect-killing substance,

synthesised DDT and revealed its amazing power*. In 1943, large scale manufacture began in the USA for use by the Armed Forces to combat tyfus and malaria.

In 1945, USA War Production Board released DDT for civilian use. Worldwide use of DDT began with special emphasis on eradicating malaria. Its effectiveness can be illustrated by data for Ceylon : 1,5 million cases and 80.000 deaths resulted from malaria in 1934-35. As a result ofwidescale use of DDT only 17 cases were reported in 1963 but when application of DDT was discontinued, malaria increased and over 600.000 cases were reported during a 15 months period in 1968-69.

On the other hand, a process of diminishing returns appeared under three different forms : Lower differential agricultural yield at higher concentration of application, appearance of resistant insect species after a relatively short period of application and exponential increase of environmental damage because of accumulation, as half life of DDT is several years, according to the special sink. Maximum observed per cent concentration (mass per mass) in water is 10^{-7} , in plankton and algae 10^{-6} - 10^{-5} , in small fish, shrimp, clams etc herbivores $2 \cdot 10^{-5}$ - 10^{-4} , in larger fish, carnivores $3 \cdot 10^{-4}$ - $6 \cdot 10^{-4}$, flesh eating birds 10^{-3} - 10^{-2} . This means that the cumulative concentration of DDT in a bird is 100.000 times greater than that of a water solution saturated with DDT.

This accumulation leads to a natural delay between application of DDTin agriculture and the appearance of its drastic concentration in a sink. According to the MIT reports, DDT concentration will continue to rise for 11 years after its assumed decline in 1970 and will not fall back to its 1970 level until 1995. AU-though there are some objections, concerning the real environmental impacts of this kind of pollution, the Environmental Protection Agency (EPA) banned the use of DDT in the USA, in the same year the MIT report was issued (1972). Since then, near total or total bans on DDT have been enacted in most industrial countries** but many less developed countries still use it.

* P. Millier received the Nobel prize in medicine for the discovery of the effects of DDT as an insecticide. However, most of the DDT produced commercially is made by Zeidler's method.

** Technological progress in methods of insect control, through a) the application of biological techniques, such as insect predators that prey on other insects, parasites and pathogens that infect specific insect pests, b) the use of induced sexual sterility and genetic defects as a means of controlling insect populations and c) the developement of insect resistant crop varieties, has favoured the DDT ban but inversely has mostly been favoured by this ban.

5. CONCLUSIONS

This article deals mainly with Carl-Goram Maler's (KGM) environmental model which serves as a paradigm for his investigation of the optimal time path that maximizes a welfare function. This model is heavily based on an unsteady-state material balance through a closed system simulated by a homogeneous lake.

Some questions raised by the aid of dimensional analysis led to the reveal of a severe shortcoming in this unsteady-state material balance : an outflow suggested to support a concept of pollutant transportation away from the lake implies a violation of the Material Conservation Law.

A general unsteady-state material balance is formulated through a lake-like environmental system, simulated by a continuous stirred tank reactor, and all possible cases are investigated to find out special solutions well fitted to KGM's model. Indeed, a solution has been found for the case of a closed system and several (depending on the assumptions to be kept) for an open or quasi-closed one. Moreover, an extension of this balance is presented to incorporate some cases closer to reality. We have proved that such an extension is possible without a change in the principles of the model under consideration, at least in four cases :

- a. Changing input in a quasi-closed lake -like environmental system.
- b. Influence of pollutant concentration on the rate constant of self-purification.
- c. Consecutive generation - transformation of pollutants within the system, when a sink is acting at the same time as a source.
- d. Coexistence of another substance in the system acting as a sink of the pollutant.

We think that these extensions may be of some help in simulating real environmental systems by a homogeneous lake-like model or its analogue, a continuous stirred tank reactor.

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