«ΣΠΟΥΔΑΙ», Τόμος 44, Τεύχος 1ο-2ο, Πανεπιστήμιο Πειραιώς / «SPOUDAI», Vol. 44, No 1-2, University of Piraeus

AN ACTUARIAL INVESTIGATION INTO THE MORTALITY OF IMPAIRED INSURED LIVES

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Abstract

This study tackles the main issues relating to a follow-up investigation of the mortality experience among insured lives with various medical impairments identified at acceptance. Statistical methods are described for analysing mortality data and for measuring the mortality experienced. It is intended to analyse the prognostic experience of impaired insured lives and to compare it with a suitable standard in order to describe the mortality pattern and to assess the prognostic significance of risk factors for various impairments. For each impairment, detailed results of the relative mortality by age at entry, duration since entry and time of entry are given. The results can be used to derive a set of ratings for use in the life insurance underwriting of impaired lives. (JEL C49,I12)

1. Introduction

The study is a follow-up observation of the mortality and survival statistics in relation to risk factors among insured lives which have been identified at acceptance. The purpose is to make available tables of comparative mortality and survival data for the convenient reference use of those involved in the risk selection process for Life Insurance. The method measures the mortality and estimates the prognosis of insured lives with various medical impairments using all the information collected up to the time of analysis. The general method for measuring the level of mortality at a time-point in the future relative to that of a set of healthy lives with the same characteristics is given in Section 4. Section 5 describes the selection of a suitable standard experience for our study and the techniques used for constructing a select control table in order to calculate the expected number of deaths. Section 6 describes the various statistical indices of

comparative mortality and gives their definitions. Statistical measures of the reliability of these indices are also discussed. The main mortality indices presented are the Interval Mortality Ratio (IMR), the Annual Exposure Cumulative Mortality Ratio (AE), the Excess Death Rate per 1000 per year (EDR), the Cumulative Mortality Ratio (CMR), the Ratio of Geometric Average Death Rates (RGAD) and the Equivalent Average Annual Excess Death Rate (EAEDR). Section 7 gives a general description of the detailed results and some points concerning their values. The results for each of the 57 impairments refer to the tables of the mortality experience, the statistical indices and the progression of the various mortality ratios by age at entry, duration since entry and calendar periods of entry. Comments are also made on the number of entrants by age at entry and on the distribution of deaths by cause of death. The paper closes with a concluding section stating the main findings and their significance and suggestions for future research on the topic. The computer program written in order to calculate all these statistical indices needed for our results is described in Appendix I. Comments and tables of results for one impairment are given in Appendix II.

2. Sources and Description of Data

The data used in the research work comes from the experience of holders of life insurance policies with various impairments effected in the ordinary branch of the Prudential Insurance Company, England. The impairments are identified at proposal. The experience is restricted to policies issued since the start of the investigation. The period of the investigation covers 35 years and includes all the new entrants in that period. So, we have a number of persons for which a certain initial event has already occured, viz. the development of an impairment prior to proposal, and which are followed with respect to survival and mortality in successive years of observations. The group of individuals is homogeneous because it comes from only one Life-Office. The statistics available for the 35 years-period were 581,726 policies (502,006 males and 79,720 females). The information that is available for each life is:

1. Policy number
2. Description of the medical impairment
   (Medical Bureau - Lists of Impairment codes)
3. Date of entry (to the nearest month)
4. Age next birthday at entry
5. Date of exit (to the nearest month)

6. Age at exit
7. Duration at exit
8. Mode (type) of exit
9. Cause of death (where appropriate)
10. Joint life marker
11. Sex (but most persons are males)

The terms of acceptance (at normal rates, with a decreasing deduction, with an extra premium or with a rating-up in age) were not included into the data, so that the experience contains both lives who were surcharged and who were accepted at standard rates. Both medically and non-medically examined lives were included without discrimination. Unfortunately, the duration since the onset of the impairment, which is vital for many conditions, is not provided by the data. This information and others such as the sum assured, the acceptance terms, the class of policy or even the class of declined lives could provide us with a useful basis for further analysis. The data are incomplete and truncated, namely individuals could be still alive at the closing date of the investigation without their subsequent times of failure being known. There are losses or withdrawals before the planned date of termination. Thus, the number of deaths and the individual times of death are random variables. The total number of policies terminated by death is 24,828 (22,898 males and 1,930 females). The data for males covered by this investigation are divided into 9 broad categories which include 57 major impairments, the most important in terms of volume of data and significance of results. Impairments with less than 100 entrants and entrants aged at issue under 16 and over 79 were excluded from the study (283 and 459 policies respectively). Just under 4,000 policies (related to codes 998-999) were excluded because of unknown impairment. Therefore, the total number of policies entering the investigation (males aged 16-79 at entry) is 497,307 with 22,689 deaths among 2,734,924 years of life exposed to risk.

3. Life Tables

3.1. Basic Definitions and Notation

The life table is a model of survival not expressed in terms of proportions but by expected number of survivors out of l_a starters at age a. A feature of the life-table method is that the time since entry is divided into intervals of convenient length. In the study we have a group of lives with a particular characteristic who are being followed-up over time x. In fact, we have a double-decrement

table considering a large number of lives subject to two independent causes of decrement (death and withdrawal). We will consider the case where the intervals are of length one year. If we let x denote the number of years since entry and if we consider the year of follow-up between x and x+1, then we can use l_x, d_x, w_x, E_x, q_x, p_x, P_x, Q_x as they are defined by the International Actuarial Notation.

It is assumed that times to loss or withdrawal are uniformly distributed. Thus, on the average individuals who are lost to follow-up or who withdraw from the study are lost for half the total interval (the withdrawal time is in the middle of the interval). Then, $E_x = l_x - \frac{1}{2} \cdot w_x$ (1), where $l_x = l_{x+1} + d_x + w_x$ (2). The life table method assumes that the withdrawals have the same mortality experience as those who remain in the study. This ignores the fact that withdrawals can be selective, as those in deteriorating health are unlikely to surrender or lapse their policies.

3.2. The Actuarial Estimator

The estimator $\hat{q}_x = \frac{d_x}{E_x}$ (3) is often called the actuarial estimate of q_x.

We must note that no probability theory is used in its derivation. The only assumption made is that, on the average, the withdrawal time is in the middle of the interval, that is the withdrawals are exposed to risk for half of the interval (Elandt - Johnson and Johnson (1980)). Breslow and Crowley (1974) have investigated the properties of the actuarial estimator under random censorship and have shown that, in general, $\hat{q}_x$ is an inconsistent estimator of q_x and $\hat{q}_x$ is a biased estimator of q_x; the bias is negative. However, conditional on l_x and w_x and for l_x sufficiently large, $\hat{q}_x$ is approximately unbiased. Maximum likelihood estimators $\hat{q}_x$ based on models with certain distributional assumptions and yielding rather complicated expressions are regarded as more scientific as distinguished from the actuarial estimator. In practice, it has been shown (Elandt-Johnson (1977)) that the discrepancies are negligible when the q_x's are small (say, <0.3) and the sample sizes are sufficiently large.

4. General Method for Measuring Relative Mortality Experience

A longitudinal prospective investigation involves the compilation of a mortality experience and the testing of hypotheses relating to the population under study. The investigation can be "retrospective" beginning at some time in the past and using records which have been available for a period of years going

back in time. Consider the case where the study group are persons proposing for life insurance with a certain impairment. Given such a group (which is not a random sample), the question is what will be the level of mortality at time t, in the future, relative to that of a "similar" set of lives without impairments. And "similar" here means controlling for the age of entry, duration since entry, sex of the lives involved and any other characteristics believed to have a significant effect on mortality rates. In order to answer this question, the observed and expected mortality must be examined in the greatest possible detail, subdividing the data by attained age, sex, duration of follow-up, severity of impairment and so on. The common method of measuring relative mortality experience in actuarial studies is by way of comparison of the actual number of deaths and the number expected if a given standard experience were applicable. As Benjamin (1980, 1983) mentions, the basic steps are:

1. Define a sufficiently homogeneous group
2. Observe the deaths in each year of experience
3. Calculate the related exposed to risk to allow for entrants and exits in the normal way
4. Apply a standard mortality table for same ages and same durations of policy to calculate the expected deaths
5. Compare actual and expected deaths

The latter is usually done by computation of the Interval Mortality Ratios (IMR), the progression of which with increasing duration is of critical importance (see Section 6.1.1.).

There are also other statistical indices for making such a comparison, which will be presented later on.

5. Selection of Standard Experience for Calculation Expected Deaths

The most appropriate basis for measuring the extra mortality experienced by the various classes of impaired lives would be the experience of healthy insured lives of comparable duration over the same period of time insured by the same insurance company. Because the experience of lives accepted at standard rates is the only available source for constructing a control table, we use the special table A1967-70 based on the experience of first-class male lives accepted at standard rates of premium by offices transacting ordinary life-insurance in the U.K. It should be mentioned that the experience of standard lives of the office concerned in the present investigation lies very close to the combined experience

of all offices and consequently the A1967-70 table is appropriate as a basis for calculating expected deaths for this insurance company. The mortality rates in this basic table are considerably lower than population mortality rates reflecting the effect of selection in individual life - insurance underwriting and propably the high proportion of persons in middle and upper income groups (who have better medical care than the average) applying for individual life-insurance.

5.1. The Standard Experience Used in Our Investigation

The investigation has been carried out in select form (with a 2 - year select period) and the A1967-70 Select Table has been used throughout as basis for calculating expected mortality for a particular life office's experience. Following the regressions mentioned in C.M.I. Report No 3 (1978), we use the relationships between observed mortality q and that of the standard table A1967-70 given for the various quadrennia that follow the model:

$$q'^{(i)}_{\{x\}} = \alpha^{(i)} \cdot q'^{(A1967-70)}_{\{x\}} + \beta^{(i)}, \qquad \text{duration "0"} \tag{4}$$

$$q'^{(i)}_{\{x\}+1} = \alpha^{(i)} \cdot q'^{(A1967-70)}_{\{x\}+1} + \beta^{(i)}, \qquad \text{duration "1"} \tag{5}$$

$$q'^{(i)}_{x+2} = \alpha^{(i)} \cdot q'^{(A1967-70)}_{x+2} + \beta^{(i)}. \qquad \text{duration "2"} \tag{6}$$

where i= 1, 2, ..., 7 for the periods 1949-52, 1953-58, 1959-62, 1963-66, 1967-70, 1971-74, 1975-78.

The parameters $\alpha^{(i)}$, $\beta^{(i)}$ are applied to both the ultimate and select rates and their values for the above mentioned periods are: 1.191, 1.131, 1.083, 1.060, 1.0, 0.94, 0.87 and + 0.00019, 0.00002, 0.00003, 0.00003, 0, 0, 0 respectively. Using linear interpolation techniques on $\alpha^{(i)}$ and $\beta^{(i)}$ we have produced a set of $\alpha^{(k)}$ and $\beta^{(k)}$, were k refers to an individual year of death. Extrapolation has been used for each year prior to 1949 or after 1978 that is part of the investigation period. With this method we have produced the values of the parameters $\alpha^{(k)}$ and $\beta^{(k)}$ for every individual calendar year (on mid-year, to be precise) of this 35-year follow-up study. After having calculated the values of the parameters for each calendar year (say, k) during which a group of lives with a particular impairment entered the study, we can use the appropriate values of $\alpha^{(k)}$ and $\beta^{(k)}$ to construct the $q'_{\{x\}}$, $q'_{\{x\}+1}$, q'_{x+2} from the regression equations mentioned above, where $q'_{\{x\}}$ is the select standard mortality rate for duration 0 and entry age x obtained at the mid-point of year k. Similarly $q'_{\{x\}+1}$ is the select standard mortality rate for duration 1 and entry age x (attained age x+1) and obtained at the mid-point of year k+1, while

q'_{x+2} is the ultimate standard mortality rate for duration 2 and entry age x (attained age x+2) obtained at the mid-point of year k+2. Thus, a select control table has been constructed from the standard experience of the same duration 0, 1, 2 and over and each year of age entry [{x} = 16, 17, ... 79 for d= 0, 1 and 19, ..., x+2 = 18,102 for d≥2] over the same period of time within each class of impaired lives. Based on this standard table we can calculate the number of expected deaths as accurately as possible by multiplying the standard mortality rate by the exposed to risk for the particular entry age and duration. The method used and the computer program we have written in order to calculate all the statistical indices needed for our results are described in Appendix I.

6. Statistical Indices for Comparison of Experience with Expected

This arises when it is desired to investigate whether the mortality experience of a group is sufficiently well described by a given life table. Regardless of the size of the study group, it does not represent a random sample from the whole population. The group is a selected part of the whole population so that any statistical procedure that assumes independence between the study population and the control population (the basis for the expected experience) is strictly inappropriate. But for practical purposes the given life table may be used as a standard with which we compare the mortality experience of the study population.

There are various important approaches to comparing the experience of the population under consideration with that expected in a standard population group of the same size and composition, by age and sex. The various indices and methods are described in the subsequent subsections.

6.1. Comparison of Actual and Expected Deaths

Let d_t, q_t, E_t be the number of deaths, the mortality rate and the initial exposed-to-risk for the population under consideration for the interval of follow-up between durations t and t+1 (say, measured in years). A "prime" shall be used to denote the corresponding functions for the standars population (d'_t, q'_t). Obviously, $q_t = \frac{d_t}{E_t}$ and $q'_t = \frac{d'_t}{E_t}$. All relative indices described in this section (IMR, AE, CMR, RGAD) are measured on a scale of 100, while the absolute indices (EDR, EAEDR) are measured on a scale of 1000.

6.1.1. The Interval Mortality Ratio-IMR

It is defined as the ratio of the actual to expected deaths in an interval (t, t+1) or as the ratio of the actual to expected interval mortality rates for that interval expressed as a percentage. Thus,

$$IMR_t = 100 \cdot \frac{d_t}{d_t'} = 100 \cdot \frac{q_t}{q_t'} \tag{7}$$

Similary, we can define the Interval Survival Ratio (ISR) as:

$$ISR_t = 100 \cdot \frac{p_t}{p_t'} = 100 \cdot \frac{(1-q_t)}{(1-q_t')} \tag{8}$$

6.1.2. The Annual Exposure Cumulative Mortality Ratio-AE

Over an n-year period, say from duration 0 to n, the comparison of actual and expected deaths is carried out by means of the cumulative mortality ratio calculated by the annual exposure method, which uses the annual exposed to risk as a basis for expected deaths. It is defined as the ratio of the actual to expected aggregate number of deaths summed over all previous intervals, i.e.

$$AE_n = 100 \cdot \frac{\sum_{t=0}^{n-1} d_t}{\sum_{t=0}^{n-1} d_t'} \tag{9}$$

Obviously, this index may be defined in respect of an interval between durations t and t+n rather between 0 and n. The corresponding notation would then need a minor modification.

6.1.3. The Excess Death Rate per 1,000 per Year-EDR

This index of comparative mortality is based on the difference between observed and expected mortality rates rather than the ratio of the rates, or the equivalent ratio of observed to expected deaths. EDR is expressed as extra deaths per thousand exposed to risk per year. In a mortality study, the EDR is defined as the ratio of the difference between actual and expected deaths in an interval to the number of persons exposed to risk and is calculated by the formula:

$$EDR = 1{,}000 \cdot \frac{(d_t - d_t')}{E_t} = 1{,}000 \cdot (q_t - q_t') \tag{10}$$

where q_t is the observed and q_t' the expected annual mortality rate. Since the standard mortality rate q' is dependent on age, the relationship of EDR to mortality ratio is also dependent on age, but whereas the mortality ratio decreases as age advances, the EDR usually increases.

6.2. Ratios Based on Cumulative Mortality

A feature of the indices discussed in Section 6.2 is that they require computation of the q_t and q_t' individually. Thus, full data on dates of death and other exits need to be recorded. But if a study only records whether each individual is alive or dead at the end of a period, then alternative indices referring only to cumulative mortality or survival rates are needed. These are discussed in this section.

6.2.1. The Cumulative Mortality Ratio-CMR

We consider a follow-up study of survival of the type described above (section 6.1) with t used to denote the duration of follow-up so that the time t=0 corresponds to acceptance of the insurance. Using the life table method for measuring survival, we let the cumulative survival rate between durations 0 and n be ${}_nP_0$ and the cumulative mortality rate be ${}_nQ_0$. Then,

$$ {}_nQ_0 = 1 - {}_nP_0 = 1 - (1-q_0) \ldots (1 - q_{n-1}) \tag{11} $$

We suppose that, as before, we have a standard life table for a population group of the same size and composition by age and sex, for which functions are denoted by a'. The Cumulative Mortality Ratio for the period 0 and n is defined as the ratio of the actual to expected cumulative mortality rates up to time n, which denotes the duration of follow-up, expressed as a percentage and is given by:

$$ CMR_n = 100 \cdot \frac{{}_nQ_0}{{}_nQ_0'} = 100 \cdot \frac{1 - {}_nP_0}{1 - {}_nP_0'} = 100 \cdot \frac{1 - (1-q_0) \ldots (1-q_{n-1})}{1 - (1-q_0') \ldots (1-q_{n-1}')} \tag{12} $$

In the case where we have complete follow-up information on a cohort, the CMR index ignores the times of deaths and considers only whether, over a particular period, death has occured or not. It should be indicated that a serious disadvantage for the use of the CMR index in follow-up studies is its dependence on the width of the time interval, namely the value of n in the above eqautions. This hinders comparison between studies where the maximum follow-up period

differs. More significantly, the dependence on n means that, as n increses, the index "naturally" tends towards 100. The reason for this is that as n increases to very long durations both Q and Q′ become close to 1, i.e. the probability of dying within a long period of time is 1 for both the study population and the standard population. Hence, CMR is biased and usually gives a completely misleading picture of the underlying pattern of IMR especially at the long durations because of its in-built tendency to move towards 100 as the duration of follow-up is extended.

6.2.2. The Ratio of Geometric Average Death Rates-RGAD

This is an index with less serious bias that the CMR but is also derived from cumulative mortality rates. RGAD is proposed as a mean of measuring the relative difference between actual and expected mortality rates over a period of several intervals (say, n years). The RGAD for the period between 0 and n is defined as follows. We assume that during this durational period of length n units the survival rate is constant within each sub-period of length 1 unit. Then, the (geometric) average death rates can be calculated from the following equations:

$$(1 - \tilde{q})^n = 1 - {}_nQ_0 \Rightarrow \tilde{p} = \sqrt[n]{{}_nP_0} \tag{13}$$

$$(1 - \tilde{q})^n = 1 - {}_nQ'_0 \Rightarrow \tilde{p}' = \sqrt[n]{{}_nP_0'} \tag{14}$$

Where $\tilde{q}$ and $\tilde{q}'$ are respectively the actual and expected geometric average death rates for the period and n is the total number of years. Then, the RGAD index is defined as the ratio, expressed as a percentage, of the actual to expected geometric average death rates, i.e.

$$RGAD_n = 100 \cdot \frac{\tilde{q}}{\tilde{q}'} = 100 \cdot \frac{(1 - \tilde{p})}{(1 - \tilde{p}')} \tag{15}$$

RGAD summarises the relative mortality experience over the n unit period from duration 0 to n.

The RGAD index for individual or grouped intervals is less dependent on the length of the period used, n, than the CMR. It also follows closely the pattern of the underlying IMR and responds more closely to the IMR at the highest durations (where the data are the most scanty) than the AE index or the CMR.

6.2.3. The Equivalent Average Annual Excess Death Rate-EAEDR

This index may be calculated when the interval of follow-up is different from one year or where several intervals are combined. If it is assumed that, during each of the n years (n is the width of the interval (say, 5 years) between durations t and t+n), the equivalent average annual survival rate, $\tilde{p}$, is constant, then $\tilde{p}$ may be calculated as the geometric mean of the interval survival rates, i.e.

$$\tilde{p}_t = \sqrt[n]{P_i} = \sqrt[n]{P_{t+n} / P_t} \tag{16}$$

if the start of the interval i is at duration t. Similarly, a geometric average expected survival rate $\tilde{p}'$ may be calculated and the difference between $\tilde{p}$ and $\tilde{p}'$ may be used to obtain an equivalent average annual excess death rate given by:

$$EAEDR = 1{,}000 \cdot (\tilde{p}' - \tilde{p}) \tag{17}$$

6.3. Statistical Measures of Reliability of Mortality and Survival Rates

Using statistical analysis it is possible to compare the values of IMR, ISR, EDR, RGAD at different points of follow-up and between different population subgroups and test hypotheses that the values are significantly different. It is of considerable importance to be able to estimate the statistical limits of random error of mortality and survival rates and the comparative indices derived there from. The usual approach for measuring the degree of reliability of the rates and related variables is to employ "standard errors" and "confidence limits".

6.3.1. Standard Errors

Assuming a Poisson distribution for the number of actual deaths, a simple formula for the standard error of the IMR may be derived. If we wish to estimate the level of extra mortality for a particular group or compare the levels of several groups, then the estimate of the standard error is given by:

$$\text{s.e.}(\hat{IMR}) = 100 \cdot \frac{\sqrt{d_t}}{d_t'} \tag{18}$$

However, if a particular group is being compared with the standard or the difference between a mortality ratio and the normal value 100 is tested, then the formula

$$\text{s.e.}(\hat{IMR}) = 100 / \sqrt{d_t'} \tag{19}$$

should be used. This formula is also a good approximation for the standard deviation of the ratio $100 \cdot \frac{d}{d'}$ (CMI Report No 8 (1986)).

6.3.2. Confidence Limits

The statistical reliability of the values of mortality ratios diminishes sharply as the number of deaths decreases. The concept of confidence limits measures the degree of reliability of mortality ratios and related variables. We use the concept of confidence limits as a criterion for the range of variability in indices when the number of observed deaths is relatively small. This concept has also been used to judge the statistical significance of the numbers of observed deaths and of the death rates derived from them. Confidence limits define an interval ranging above and below an observed sample estimate (such as the mortality ratio); both the observed value and its associated confidence limits are subject to sampling variation. Where repeated sample estimates have been made and confidence limits computed for each estimate, then the confidence levels indicate the proportion of the estimates for which the confidence limits will enclose the "true" value in the uderlying population. The confidence limits for confidence levels of 50% and 95% can be calculated by the formulae indicated below.

i) When the number of deaths exceeds 35, a normal distribution may be assumed to provide a satisfactory approximation for the reliability of the mortality ratios. The confidence limits at a confidence level of 50%, say, for the IMR, are given by the formula:

$$CL_{50\%} = IMR \cdot (1 \pm 0.67 \cdot \frac{1}{\sqrt{d_t}}) \tag{20}$$

while at the confidence level of 95% they are given by:

$$CL_{95\%} = IMR \cdot (1 \pm 1.96 \cdot \frac{1}{\sqrt{d_t}}) \tag{21}$$

Similar formulae can be given for the confidence limits of other indices as well.

ii) When the number of deaths is less than 35, the errors in the above formulae become appreciable and a better estimate can be obtained by assuming a Poisson probability distribution rather than the normal approximation used above.

7. Description of Detailed Results

The degree of detail in which the statistics have been analysed has been determined by the quantity of data within the various impairment groups. For some impairments it has been necessary to show the statistics in fairly broad groups. Where the volume of data for a particular impairment classification was reasonably large, comment has been made on the number of policies by age at issue, on the mortality trends by age at issue and duration since entry, on the excess mortality by duration from entry (in interval and cumulative form) for all ages combined, on the mortality experience by 5-year calendar periods of entry, on the distribution of deaths by cause. Another point that should be made concerns the table which shows the mortality ratio by time (5-year calendar periods of entry).

In general, higher values of the mortality ratio are experienced in the last twenty years of the 35-year period than in the first 15 ones. Possible reasons for this feature could be the following:

1. Persons with impairments previously considered unsuitable became acceptable as substandard risks, thus increasing the proportion of highly impaired risks.
2. Persons with impairments previously classified substandard became acceptable as standard risks, thus decreasing the proportion of slightly impaired risks.
3. Improvement of mortality on medically impaired risks was not as great as that on unimpaired standard risks.
4. There may be distortions arising from the particular control experience used based on modifications to the A1967-70 Life Table.

As far as the IMR index is concerned, we must point out its significance in analysing the mortality experience by age at issue and duration since entry simultaneously. By doing this we avoid disadvantages like the following:

1. When comparing mortality for all ages combined, the mortality ratio usually falls with increasing age, so that, for all ages combined, it may be materially affected by the age composition of the lives with a particular impairment.

2. For some impairments, the volume of data is proportionally greater in the early durations and at the younger ages and fewer in the high durations and the older ages. As a result, the aggregate excess mortality for all durations or ages combined might not be excessive giving a misleading picture of excess mortality.

8. Overall Results

The data for males covered by this investigation are divided into 9 broad categories which include 57 major impairments. The nine categories and the impairments included in each of them are the following:

1. **Circulatory Diseases**
   Arteriosclerosis, Cerebrovascular Disorders, Hypertension with weight "standard + 19%", Hypertension with weight "standard + 20% or over", Hypotension, Coronary Arteries, Rheumatic and Congenital diseases
2. **Diseases of Stomach and Intestines**
   Acute Peptic Ulcer, Chronic Peptic Ulcer, Dyspepsia, Gastritis, Cholecystitis, Amoebic Dysentery, Hernia, Varicose Veins, Ulcerative Colitis, Crohn's Disease, Fistula in Ano
3. **Nervous Disorders: Head and Ear Impairments**
   Epilepsy, Head Injuries, Psychoneuroses, Attempted Suicide, Migraine, Attacks of Unconsciousness, Disseminated Sclerosis, Otitis Media
4. **Tuberculosis**
   Non-Pulmonary Tuberculosis, Pulmonary Tuberculosis, Family History of Tuberculosis
5. **Endocrine Group**
   Diabetes Mellitus, Goitre (Thyroid Dysfunction), Glycosuria
6. **Underweight and Overweight**
   Underweight-weight greater than 20% under standard, Overweight-weight 20%-30% over standard, Overweight-weight 30%-40% over standard, Overweight-weight greater than 40% over standard
7. **Respiratory Impairments**
   Hay Fever (simple), Bronchial Asthma, Chronic Bronchitis without Emphysema, Chronic Bronchitis with Emphysema, Emphysema without Bronchitis, Pleurisy, Spontaneous Pneumothorax
8. **Urinary Impairments**
   Urinary Calculus, Cystitis-Pyelitis, Pyouria-Haematuria, Albuminuria-Nephritis, Other Renal Disorders (Hydronephrosis-Nephrectomy)
9. **Tumours and Miscellaneous**
   All Malignant Tumours, Innocent Tumours-Skin and Superficial Tissue, Innocent Tumours - Lips, Mouth and Salivary Glands, Innocent Tumours -Lymphatic System, All Non-Malignant Breast Tumours, Innocent Tumours-Male Genital Organs, Miscellaneous Innoc. Tumours (not Classi-

fied), Enlargement of the Prostate, Anaemia (Blood Conditions), Osteomyelitis

Because of the bulk of the data all the results can not be presented in this paper analytically. As an example, we refer to the impairment of Diabetes Mellitus giving comments and tables of the results produced in Appendix II.

9. Conclusions

The project has considered the problem of measuring the mortality experience of a group of impaired insured lives, relative to a suitable standard. It provides a full analysis of the excess mortality experienced by these lives. The results cover all the major impairment categories in detail and facilitate the making of comparisons across a number of different dimensions. Various mortality indices, and in particular the interval mortality ratios, have been used to assess the prognostic significance of the main effects of individual factors (like the medical status, the age at entry, the policy duration since entry, the calendar year at entry) and their interactions on the level of excess mortality. The approach to the analysis of mortality has examined the variation in excess mortality with all the covariables present simultaneously. It has also tested whether certain factors were contributing to excess mortality to a significant extent. We have investigated the progression of the level of relative mortality with duration since entry, age at entry and other covariables (like the level of blood pressure and weight for hypertensives). Two important factors need to be considered when using the results from these mortality data for underwriting purposes and for the rating of impaired lives. Firstly, the past experience may not be a reliable indication of future experience. Clearly, significant changes have occured in the years since the early experience of the Office concerned. Some of the impairments are no longer serious or have ceased to be a problem as an underwriting risk. As a result, changes in underwriting practice have been made. Secondly, data underlying the standard table have a different intercompany mix than the data in this study. The method of adjustment used to introduce a secular trend in the underlying standard mortality rates can also be an important factor. There may be distortions arising from the particular standard experience used based on regression-type modifications to the A1967-70 Life-Table. The results reported in the study indicate that age at entry and duration since entry (and the interaction between them) are significant factors regarding the trend in mortality ratios for certain impairments and should be taken into account in the rating process. For example, the age factor is dominant in obtain-

ing the IMR values for entrants with hypertension, other circulatory diseases, diabetes, peptic ulcer, while policy duration is an important factor for impairments like malignant tumours and epilepsy, with high IMR values at the early durations, and for urinary impairments, with high IMR values only at the long durations. The results can be applied to current or future mortality evaluations as an indispensable tool in the development of risk selection and rating procedures to be used in life insurance underwriting. The results obtained should help to provide a basis for assessing appropriate surcharges in the future or, in some cases, a justification for accepting the risk at standard rates of premium.

Regarding future research, further analysis on this group of lives should be planned, using each medical code and other covariates, in order to examine in more detail the influence of these covariates on the level of extra mortality. It should then be possible to obtain more precise results for some impairments. Duration since the onset of the disease, which was not provided by the data in this study, is of vital importance in obtaining more meaningful results and it would be particularly helpful if future studies were planned bearing this in mind. Under the heading of supplementary data, inclusion of the terms of acceptance or the sum assured could also provide a more precise basis for understanding and explaining the pattern and trend of the mortality ratios for this experience of impaired insured lives.

Appendix I

The Computer Program Used to Calculated all Statistical Indices

We refer briefly to the computer program we have written in order to produce all the statistical indices neede for each impairment. The FORTRAN program reads from two sources, a tape and a disc. Into the tape we have allocated all the life tables produced for each impairment in the most detailed form, namely, for each calendar entry-year of the 35-year period, by yearly duration since issue (0-34 years) and by twelve age groups at entry (16-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-79). Each life table gives the number of entrants, withdrawals, exposed to risk, observed deaths, observed interval mortality rate, observed interval survival rate, observed cumulative survival rate for each single interval of yearly duration (i= 0, 34) of a certain age group (j= 1, 12) for each calendar year of entry (1= 1, 35). All life tables were produced and written into the tape by a computer program based on the survival analysis procedure of the S.P.S.S.-X package after having read the data from an initial tape. We have written the standard mortality rates

q' onto the disc (in the select form for durations 0 and 1 and ultimate for duration over 2) for each calendar entry-year by each single age at entry (16--00). These q' have been calculated by the method described in Section 5.1. We must mention that we have used the mid-points of the twelve age groups mentioned above for the calculation of the q's, i.e. 18, 22, 27, 32, 37, 42,47, 52, 57, 62, 67, 72. With our FORTRAN program and with the information we have obtained from our sources, we can calculate the number of expected deaths, d', as well as the Q, p', P', Q', IMR, ISR, CMR, CSR, EDR, AE, RGAD and the s.e (IMR), C.I.(IMR), if needed, for each combination (i, j, 1). For presentation reasons, we produce our final results after having condensed the duration since entry into seven groups, the age groups at entry into five and the calendar years of entry into seven 5-year periods.

Appendix II

Diabetes Mellitus

This impairment is one of the most important in our study. Codes 520-528, which specifically relate to age at entry and weight, were discontinued on 31.12.1979 and the new codes 570-575 replaced them since 1.1.1980 for diabetes according to entry age. The total number of entrants is 6,080 with 63% of them aged under 40 at entry and just over 15% aged over 50.

Table 1.1. shows the mortality ratios by age at entry and duration since entry. For all durations combined, the IMR trend decreases dramatically by age from the very heavy excess mortality at younger ages to much lower levels for the over 50 entry ages. The figures are 1242%, 727%, 316%, 166% for the under 30, 30-39, 40-49, over 50 age groups respectively. From these figures, it appears that the first two age groups (entry ages under 40) are uninsurable under the present practical level of insurability that companies tend to follow (over 500%) (Brackenridge (1985)). Moreover, these two age groups show excess mortality higher than this level of insurability at all individual durational periods. For the 16-29 age group, the IMR values are increasing by duration since entry with figures of 530%, 946%, 1184%, 1694%, 2045% at the durations 0-1, 2-4, 5-9, 10-14, 15 and over respectively, while for the 30-39 age group the IMR trend reaches the highest level at duration 2-4 (504% and 954% at 0-1,2-4 respectively) decreasing at the higher durations (850%, 722%, 643% at 5-9, 10-14, over 15 respectively). Looking at the IMR by duration for the age group 40-49, we see that the durations 2-9 experience the heaviest excess mortality (344%, 340%, at 2-4, 5-9 respectively), which decreases afterwards by duration to 315%, 287% at

10-14, over 15 respectively. For the over 50 age group, the highest value of IMR is shown at the duration 0-1 (200%) and the lowest at the durations over 10 (136%). The EDR index increases by duration for each age group, while, for all durations combined, it has values 9.9, 17.1, 16.5, 12.7 for the under 30, 30-39, 40-49, over 50 age groups respectively. **Table** 1.2. shows the mortality indices by duration since entry, for all ages combined. All four ratios decrease slightly during the first 10 years since entry but they increase at the remaining durational periods. The IMR values are 281%, 276%, 271%, 305%, 457%, 585% at the durations 0-1, 2-4, 5-9, 10-14, 15-19, over 20 respectively. We should point out that the RGAD index follows the IMR values more closely at the long durations than the CMR and AE indices. The EDR trend is upwards as duration increases. We can also draw the same conclusion about the insurability of younger ages from **Table 1.3.** The **IMR** for the first five 5-year age groups, namely 16-19, 20-24, 25-29, 30-34, 35-39, is 1106%, 1249%, 1262%, 896%, 644% respectively, for all durations combined. In then decreases sharply by age for the rest age groups. **Table 1.4** gives the distribution of deaths by cause. Almost half of the deaths (49.6%) have been caused by circulatory diseases and in particular by ischemic heart disease (over 40% of deaths). Diabetes is the cause of 10% of the deaths, a percentage that is much higher than that for all deaths in this study (0,35%). Cancer is the third main cause of death with 7.2%, a percentage which is only one third of that for all deaths in the study. We should also mention that 38.3% of the deaths occurred among entrants aged under 40 and only 30% among entrants aged over 50.

TABLE 1.1

By Age at Entry and Duration Since Entry

Age at Entry	Duration Since Entry	E	d	d′	IMR	CMR	AE	EDR
	0-1	2393	7	1.32	530	540	530	2.4
	2-4	1709	11	1.16	946	845	727	5.8
	5-9	1308	12	1.01	1184	977	860	8.4
16-29	10-14	629	13	0.77	1694	1210	1010	19.4
	15-30	474	27	1.32	2045	971	1254	54.2
	ALL	6518	70	5.63	1242	711	1242	9.9
	0-1	1675	6	1.19	504	467	504	2.9
	2-4	1282	18	1.89	954	850	780	12.6
	5-9	1335	27	3.18	850	804	815	17.8
30-39	10-14	879	27	3.74	722	700	780	26.5
	15-30	684	38	5.91	643	404	729	46.9
	ALL	5859	116	15.95	727	330	727	17.1
	0-1	1462	8	2.96	270	321	270	3.4
	2-4	1380	23	6.68	344	335	322	11.8
	5-9	1732	44	12.94	340	321	332	17.9
40-49	10-14	1133	44	13.99	315	290	325	26.5
	15-30	621	34	11.83	287	204	317	35.7
	ALL	6327	153	48.39	316	188	316	16.5
	0-1	1091	14	7.00	200		200	6.4
	2-4	1252	37	22.48	165		173	11.6
	5-9	1559	67	38.18	175		174	18.5
50-79	10-24	566	24	17.61	136		166	11.3
	ALL	4467	142	85.30	166		166	12.7

TABLE 1.2

By Duration Since Entry (all ages combined)

Duration Since Entry	E	d	d´	IMR	s.e.(IMR)	EDR	EAEDR	RGAD
0-1	6620	35	12.48	281	47.4	3.4	7.8	281
2-4	5623	89	32.20	276	29.3	10.1		
5-9	5935	150	55.31	271	22.1	16.0	16.2	270
10-14	3170	105	34.39	305	29.8	22.3	23.6	316
15-19	1358	70	15.32	457	54.6	40.3	41.3	464
20-24	456	32	5.47	585	103.4	58.2	58.4	594
25-29								
ALL	23170	481	155.28	310	14.1	14.1		

Duration Since Entry	IMR	AE	CMR	RGAD
2	281	281	283	284
5	276	278	277	281
10	271	274	260	274
15	305	282	263	293
20	457	300	284	348
25	585	308	296	410
30		310	302	513

TABLE 1.3

By Age at Entry (all durations combined)

Age at entry	E	d	d′	IMR	EDR
16-19	772	6	0.54	(1106)	7.1
20-24	2772	26	2.08	1249	8.6
25-29	2974	38	3.01	1262	11.8
30-34	2792	47	5.25	896	15.0
35-39	3066	69	10.71	644	19.0
40-44	3319	77	19.79	389	17.2
45-49	3008	76	28.60	266	15.7
50-54	2470	66	33.10	199	13.3
55-59	1289	37	26.11	142	8.4
60-64	482	23	15.18	151	16.2
65-69	187	12	8.79	137	17.2
70-79	38	4	2.12	(189)	50.2
ALL	23,170	481	155.28	310	14.1

TABLE 1.4

By Cause of Death

Cause of Death	No. of Deaths	%	No of all Deaths in Study	%
Tuberculosis	3	0.6	61	0.27
Lung Cancer	8	1.6	1844	8.11
Other Cancer	27	5.6	3304	14.53
Leukaemia	6	1.2	283	1.24
Diabetes	48	9.9	80	0.35
Vascular Lesions	24	4.9	1456	6.4
Isch. Heart Dis.	195	40.2	7866	34.6
Other Circul. Dis.	22	4.5	1920	8.44
Influenza	-	-	14	0.06
Pneumonia	3	0.6	243	1.07
Bronchitis	1	0.2	222	0.98
Peptic Ulcer	1	0.2	114	0.50
Nephritis	5	1.0	94	0.41
Motor Accident	6	1.2	395	1.74
Other Accidents	7	1.4	664	2.92
Suicide	4	0.8	450	1.98
Other Causes	125	25.8	3725	16.38
ALL	485	100%	22735	100%

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