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SOME TESTS FOR SPECULATIVE EXCHANGE RATE BUBBLES BASED ON UNIT ROOT TESTS*

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By

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Abstract

In this paper we conduct an indirect test for speculative bubbles in the exchange value of the currencies of Germany and the United Kingdom relative to the U.S, dollar. Our test is general enough to include models that either assume the validity of purchasing power parity (PPP) or arrive at a PPP-type relationship. On the empirical side, the test is based on a unit root test appropriate for general ARMA representations of the underlying time series. We obtain strong evidence against the presence of bubbles over the free floating period 1974-87 (JEL C22, F31).

1. Introduction

In a paper that appeared recently in this Journal, Kirikos (1991) employed stationarity and cointegration tests to check the empirical relevance of exchange rate speculative bubbles for a class of models that either assume the validity of purchasing power parity (PPP) or arrive at a PPP-type relationship. Even though his results were consistent with the *possible* existence of bubble paths in the dollar/deutschemark and the dollar/pound exchange rates over the post-1973 free floating period, the author was reluctant to conclude that the no bubbles hypothesis could be definitely rejected and suggested a number of extensions of the tests on the basis of power considerations and the adequacy of the underlying stochastic representations. A testing procedure that improves on Kirikos' (1991) approach is considered in this study.

While the possibility of rational speculative bubbles in linear asset market models and in general equilibrium asset pricing models cannot be excluded on

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purely theoretical grounds [see Blanchard (1979), Blanchard and Watson (1982), Obstfeld and Rogoff (1983, 1986), Singleton (1987), Diba and Grossman (1988), Kirikos (forthcoming)], the available work on the empirical relevance of bubbles has produced mixed results [see Meese (1986), Evans (1986), Woo (1987), West (1987), Kearney and MacDonald (1990), Kirikos (1991)]. Nevertheless, a rejection of the no bubbles hypothesis does not necessarily imply that bubbles exist. As Flood (1987) and Flood and Hodrick (1990) have pointed out, bubbles are model-specific and therefore their presence is tested jointly with the hypothesis that the underlying model, which generates the fundamental values of the relevant asset price, is correctly specified. In addition, bubble paths can be easily confused with asset price paths generated by expected violent changes in fundamental determinants which are not realized in the sample.

In this paper we exploit the relationship between nominal exchange rates and relative prices, implied by Purchasing Power Parity (PPP) or by a PPP-type relationship [see, e.g., Stockman (1980) and Lucas (1982)], to test the no bubbles hypothesis for the currencies of Germany and the United Kingdom relative to the U.S. dollar. The approach taken here is indirect in the sense that we do not assume the validity of a particular model of exchange rate determination. Instead, we examine the relevance of price level bubbles for the underlying currencies. On the empirical side, our test is based on a Dickey-Fuller unit root test appropriate for general Autoregressive Moving Average (ARMA) representations of the underlying time series.

The paper is organized as follows. In section 2 we describe the test and discuss its properties. The estimation method is presented in section 3, and our results are reported in section 4. Section 5 contains concluding remarks.

2. Testing for Speculative Exchange Rate Bubbles

Diba and Grossman's (1984) characterization of a divergent bubble provides the basis of the bubble test considered in this study. Specifically, Diba and Grossman observe that the presence of an explosive bubble implies that the order of integration (i.e. the number of times a series must be differenced to induce stationarity[1]) of the relevant asset price series exceeds that of the underlying fundamentals. In addition, Hamilton and Whiteman (1985) have shown that tests for bubbles that depend on the validity of a linear rational expectations model impose untestable restrictions on the dynamics of variables that may be taken into account by market participants but not observed by an econometrician. Once such restrictions are relaxed, the only statistically valid test for bubbles involves a test of stationarity for the asset price series and the underlying fundamentals.

The stationarity test for bubbles is based on the following observation. If a bubble does not exist, the order of integration of the relevant price series is equal to the order of integration of the driving exogenous variables. However, if a price series does not exhibit stationary behavior after differencing as many times as necessary to induce stationarity of the underlying fundamentals, we cannot conclude that a bubble is actually present. Indeed, Hamilton and Whiteman (1985) note with regard to the interpretation of a stationarity test for bubbles:

> "... in potentially explosive systems, the 'trans versality', 'forward-looking', and 'no bubble' restrictions are identical. Thus, not even the proposed stationarity test has anything to say about whether bubble terms... enter with non-sero coefficients; rejection of stationarity is subject to varying interpretations." [pp. 371-372]

Alternative interpretations of a rejection of the equality of the orders of integration of a price series and the underlying market fundamentals include a misspecification of the underlying model, "peso problem" effects, a bubble path, and even irrationality of expectations.

The appropriate null hypothesis here is the no bubbles hypothesis because in the presence of a bubble the test is not uniquely defined. Furthermore, the test can only provide conclusive evidence against the existence of divergent or explosive bubble paths. Non-explosive bubbles [see, e.g., Miller and Weller (1990)] generate stationary deviations of asset prices from their fundamental values and therefore a stationarity test would always fail to detect them (i.e. the test has no power in the case of non-explosive dynamic indeterminacies).

Meese (1986) argues that the stationarity test for explosive bubbles may give misleading results because the bubble term may not exhibit nonstationary behavior that is discernible from the sample Autocorrelation Function (ACF) of the asset price series. Specifically, Meese considers Blanchard and Watson's (1982) specification of a stochastic bubble:

$$d_t = (b\pi)^{-1} d_{t-1} + q_t, \text{ with probability } \pi$$

$$d_t = q_t, \text{ with probability } 1-\pi, \tag{1}$$

where $E_t q_t = 0$. Using the specification in equation (1), he generates artificial bubble series with 112 pseudo-observations and estimates the corresponding

ACF and Partial Autocorrelation Function (PACF). Upon inspection of the estimated functions, Meese concludes that the artificial bubble series can be identified as low order stationary autoregressive processes in spite of the fact that they are generated by the nonstationary model given in equation (1).

We repeat Meese's experiment, but we restrict the bubble to be non-negative because a negative asset price bubble is not possible under free disposal [see Diba and Grossman (1988)]. Specifically, we use the same values of b and π as in Meese (1986) and generate 200 pseudo-observations while the bubble lasts. Because the implied autocovariances of the nonstationary artificial series are infinite, we take the logarithms of our pseudo-observations[2] and estimate their ACF and PACF. The results are reported in Table 1 (see appendix) and suggest that the restricted artificial bubbles exhibit nonstationary behavior[3].

In section 4 we employ the stationarity test to investigate the relevance of rational exchange rate bubbles in the currencies of Germany and the United Kingdom relative to the U.S. dollar over the free floating period 1974-87. Our approach is indirect in the sense that we do not test for the presence of exchange rate bubbles in the context of a particular model of exchange rate determination. Instead, we use the stationarity test to examine the relevance of price level bubbles for the nuderlying currencies. Evidence of divergent price level bubbles should be taken as evidence of exchange rate bubbles, given that the exchange rate is a relative price between two currencies.

We choose the stationarity test to assess the relevance of speculative exchange rate bubbles for two reasons. First, the possibility of omitted fundamental variables or a misspecification of a model leaves the equality of the orders of integration of prices and market fundamentals as the only observable implication of the absence of bubbles. Second, we avoid conditioning the test on the assumption that a particular model of exchange rate determination is correct and thus we reduce the number of maintained hypotheses that such a dependence entails. The stationarity test is general enough to include models that assume some form of PPP (e.g. monetary models) as well as general equilibrium models that arrive at a PPP-type relationship. In view of the strong econometric evidence against the adequacy of the available structural exchange rate models, this property of the test appears to be particularly important.

The test for explosive price level bubbles is based on a comparison of the orders of integration of the (logarithm of the) price level [Consumer Price Index (CPI)] and the (logarithm of the) money stock (M1) series for the United States, Germany, and the United Kingdom. If the two series are integrated of the same order, we conclude that the price level does not exhibit bubble behavior, which implies that the relevant exchange rate also does not exhibit bubble behavior.

The use of a reduced-form equation for the price level, in which the only predetermined variable is the money stock series, will not affect the ability of our approach to provide evidence against the presence of bubbles. Indeed, if the price level series is integrated of the same order as the money stock series, a divergent bubble cannot exist because the order of integration of omitted fundamental variables cannot exceed that of the price level series. However, a rejection of the no bubbles hypothesis may be due to nonstationary omitted fundamental variables and therefore it does not necessarily provide evidence of bubbles.

3. Estimation Method and Test Statistics

To determine the order of integration of a time series, we examine plots of the series against time and of its sample ACF. A stationary time series does not wander extensively and its ACF dies out quickly. The same is true for the errors of a trend stationary series. While an examination of the series and its sample ACF provides some information about the stationarity of a series, this study also conducts statistical tests for stationarity of a series.

If a series is nonstationary, then the autoregressive (AR) lag polynomial of its ARMA representation has one or more unit roots. For example, let the series Y_t have the representation:

$$\varphi(L)\,(1-\rho L)Y_t = c + \theta(L)\varepsilon_t,\ t= 1, 2, \ldots; \tag{2}$$

where the error term ε_t is i.i.d. with $\varepsilon_t \sim (0, \sigma^2)$, c is a constant, L is the lag or backshift operator ($L^iY_t= Y_{t-i}$), $\varphi(L)= 1-\varphi_1L-\varphi_2L^2 - \ldots - \varphi_{p-1}L^{p-1},$ $\theta(L) = 1-\theta_1L-\theta_2L^2 - \ldots - \theta_qL^q$, and all roots of $\varphi(L)$ lie outside the unit circle. If $0<\rho<1$, then Y_t is stationary [see, e.g., Wei (1990, chapter 3)]. However, for $\rho= 1$, equation (2) can be written as:[4]

$$\varphi(L)\Delta Y_t = c + \theta(L)\varepsilon_t \tag{3}$$

where Δ [$\Delta = (1-L)$] is the difference operator. Thus, when the autoregressive lag polynomial in equation (3) has a unit root, the first differences of Y_t exhibit stationary behavior so that Y_t is difference stationary. Finally, for $\rho>1$, the series Y_t exhibits explosive behavior. Similar results obtain for negative values of ρ ($-1<\rho<0$, $\rho= -1$, $\rho<-1$).

Equations (2) and (3) show that testing the stationarity of a series is equivalent to testing for a unit root in the AR polynomial of its ARMA representation. In the presence of a unit root, the series is difference stationary and has the representation given in equation (3). Thus, we can determine the order of integration of a series by testing the null hypothesis H_0:p=1 against the alternative hypothesis Hi:p<1. This is equivalent to testing the validity of the nodel given in equation (3) against that of the model given in equation (2).

Under the null hypothesis of nonstationarity, standard asymptotic theory does not apply. Nevertheless, Dickey and Fuller (1979) have derived the limiting distribution of the estimator of ρ and of the relevant t-ratio when the hypothesis H_0:p=1 is true and $\theta(L)$= 1. Dickey and Fuller (1981) have extended these results to likelihood ratio statistics and Said and Dickey (1985) have shown that the same limiting distributions apply for Autoregressive Integrated Moving Average [ARIMA (p, 1, q)] models when the latter are estimated by a one-step Gauss-Newton non-linear estimation with proper initial values for the parameters of $\varphi(L)$ and $\theta(L)$. Percentiles of the distributions are given in Fuller (1976) and applications of the tests are described in Dickey, Bell and Miller (1986).

The presence of unit roots in macroeconomic time series has important implications regarding the persistence of random shocks, forecasting, regression analysis, and real business cycle theory [see Nelson and Plosser (1982), Stock and Watson (1988), Christiano and Eichenbaum (1990)]. Nelson and Plosser (1982) use annual data from 1860 to 1970 for fourteen U.S. macroeconomic time series (including the CPI and the money stock series) and perform Dickey-Fuller (1979) tests for unit roots. In all but one case (the unemployment rate) they cannot reject the null hypothesis of a unit root against the alternative hypothesis of a linear trend and stationary errors and conclude that the series are difference stationary.

Nelson and Plosser's results cannot be taken as evidence for speculative exchange rate bubbles because their data cover a period of fixed exchange rates (except for a short period in the 1920s) which is incompatible with explosive bubble behavior. In addition, they conduct the unit root tests under the assumption that only autoregressive terms are required to obtain satisfactory representations of the series.

Here we use the results of Said and Dickey (1985) to test for a unit root in a general ARIMA (p, 1, q) representation of a series[5]. Although the test is appropriate for testing the null hypothesis of a single differencing of the data against the alternative hypothesis of no differencing, the series under consideration may have already been differenced on the basis of information provided by plots of the series and sample ACFs. Thus, the unit root test can help us decide whether a second differencing of the data is needed or not.

The simplest case obtains when a series is satisfactorily represented by an autoregressive process so that $\theta(L)= 1$ in equation (2). Then, the appropriate test statistic for testing the hypothesis $H_o:\rho=1$ against the alternative $H_1:\rho<1$ is the t-ratio for the coefficient of Y_{t-1} in the regression of ΔY_t on 1, Y_{t-1}, ΔY_{t-1}, , ..., ΔY_{t-p}, where the lag length p is selected so that the residuals are approximately a white noise process. This statistic, denoted by τ_μ, does not have the usual Student-t distribution under the null hypothesis of a unit root. As mentioned above, the limiting distribution of τ_μ is derived in Dickey and Fuller (1979) and its percentiles at various significance levels and sample sizes are given in Fuller (1976)[6].

When 0(L)#1 in equation (2), Said and Dickey (1985) show that the τ_μ statistic has the same limiting distribution as long as ρ is estimated by one iteration of the Gauss-Newton algorithm. Said and Dickey (1985) and Dickey, Bell and Miller (1986) point out that empirical power studies emphasize the importance of good initial estimates of the autoregressive and moving average parameters on which the one-step Gauss-Newton non-linear estimation is based. We can obtain initial estimates of the constant term and the MA parameters by estimating the model given in equation (2). Good initial estimates of the AR parameters are obtained by estimating equation (2) under the constraint p=l. The estimates of $\varphi(L)$, c, and $\theta(L)$ along with p=l are then used as initial parameter values in the one-step Gauss-Newton iteration and the resulting parameter estimates are then used to calculate the value of the statistic $\tau_\mu = (\rho-1)/\sigma$, where ρ and σ are the one-step Gauss-Newton estimates of ρ and its standard deviation, respectively. The procedure is further illustrated by applications on price level and money stock series in the next section.

4. Data and Results

As in Kirikos (1991), our monthly data are the logarithms of the CPI (base year= 1980) and the Ml series of the United States (US), Germany (G), and the United Kingdom (UK) from January 1974 to December 1987. The levels of the series are taken from various issues of the *International Financial Statistics.*

We use the following general notation: P and M denote the logarithms of the CPI and M1, respectively, while the superscripts (US, G, UK) denote the country (e.g. M^G stands for the logarithm of the money stock series of Germany). A differencing factor Δ [Δ= (1-L)] and/or a seasonal differencing factor Δ_{12} [Δ_{12}= (1-L^{12})] precedes any differenced and/or seasonally differenced series (e.g. Δ_{12}–ΔP^{US} denotes the seasonal differences of the first differences of the U.S. price level). Since the logarithms of the series are used throughout the analysis, we drop the term logarithm in the following discussion.

Because the sample autocorrelations of the series P and M, reported in Table 2 (see appendix), are significant at long lags and decline very slowly, this evidence suggests that the series are not stationary. The sample ACFs of the differenced series, also reported in Table 2, have significant spikes for the U.S. data while they exhibit significant spikes and strong seasonal patterns for the German and the British data. For the U.S. series we do not observe a discernible seasonal behavior. Clearly, simple inspection of the sample ACFs cannot help us determine whether a second differencing is necessary to induce stationarity in the data, so we now present the results from Dickey-Fuller tests for unit roots.

In Table 3 we report the values of the Dickey-Fuller τ_μ statistic for the differenced series and their seasonal differences. These values are derived under the assumption that the series have low order AR representations [AR(1), AR(2), and AR(3)][7] and serve as a preliminary test of the hypothesis of a unit root against the alternative hypothesis of stationarity. More specifically, the τ_μ statistic is the t-ratio for the coefficient of Y_{t-1} in the Dickey-Fuller regression:

$$\Delta Y_t = c + \alpha Y_{t-1} + \sum_{i=1}^{P} \beta_i \Delta Y_{t-i} + \varepsilon_t \qquad (4)$$

where c is a constant, α and β_i (i= 1, ... p) are regression parameters, and Y_t is series which has already been differenced. The critical values of the τ_μ statistic for testing the hypothesis H_o:ρ=1 against the alternative H_1:ρ<1 are approximately -3.51, -2.89, and -2.58 at the 1%, 5%, and 10% significance levels, respectively [see Fuller (1976, p. 373)]. Thus, according to the values reported in Table 3, we reject the hypothesis of a second unit root in all but one case.

Although the results of Table 3 suggest that the first differences of the series and their seasonal differences exhibit stationary behavior, they are based on the arbitrary assumption that the series are represented by low order autoregressive processes. In what follows, we relax this assumption and use the sample ACFs and PACFs to identify the ARMA representations of the differenced series and/or their seasonal differences. Then we estimate the corresponding models and use the value of the τ statistic to test the hypothesis of a second unit root according to the procedure discussed in the previous section.

The ACFs and PACFs of the seasonal differences of our differenced data are reported in Table 4. Based on these sample functions and those of table 2, we identified the representations described in Table 5. The estimated ARMA models are reported in Table 6.

To understand Said and Dickey's (1985) method for estimating the τ_μ Dickey-Fuller statistic, we describe the estimation procedure for the indentified model for the series $\Delta_{12}\Delta P^{US}$. Similar procedures apply for all other series.

The identified model for the series $\Delta_{12}\Delta P^{US}$ [see Table 5] is estimated as follows[8]:

$$(1+0.24L)\,(1\text{-}0.71L)\,\Delta_{12}\Delta P^{US} = -0.00012 + (1\text{-}0.67L^{12})\,\varepsilon_t, \tag{5}$$

$$Q(36) = 41.34,\ p = 0.248,$$

where Q is the Box-Ljung statistic and p the associated p-value. Under the null hypothesis that the residuals follow a white noise process, the Q statistic has a χ^2 distribution with 36 degrees o freedom. To estimate the τ_μ statistic, we set the potential unit root equal to one and derive an initial estimate of the autoregressive parameter as $\hat{\varphi} = -0.35$. Then the model is estimated by a one-step Gauss-Newton iteration with the initial values $\rho^{(o)}= 1$, $\varphi^{(o)}= -0.35$, $c^{(o)}= -0.00012$, $\theta^{(o)}= -0.67$:

$$\underset{(0.07)}{(1 + 0.35L)}\ \underset{(0.052)}{(1 - 0.75L)}\ \Delta_{12}\Delta P^{US} = -0.00012 + \underset{(0.02)}{(1 - 0.69L^{12})}\ \varepsilon_t \tag{6}$$

where the numbers in parentheses are standard errors. Thus, $\tau_\mu= (0.75 - 1)\ /\ 0.052 = -4.80$ which is lower than the critical value -3.51 at the 1% significance level. The hypothesis of a unit root for the seasonally adjusted series ΔP^{US} is rejected.

Our results are summarized in Table 6. In all cases the value of the τ_μ statistic implies a strong rejection of the hypothesis of a unit root against the alternative hypothesis of stationarity. We conclude that, except for a seasonal adjustment, the first differences of the price level and money stock series of the United States, Germany, and the United Kingdom need no further differencing to induce stationary behavior. This equality of the orders of integration suggests that divergent price level bubbles and exchange rate bubbles did not occur over the free floating period 1974-87.

5. Conclusions

The unit root tests provide strong evidence that, except for a seasonal differencing, the price level and money stock series of the United States, Germany, and the United Kingdom are integrated of order one. Thus, the price level of these countries did not exhibit bubble behavior over the period 1974-87 which implies that the deutschemark/dollar and the pound/dollar exchange rates also did not exhibit bubble behavior.

While a stationarity tests appears to be appropriate for testing for speculative asset price bubbles, the ability of unit root tests to discriminate between borderline stationary alternatives has been questioned [see, e.g., Christiano and Eichenbaum (1990) and Cochrane (1991)]. Specifically, Cochrane (1991) shows that a unit root test has arbitrarily low power in borderline cases. In addition, by constructing local unit root alternatives to a stationary series, he also shows that even when a test can distinguish between alternative stationary models, it cannot provide any information about the best small sample distribution theory because the likelihood finctions of the local alternatives are arbitrarily close.

Notwithstanding their limitations, the unit root tests can help us avoid some serious problems implied by the presence of a unit root. In particular, our results provide strong evidence against the presence of unit roots in the series of inflation and the rate of monetary growth for three countries and, consequently, against the presence of speculative bubbles in the relevant exchange rates. These findings are important for several reasons. First, while they do not provide support for the hypothesis of market efficiency, they show that inefficiencies are not likely to be the outcome of speculative bubble behavior. Second, in the absence of bubbles, direct government intervention in the foreign exchange market cannot be justified on the basis that bubbles represent macro shocks that must be offset [see Dornbusch (1982)]. Third, bubble-augmented structural exchange rate models are not likely to explain the observed variability of exchange rates over the recent free floating period.

Our tests and results pertain to divergent rational exchange rate bubbles. However, the possibility of stationary bubbles has important implications for market efficiency [see Miller and Weller (1990)]. Similarly, the presence of bubbles in exchange rate target zones affects the choice of an intervention rule which is consistemt with the viability of the system [see Willem Buiter and Paolo Pesenti (1990)]. Also, recent work by Evans (1991) has shown that conventional stationarity tests cannot detect an explosive bubble that collapses periodically, because such indeterminacies do not fall into the vategory of linear autoregressive alternatives considered by these tests[9]. Future research ought to assess the empirical relevance of such indeterminacies.

Appendix

TABLE 1
ACF and PACF of Artificial Bubble Series

			Lag										τ_m		
π	b		1	2	3	4	5	6	7	8	9	10	0	1	2
0.1	0.9	ACF	.985	.969	.955	.939	.925	.910	.895	.880	.865	.850	-1.8	1.7	0.6
		PACF	.985	-.006	-.007	-.007	-.007	-.007	-.007	-.007	-.007	-.007			
0.5	0.9	ACF	.984	.969	.954	.939	.924	.909	.894	.879	.864	.849	-1.7	-2.3	0.5
		PACF	.984	-.007	-.007	-.007	-.007	-.007	-.007	-.007	-.007	-.007			
0.9	0.9	ACF	.976	.961	.947	.932	.917	.902	.887	.873	.858	.844	-2.4	1.1	-0.5
		PACF	.976	.173	.040	-.004	-.016	-.005	-.002	-.002	-.006	-.009			

Notes: The data are the logarithms of the artificial bubble series generated from the form given in equation (1) while the bubble persists and is non-negative. The starting value of the bubble (d_o) is set at .0001 and q_t is drawn from a standard normal distribution. τ_{μ} is the Dickey-Fuller statistic estimated with 0, 1, and 2 lags of the Dickey-Fuller regression [see equation (4)]. The critical values of τ_{μ} for testing the null hypothesis of a unit root at the 1%, 5%, and 10% significance levels are approximately -3.46, -2.88, and -2.57, respectively [see Fuller (1976, p. 373)].

TABLE 2
ACF of Price Level and Money Stock Series

						Lag						
	1	2	3	4	5	6	7	8	9	10	11	12
Series												
P^{US}	.983	.967	.951	.935	.919	.903	.887	.872	.857	.841	.825	.810
ΔP^{US}	.686	.549	.491	.440	.389	.335	.389	.406	.467	.444	.440	.397
M^{US}	.978	.953	.924	.894	.864	.832	.799	.763	.726	.689	.650	.610
ΔM^{US}	.491	.311	.314	.131	.194	.173	.185	.265	.279	.141	.149	.159
P^{G}	.984	.969	.953	.937	.922	.906	.890	.874	.858	.842	.826	.810
ΔP^{G}	.463	.295	.273	.210	.150	-.014	.134	.208	.220	.242	.379	.439
M^{G}	.969	.935	.911	.888	.866	.844	.821	.796	.773	.750	.737	.723
ΔM^{G}	-.039	-.386	-068	-.079	.115	.132	.087	-.068	-0.82	-.395	.009	.835
P^{UK}	.980	.961	.941	.922	.903	.884	.865	.845	.825	.805	.786	.767
ΔP^{UK}	.455	.327	.321	.164	.249	.348	.235	.140	.225	.150	.245	.452
M^{UK}	.978	.956	.934	.912	.891	.869	.847	.825	.804	.783	.763	.743
ΔM^{UK}	-.167	-.107	-.135	.154	.000	-.085	.091	.064	-.030	-.191	.060	.363

TABLE 3
Values of the Dickey - Fuller τ_μ Statistic

	Lags (p)		
	0	1	2
Series			
ΔP^{US}	-5.46	-4.33	-3.38*
ΔM^{US}	-7.48	-5.83	-4.45
ΔP^{G}	-7.89	-5.88	-4.72
ΔM^{G}	-13.27	-13.92	-10.54
ΔP^{UK}	-7.79	-5.63	-4.58
ΔM^{UK}	-15.27	-11.33	-10.10
$\Delta_{12}\Delta P^{US}$	-7.59	-6.13	-5.41
$\Delta_{12}\Delta M^{US}$	-7.26	-6.04	-4.93
$\Delta_{12}\Delta P^{G}$	-10.14	-7.96	-5.78
$\Delta_{12}\Delta M^{G}$	-12.98	-8.57	-4.90
$\Delta_{12}\Delta P^{UK}$	-7.61	-5.60	-4.94
$\Delta_{12}\Delta M^{UK}$	-16.64	-9.45	-8.53

Notes: The lag length p is the lag length of the Dickey-Fuller regression given in equation (4). A * denotes non-rejection of the null hypothesis of a unit root at the 1% significance level.

TABLE 4
ACF and PACF of Seasonally Differenced Series

		Lag											
		1	2	3	4	5	6	7	8	9	10	11	12
Series													
$\Delta_{12}\Delta P^{US}$	ACF	.45	.26	.13	.10	.16	.16	.13	.11	.26	.22	.09	-.24
	PACF	.45	.07	-.01	.04	.11	.05	.01	.03	.23	.01	-.11	-.41
$\Delta_{12}\Delta M^{US}$	ACF	.43	.18	.13	-.08	.03	.06	.09	.28	.20	-0.2	-.09	-.37
	PACF	.43	-.01	.06	-.19	.16	.00	.10	.21	.00	-.19	-.09	-.35
$\Delta_{12}\Delta P^{G}$	ACF	.19	.01	.11	.06	.07	.03	.00	.10	.08	.08	.00	-.47
	PACF	.19	-.02	.12	.02	.06	-.01	-.01	.09	.03	.07	-.05	-.51
$\Delta_{12}\Delta M^{G}$	ACF	-.06	.03	.32	-0.2	.04	.14	-.09	.02	.14	-.14	-.06	-.10
	PACF	-.06	.03	.33	.02	.01	.04	-.09	-.01	.10	-.08	-.11	-.21
$\Delta_{12}\Delta P^{UK}$	ACF	.44	.31	.21	.05	.09	.11	-.01	-.07	-.09	-.22	-.19	-.42
	PACF	.44	.14	.03	-.10	.09	.08	-.12	-.11	-.02	-.15	-.05	-.37
$\Delta_{12}\Delta M^{UK}$	ACF	-.29	.12	-.17	.09	-.05	-.01	.14	-.13	.17	-.08	.18	-.45
	PACF	-.29	.04	-.13	.00	-.01	-.06	.15	-.07	.11	.03	.12	-.37

TABLE 5
Identified ARIMA Models

Series	Model
P^{US}	ARIMA (2, 1, 0) x (0, 1, 1)
M^{US}	ARIMA (1, 1, 0)
P^{G}	ARIMA (1, 1, 0) x (0, 1, 1)
M^{G}	ARIMA (3, 1, 0) x (0, 1, 0)
P^{UK}	ARIMA (1, 1, 0) x (0, 1, 1)
M^{UK}	ARIMA (1, 1, 0) x (0, 1, 1)

Note: The identification is based on the sample ACFs and PACFs reported in Tables 2 and 4.

TABLE 6
Estimated ARMA Models

Series	AR Polynomial	MA Polynomial	Q	p	τ_μ
$\Delta_{12}\Delta P^{US}$	$(1\text{-}.47L\text{-}.17L^2)$	$(1\text{-}.67L^{12})$	41.33	.25	-4.80
	(0.8) (.08)	(.06)			
ΔM^{US}	$(1\text{-}.49L)$		40.20	.28	-7.48
	(.068)				
$\Delta_{12}\Delta P^{G}$	$(1\text{-}.32L)$	$(1\text{-}.73L^{12})$	45.53	.13	-8.95
	(.076)	(.057)			
$\Delta_{12}\Delta M^{G}$	$(1\text{+}.073L\text{-}.052L^2\text{-}.34L^3)$		42.43	.21	-4.90
	(.07) (.07) (.07)				
$\Delta_{12}\Delta P^{UK}$	$(1\text{-}.39L)$	$(1\text{-}.63L^{12})$	41.31	.25	-8.26
	(.075)	(.064)			
$\Delta_{12}\Delta M^{UK}$	$(1\text{+}.27L)$	$(1\text{-}.83L^{12})$	14.43	.99	-16.2
	(.078)	(.054)			

Notes: L is the lag operator. Numbers in parentheses below the estimated coefficients are standard errors. Q is the Box-Ljung statistic and p is the associated p-value [under the hypothesis that the residuals follow a white noise process, Q has a χ^2 distribution (with 36 degrees of freedom here)]. τ_μ is the Dickey-Fuller statistic estimated by Said and Dickey's (1985) method. The critical values of the Dickey-Fuller statistic for testing the null hypothesis of a unit root are approximately -3.51, -2.89, and -2.58 at the 1%, 5% and 10% significance levels, respectively [see Fuller (1976, p. 373)]. All models include a constant not reported here.

Footnotes

1. We use the term stationarity in a second order weak sense. A second order weakly stationary process or covariance stationary process has a constant mean and variance with covariances and the correlations being functions only of the time difference. For alternative definitions of a stationary stochastic process see Wei (1990, chapter 2).

2. Also, by taking the logarithms of the artificial bubble series, we account for the Siegel paradox [see Richard Baillie and Patrick McMahon (1989, pp. 166-167)] in the case of exchange rate bubbles.

3. The following program for the econometrics software RATS (version 3.10) generates artificial bubble series of the form given in equation (1) when the bubble does not crash:

```
all 0 200
declare vec q(200)
matrix q= ran (1.0)
clear d
   if q(1)>-(1/ (b*π) )* .0001
      eval d(1) = (1/ (b*π))* .0001+q(1)
   else
      eval d(1) = (1/ (b*π) )* .0001
   do i= 2,200
      while q(1)> - (1/b*π) )* d(i-1)
      {
      eval d(i) = (1/ (b*π) )* d(i-1) + q(i)
      break
      }
      end while
   end do i
end
```

The values of b and π are given in Table 1.

4. Letting μ denote the mean of Y_t, we have $c= \varphi(1)(1-\rho)\mu$ for equation (2). Thus, for $\rho= 1$ we take $c= 0$ in equation (3). If $c\neq 0$ in equation (3), then Y_t follows a linear trend with nonstationary errors and $c= \beta\varphi(1)$ where β is the slope of the trend. If Y_t follows a linear trend with stationary errors (i.e. $|\rho|<1$), then its ARMA representation is $\varphi(L)(1-\rho L)(Y_t-\alpha-\beta t) = \theta(L)\varepsilon_t$ where α and β are the trend parameters.

5. Said (1991) has further extended the unit root test to ARIMA (p, 1, q) models with a linear trend.

6. Dickey, Bell and Miller (1986) recommend the use of the τ_μ statistic over other statistics on the basis of its stability in empirical power studies. See also Said and Dickey (1985).

7. If a series has an AR(p) representation, then the lag length of the Dickey-Fuller regression [equation (4)] is p-1.

8. Equation (5) is estimated by the Gauss-Newton algorithm.

9. Based on a decomposition of market noise, Kirikos (1993) has suggested a weak test for bubbles that overcomes the problems discussed in Evans (1991).

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