ON THE VALUE OF THE OPTIMAL TAX RATE

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Abstract

The value of studies for reducing uncertainty about the revenue maximizing tax rate is considered from the viewpoint of a decision-maker who can set the rate. A simple approximation of the maximum value of such studies for a decision - maker who thinks the existing rate is optimal is given by $(1/4)VAR(\Theta)T_0$ where $VAR(\Theta)$ is the variance of the decision-makers prior distribution on the elasticity of income with respect to the tax rate and T_0 is current tax revenue. (JEL C44, H21)

1. Introduction

The theoretical progress and empirical controversy that continue to characterize perceptions of relations among tax rates, incomes and tax revenues are illustrated by J. Gwartney and R. Stroup (1983, 1984) and J. A. Wilde (1984). The theoretical implications of tax rate changes on labor supply, for example, are better developed, but the quantitative implications remain relatively unconstrained. Since it is perceptions of numerical effects that influence tax policy, and mistakes in tax policy can be costly, better information to constrain numerical perceptions might be worth its costs.

The problem considered is the maximization of income tax revenue through choice of income tax rate. This hypothetical problem is of some practical interest because of its relation to familiar current questions such as what are the effects of changes in compensation on incentives and incomes, what is average tax rate that will most effectively reduce a deficit and what is the appropriate graph for the relation between tax rates and taxes identified with the Laffer curve.

J. E. Stiglitz (1982) considered the optimal tax problem as one of self-

selection. That is, a government has to set a tax rate to provide incentives to individuals to reveal their characteristics. It has been shown that a randomized income tax rate may serve as an effective screening device.

In a recent study, L. B. Lindsey (1987) provided direct policy implications regarding the tax rate which would theoretically maximize total tax revenue. In particular he showed that tax rates of 35% to 40% may maximize income tax revenues.

This paper considers the possibility of value for randomized assignment of taxation policies by approximating the expected value of precise information for a simple taxation problem. Therefore, from the viewpoint of persons setting tax rates, it is of interest the value that a study about the optimal tax rate might have.

Speculations regarding the practical influence of tax rate studies may be facilitated by expressing them in the quantitative form suggested by conventional decision theory. The next section provides an illustration.

2. The Value of Tax-Rate Information to Decision-Makers

Suppose that Y will be the total income established for some relevant period by the exogenous proportional tax rate r so that tax revenue for that period will be given by

$$T = rY \tag{1}$$

Suppose also that if the existing tax rate r_o is left unchanged it is known that this will result in a total income of Y_o and a total tax revenue of $T_o = r_o Y_o$.

To focus the problem, it is to be additionally assumed that the utility function of the decision-maker is not only linear with tax revenue but measured in dollars, and that the decision-maker is faced with only two options. He can set the tax rate based on current knowledge and uncertainty; alternatively he can set the tax rate after a costless timeless perfect experiment in which the revenue maximizing tax rate will be learned soon exactly. Of interest is the expected dollar value of such idealized information.

The value of the experiment depends upon the decision-maker's current uncertainty regarding the way Y depends upon r. One point of this relation is provided by the values r_o and Y_o, and it is presumed that for the decision-maker

the elasticity of Y with respect to r is at least negative at this point. In particular, letting $-\theta$ represent the elasticity at r_o so that $\theta > 0$, the decision maker's knowledge of elasticity without an experiment is summarized by a prior distribution on $\log\theta$ which is normal with expected value μ and variance σ^2. If there is an experiment the posteriori distribution will become concentrated on a single value of θ.

Identifying the derivative of Y with respect to r as $-\beta$ at r_o, the relation between Y and r at r_o is approximated by

$$Y = Y_o + \beta (r_o - r) \tag{2}$$

with $\beta > 0$, and this approximation is considered accurate with negligible error at least within the interval contained by r_o and the unknown value of r which maximizes rY. Under this approximation, total tax revenue is represented by

$$T = r [Y_o + \beta(r_o - r)] \tag{3}$$

or, since $\beta = \theta(Y_o/r_o)$, as

$$T = rY_o [1 + \theta (1 - r/r_o)]. \tag{4}$$

If there is an experiment, β and thus θ will be accurately known, and the decision maker maximizing tax revenue will set r to $\delta_e r_o$ with $\delta_e > 0$ inderxing the change from r_o. Thus with an experiment the revenue will be

$$T (\delta_e) = \delta_e r_o Y_o [1 + \theta (1 - \delta_e)] \tag{5}$$

and δ_e will satisfy

$$(1 + \theta - 2\delta_e\theta) = 0 \tag{6}$$

establishing

$$\delta_e = (1/2) (1/\theta + 1) \tag{7}$$

with $\delta_e > 1/2$.

If there is no experiment, tax rate r will depend on the decision maker and will be set so that $r = \delta_d r_o$. In this case the tax revenue is represented by

$$T(\delta_d) = \delta_d r_o Y_o [1 + \theta (1 - \delta_d)] \tag{8}$$

with δ_d chosen to maximize the decision maker's expected value of this expression. Thus δ_d must satisfy

$$1 + E(\theta) - 2\delta_d E(\theta) = 0 \tag{9}$$

so that

$$\delta_d = (1/2)(1/E(\theta) + 1) \tag{10}$$

For the decision-maker the value of the experiment is the expected difference between $T(\delta_e)$ and $T(\delta_d)$. Taking expected values after substituting (7) into (5) and (10) into (8) shows (details in the Appendix)

$$VAL_d(\delta_e, \delta_d) = E(T(\delta_e)) - E(T(\delta_d))$$

$$= (1/4) T_o [E(1/\theta) - 1/E(\theta)]$$

$$= \frac{T_o VAR(\Theta)}{4(E(\theta))^3} \tag{11}$$

The value of the experiment is thus directly proportional to the variance and inversely proportional to the expected value of the decision-maker's prior distribution on θ.

As an example, if the decision-maker sees the existing tax rate r_o as optimal, then $\delta_d = 1$ and (10) shows $E(\theta) = 1$. In this case the value of the perfect experiment reduces to

$$VAL_d(\delta_e, 1) = (1/4) T_o VAR(\theta). \tag{12}$$

Ιφ ιν αδδιτιον, τηε δεψισιον-μακερ τηινκσ τηε τρθε οπτιμιζιγν δ_e could be greater than 1.20 with probability 0.10, routine calculations show the implied value of VAR(θ) is 0.06 and thus that the value given by (12) is $0.015T_o$.

As another example, suppose that $\delta_d = 1.20$ so that a 20% increase from r_o is seen as optimal then $E(\theta) = 0.714$. If uncertainty is expressed by a subjective probability of 0.10 that the true δ_e might even be above 1.40, then VAR(θ) = 0.175 and the value given by (11) is $0.012T_o$.

3. Conclusions

Numerical values implied by less simple assumptions may modify the values for the large tax rate differences used for the illustrations. But even simple

quantitative considerations appear to help explain why decision-makers some-times disregard conventional economic research.

Since the assumptions are such as to suggest an upper bound for the value of experimentation, the values for the decision-makers implied by the model may be too small to be influential. If the decision-maker is quite certain of his or her views, this is of course to be expected.

In particular, numerical examples suggest that the value of even a perfect experiment for a decision-maker seeking to maximize tax revenue and who is far from certain about the maximizing tax rate may never - the - less be small. A maximum of one or two percent of tax revenue for a perfect experiment implies that the smaller value to be expected for imperfect experimentation may not seem sizable enough even to offset the cost of reading about the experiment from the point of view of a rational rate - setting decision-maker.

Examples of governments randomly assigning tax rates are rare. Although maximizing tax revenue is an oversimplified objective, raising tax revenue is usually an important objective, and yet experimentation to better estimate the most basic incentive effects of tax incentives is not observed. This lack of exper-imentation is consistent with, and this may be partially explained by, the rela-tively unimpressive values for experimentation implied from the point of view of the decision-makers who set taxes.

Thus, under simple assumptions, the rarity of experimentation for better taxation policies is not explained by economic argument.

Appendix

Details of Computations

From (4), period 2 tax revenue given $\delta_e = 1/2 \, (1/\theta+1)$

From (5) is
$$T(\delta_e) = T_o(1/2) \, (1/\theta+1) \, [1+\theta \, (1 - (1/2) \, (1/\theta+1)) \,]$$
$$= T_o(1/2) \, (1/\theta+1) \, [\, (1/2) + \theta/2 \,]$$
$$= T_o \, (1/4) \, [\, (1/\theta) + 2 + \theta \,]$$

taking the expectation of this value:

(A1) $E_d T(\delta_e) = T_o(1/4) \, [E_d \, (1/\theta) + 2 + E_d \theta]$

from (6), period 2 tax reveuce given that $\delta_d = (1/2)(1/E_d\theta + 1)$ is

$$T(\delta_d) = T_o(1/2)(1/E_d\theta + 1)[1 + \theta(1-(1/2)(1/E_d\theta + 1))]$$

and the expectation of this value is

$$E_dT(\delta_d) = T_o(1/2)(1/E_d\theta + 1)[1 + E_d\theta(1 - (1/2)(1/E_d\theta + 1))]$$

$$= T_o(1/2)(1/E_d\theta + 1)[(1/2) + E_d\theta/2]$$

so

(A2) $E_dT(\delta_d) = T_o(1/4)[(1/E_d\theta) + 2 + E_d\theta]$.

Substracting (A2) from (A1) this shows

(A3) $E_dT(\delta_e) - E_dT(\delta_d) = (T_o/4)[E_d(1/\theta) - (1/E_d\theta)]$ and provides (8).

The result for (9) reflects the assumption that the distribution of θ is log-normal. In particular, since $\log\theta$ is normal with expected value μ and variance σ^2, $\log(1/\theta) = -1\log\theta$ is normal with expected value $-\mu$ and variance σ^2. Then properties of the lognormal distribution, as given for example by G. S. Maddala (1977), establish

$$E_d(\theta) = \exp(\mu + \sigma^2/2)$$

and

$$VAR_d(\theta) = [\exp(2\mu + \sigma^2)][\exp(\sigma^2) - 1].$$

Thus also,

(A4) $\qquad 1/E_d(\theta) = \exp(-\mu - \sigma^2/2)$,

$$E_d(1/\theta) = \exp(-\mu + \sigma^2/2)$$

(A5) $\qquad\qquad = \exp(\sigma^2)(1/E_d\theta)$,

and

(A6) $\qquad\qquad VAR_d(\theta) = (E_d\theta)^2(\exp(\sigma^2) - 1)$.

Expression (9) is then provided by substituting from (A4), (A5), and (A6) to get

$$E_d(1/\theta) - 1/E_d\theta = (\exp(\sigma^2) - 1)/E_d\theta$$

(A7)
$$= VAR_d\theta/(E_d\theta)^3.$$

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