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# A BARGAINING GAME IN A PARTIALLY UNIONIZED FIRM WITH MONOPOLISTIC POWER

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**Abstract**

The present paper develops an equilibrium model of a partially unionized firm that is a monopolist in product market. The firm - union bargaining follows the efficient bargaining process. They bargain simultaneously over the employment level and the unionized wage taking the non-unionized labor employed by the firm as given, while the agent takes the bargaining process as given and chooses the quantity of labor which maximizes his profits. We use the model to examine the comparative statics effects of the unions bargaining power, the "competitive" wage paid to the non-unionized workers and the union's orientation on the bargained wage and the employment level. Applying a Nash cooperative bargaing process we found that the effect of a change in the competitive wage and the increase in the wage elasticity of the union will be posistive both on equilibrium wage and employment. The effect, however, of a change in the bargaining power of the firm, will be positive for the wage rate but for the employment depends on whether the union is wage or employment-oriented. (JEL C71, J21)

## 1. Introduction

The experience of high unemployment in most market economies since the first oil crisis has prompted a large amount of research into problems of labor markets and the interest in bargaining models began to gain momentum.

Contract negotiations between a union and a firm have been analysed in a bargaining model of a two-person cooperative game. But as Friedman (1989, p. 159) notes "... this most strictly correct if the union is the only source of labor for

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the firm...". In conjunction with this, traditional models focus on the negotiation over a single wage and do not consider the possibility of wage dispersion.

The homogeneity assumption, the common pressumption in bargaining models can be removed earlier by assuming that a monopoly union sets the wage levels for two groups of workers or that the labor union is not a monopolist in the labor market. In the later case the firm can hire "homogeneous" workers, at a possibly lower wage rate, through an agent called subcontractor. Thus the situation in the labor market is not longer one of bilateral monopoly. The bargaining process can be modelled as a two-stage game between the government, the firm, the union and the agent.

The modelling of a bargaining process as a two-stage game has been used by international trade theorists to analyse the link between international trade nad unions. In the Brander and Spencer (1988) model, the union and the firm bargain over the unionized wage first and then the firm sets its outpout unilaterally as part of its product game with the foreign firm. In the Mezzetti and Dinopoulos (1990) model, the government announces a specific tariff imposed on the output of the foreign firm first and then each firm chooses its output in a Nash-Cournot fashion, taking the action of the government as given.

The present model develops an equilibrium model of a partially unionized firm that is a monopolist in product market. The firm-union bargaining follows the efficient bargaining process analysed by Hall and Lilien (1979) and MacDonald and Solow (1981): they bargain simultaneously over the employment level and the unionized wage taking the non-unionized labor employed by the firm as given, while the agent takes the bargaining process as given and chooses the quantity of labor which maximizes his profits.

We use the model to examine the comparative statics effects of the unions bargaining power, the "competitive" wage paid to the non-unionized workers and the union's orientation on the bargained wage and the employment level.

## 2. A Model of a Partially Unionized Firm

In this section we develop a partial equilibium model of a firm that is monopolist in the market for its output. There are two homogeneous worker categories competing to each other: the unionized and the non unionized. The Nash bargaining solution is determined in a two-stage game. In the first stage the government imposes a specific profit-tax which is taken as given by the firm, the union and the "agent". The output and the negotiated wage are determined simultaneously in the second stage. The union and the firm bargain over the employment level and the wage taking the non-unionized labor employed by the firm as given. The agent chooses the non-unionized labor taking the bargaining process as given. The rest of the section is organized as follows: i. we provide the formal structure of the model and ii. we describe the Nash bargaining game.

### 2.1. Nash Bargaining Game

We assume that labor is the only factor of production and one unit of labor is required to produce one unit of output, Q= L. The firm operates in a monopolistic market and the inverse demand function for its product is assumed to be linear and negatively sloped, P= D(Q), [primes denote first and second order derivatives]. The profit function of this monopolistic firm is given by (1).

$$\Pi = L\,D(L) - w\lambda L - \overline{w}\,(1-\lambda)\,L \tag{1}$$

where, $0 \leq \lambda \leq 1$, the proportion of the labor force provided by the union.

The firm uses two categories of homogeneous labor, the unionized and the non-unionized provided by a subcontractor called "agent". The firm pays a wage rate w to the unionized workers, and this is the result of the bargaining process between firm and union and a "competitive" wage $\overline{w}$ to the workers who are provided by the agent. If the firm is profit maximizer, then it is indifferent among (w, L) combinations that leave $\pi$ constant. Thus, isoprofit contours in the (w, L) space have a slope

$$\frac{dw}{dL} = \frac{1}{\lambda}\,\frac{LD'(L) + D(L) - w\lambda L - \overline{w}\,(1-\lambda)}{L}$$

These isoprofit contours have a relative maximum at

$$w = \frac{1}{\lambda}\,[LD'(L) + D(L)] - \left(\frac{1-\lambda}{\lambda}\right)\overline{w} \tag{2}$$

which means that the isoprofit contours intersect the marginal revenue product curve at linear combinations of bargaining wage w and the competitive wage w. The lower the isoprofit contour is, the better for the firm is, $\Pi_2 > \Pi_1$.

We assume that labor unions maximize a utility function which depends on employment and real wage, described by a Stone-Geary type function

$$u(L, w) = (w - \overline{w})^{\alpha} (\lambda L)^{\beta} \tag{3}$$

where $\alpha$, $\beta \leq 0$, correspond to the excess wage and union employment elasticities of the utility function. Depending on the relative size of $\alpha$ and $\beta$ we may say that the union is wage oriented if $\alpha > \beta$ and employment oriented if $\alpha > \beta$ [McDonald and Solow; 1981, Oswald; 1982, 1985].

Union's indifference curves, since they are derived from a standard sort of concave utility function, have the normal downward-sloping convex shape in the wage-employment space. Their slope is taken if in (3) set $U = \overline{u}$ and taking the total differential we get

$$\frac{dw}{dL} = -\frac{\beta}{\alpha} (w - \overline{w}) L^{-1} \tag{4}$$

Furthermore, they are asymptotic to the horizontal at $w = \overline{w}$, since

$$\lim_{L \to \infty} \frac{dw}{dL} = -\frac{\beta}{\alpha} \lim_{L \to \infty} (w - \overline{w}) \lim_{L \to \infty} \frac{1}{L} = 0 \tag{5}$$

The higher the indifference curve is, the better for the union is, $U_2 > U_1$.

In diagram 1, we present the isoprofit contours and the unions indifference curves with the properties described previously.

The union and the firm bargain over the wage rate w and the employment level L, through a cooperative Nash bargaining process [Nash; 1950]. If we assume that firm must hire at least one unionized worker in order to produce its output, then the threat point is at zero employment where both union utility and firm profits are zero.

If this is the case, then the generalized Nash product is given by (6). the Nash bargaining product has been defined by Nash (1950) as the product of parties' gains over and above the non-contract outcome or what we call threat point outcome. The bargaining Power of the parties were equal in Nash's paper. In recent literature, however, and in our paper too, we incorporate different bargaining power for the parties defining that way the generalized Nash product [Mezzetti and Dinopoulos; 1990].

$$F(L, w) = \pi^{\rho}\, u^{(1-\rho)} = [\, LD(L) - w\lambda L - \overline{w}\,(1-\lambda)\, L]^{\,\rho}\, [\, (w - \overline{w})^{\alpha}\, (\lambda L)^{\beta}\, ]^{1-\rho} \qquad (6)$$

where $0 \leq \rho \leq 1$, a parameter which denotes the relative bargaining power of the firm. Maximizing F (L, w) with respect to w and L we get the first order conditions

$$F_L = F(L, w)\left[\frac{\beta(1-\rho)}{L} - \frac{\rho\,[w\lambda + \overline{w}\,(1-\lambda) - D(L) - LD'(L)\,]}{[\, LD(L) - w\lambda L - \overline{w}\,(1-\lambda)\, L\,]}\right] = 0 \qquad (7)$$

$$F_w = F(L, w)\left[\frac{\alpha(1-\rho)}{w - \overline{w}} - \frac{\rho\lambda L}{[LD\,(L) - w\lambda L - \overline{w}\,(1-\lambda)\, L\,]}\right] = 0 \qquad (8)$$

Subscripts denote the corresponding partial derivatives.

From (7) and (8) we obtain the contract curve (CC) which is the locus of the tangencies between isoprofit contours and indifference curves in the wage-employment space.

$$\frac{\beta}{\alpha}(w - \overline{w}) = \frac{1}{\lambda}[\, w\lambda + \overline{w}\,(1-\lambda) - D(L) - LD'(L)\,] \qquad (9)$$

The contract curve is defined as the locus of the tangencies between the isoprofit contours and the indifference curves with a slope

$$\frac{dw}{dL} = -\frac{2\alpha}{\lambda\,(\beta-\alpha)} D'(L) \qquad (10)$$

If $\alpha > \beta$, the union is wage oriented, the slope of the contract curve is positive while if $\alpha < \beta$, the union is employment oriented, the slope is negative, and if $\alpha = \beta$, then the contract curve is vertical.

Finally, equation (7) defines another locus in the wage-employment space, the Nash Bargaining Locus, NBL [Mezzetti and Dinopoulos; 1990]. The Nash bargaining locus is the efficient frontier of the bargaining set. The bargaining set is related to the contract curve in the way the production possibility set is related to contract curve in production, in an Edgeworth box [McDonald and Solow; 1981]. Axiomatic bargaining theories require that Nash bargaining set be convex so the Nash bargaining locus to be a concave function in the (w, L) plane. The selection of the U and $\Pi$ functions guarantee the well behavior of the NBL.

Thus, the Nash bargaining locus is given by

$$w = -\theta \overline{w} + \frac{1}{\lambda} D(L) + \frac{\rho}{\lambda [\rho + \beta (1-\rho)]} LD'(L) + D(L) \qquad (11)$$

where $\theta = \dfrac{(1-\lambda)[\beta(1-\rho) - \rho]}{\lambda[\beta(1-\rho) + \rho]}$

The slope of the NBL is negative and it could be characterized as a weighted average of the slope of the demand curve and the slope of the marginal revenue curve. Figure 1, shows the case of a union which is wage oriented. Point A, the intersection between contract curve and Nash bargaining locus determines the equilibrium levels of wage and employment. This equilibrium point is located in the area defined by the inverse demand curve, the marginal revenue level and the competitive wage level $\overline{w}$. Thus if the outcome of the bargaining is w* and L*, then the firm sells OL* and charges a price OP*, collecting profits OP*BL*. The union provides OL* employment and it takes a wage rate OW* thus receiving a reward OW*AL*.

## 3. Comparative Statics

This section examines the comparative statics effects of the union relative bargaining power, the wage rate paid to the non-union workers and the wage orientation of the union. We apply the comparative statics at the system of the equations (12) and (13) where 12 stands for equation 9 and 13 for equation 2:

$$\frac{\beta}{\alpha} (w - \overline{w}) = \frac{1}{\lambda} [\, w\lambda + \overline{w} (1-\lambda) - D(L) - L\, D'(L) \,] \qquad (12)$$

$$w = \frac{1}{\lambda} [LD'(L) + D(L)] - \left(\frac{1-\lambda}{\lambda}\right) \overline{w} \qquad (13)$$

Taking the total differential of (12) and (13) we get the Jacobian of the system

$$|J| = \begin{vmatrix} \dfrac{\beta - \alpha}{\alpha} & \dfrac{1}{\lambda} 2D' \\ 1 & \dfrac{1}{\lambda} D' + \dfrac{\rho}{\lambda[\rho + \beta(1-\rho)]} D' \end{vmatrix} \qquad (14)$$

The sign of the Jacobian is positive if we assume that the demand function is linear, an assumption which is not strong in general.

### 3.1. The Effect of Change in w

The effect of a small change in w will be positive for the wage rate that is asked by the union and negative for the employment level. This effect is independent to the orientation of the union towards excess wage or employment. Thus, in the case where the agent who provides external labor to the firm will ask a higher wage for his workers then the union will have more room to ask for an increased wage rate for the unionized workers. On the other hand, the firm seeing that its labor cost increases both for union workers but also for agent's workers will decide to reduce employment at least in the short rum. (see equations Al and A2 in appendix).

### 3.2. The Effect of a Change in the Wage Elasticity

Suppose that the elasticity of the utility function with respect to wage rate increases. Then the wage rate will increase but also the labor employment will increase too. This result is independent of the union's orientation e.g. whether $\alpha<\beta$ or $\alpha>\beta$. (see equations A3 and A4 in appendix). If we assume that $\alpha+\beta= 1$, then as $\alpha$ increases, the NBL locus becomes steeper and shifts upwards. On the other hand, if the union is wage oriented the CC curve is positively sloped and if it is employment oriented the CC curve is negatively sloped. In the first case, the CC curve becomes flatter and shifts downwards and the result of a change in $\alpha$ on w and L is unambiguous, since the new equilibrium point will be on the CC curve in the area BAC. In the second case, however, the CC curve becomes steeper and shifts outwards and even though the effect on w is unambiguous the effect on L depends on whether the new equilibrium point is in the area BAC or not. The above results are due to the increased on competitiva in the labor market because of the increases in the number of firms. The equilibrium point becomes closer to the demand curve which means that the price that the firm will charge for its product becomes lower and near to the competitive one. On the other hand, the labor union having to do with small individual firms gets monopolistic power in the labor market and has the ability to charge a higher wage but also to pursue a higher level of employment.

### 3.3. The Effect of a Change in Firm's Bargaining Power

The effect of the change in firm bargaining power, as far as it is concern the nominal wage, it is positive, indepedently on whether the union is employment or wage oriented. The effect, however, on labor employment depends on the union orientation toward excess wage and the employment level. An increase in the bargaining power of the firm in bargainin process, with a wage-oriented union, will decrease employment while the employment will increase in a bargaining process with labor employment-oriented union (see equations A5 - A7 in appendix). As the bargaining power of the firm increases, the slope of the NBL becomes stteper. This means that the monopolistic power of the firm in the product and the labor market increases since the NBL moves toward and closer to the MR curve and the share of the labor in the revenues made by the firm decreases and the share of profits increases. A wage-oriented union prefers to reduce employment in order to keep stable its standard of living. On the other hand, an employment-oriented union would prefer to keep employment or increase employment even in the cost of a lower wage rate which may reach the competitive level. Then firm might hire unionised workers than agent's workers.

## 4. Conclusion

A reaction of the firm to the increasing bargaining power of the union could be the cooperation with an external agent who provides "cheap" labor to the firm in a competitive wage. Our objective in this paper was to discuss the effects that a change in competitive wage, the bargaining power of the firm and the wage elasticity of the union might have on the bargaining outcome e.g. wage and employment. Applying a Nash cooperative bargaging process we found that the effect of a change in the competitive wage and of an increase in the wage elasticity of the union will be positive both for equilibrium wage and employment. The effect however of a change in the bargaining power of the firm, will be positive for the wage rate but for the employment depends on whether the union is wage or employment-oriented.

## Appendix

From the total differentiation of (12) and (13) and the use of the Cramer's rule we can calculate the effects on w and L that result from a small change in w, $\alpha$ and $\rho$.

$$\frac{dw}{d\overline{w}} = -\frac{\frac{1}{\lambda} D' \left[ \left( \frac{\beta}{\alpha} + \frac{1-\lambda}{\lambda} \right) \left( \frac{2\rho + \beta(1-\rho)}{\rho + \beta(1-\rho)} \right) + (1-\lambda) \right]}{|J|} > 0 \qquad \text{(A1)}$$

$$\frac{dL}{d\overline{w}} = \frac{-\frac{\beta}{\alpha\lambda}}{|J|} < 0 \qquad \text{(A2)}$$

$$\frac{dL}{d\alpha} = \frac{\frac{\beta(w - \overline{w})}{\alpha^2}}{|J|} > 0 \qquad \text{(A3)}$$

$$\frac{dw}{d\alpha} = \frac{\begin{vmatrix} \frac{-\beta(w - \overline{w})}{\alpha^2} & \frac{1}{\lambda} 2D' \\ 0 & -\left( \frac{1}{\lambda} + \frac{\rho}{\lambda(\rho + \beta(1-\rho))} \right) \end{vmatrix}}{|J|} > 0 \qquad \text{(A4)}$$

$$\frac{dw}{d\rho} = \frac{\frac{2D}{\lambda} k}{|J|} > 0 \qquad \text{(A5)}$$

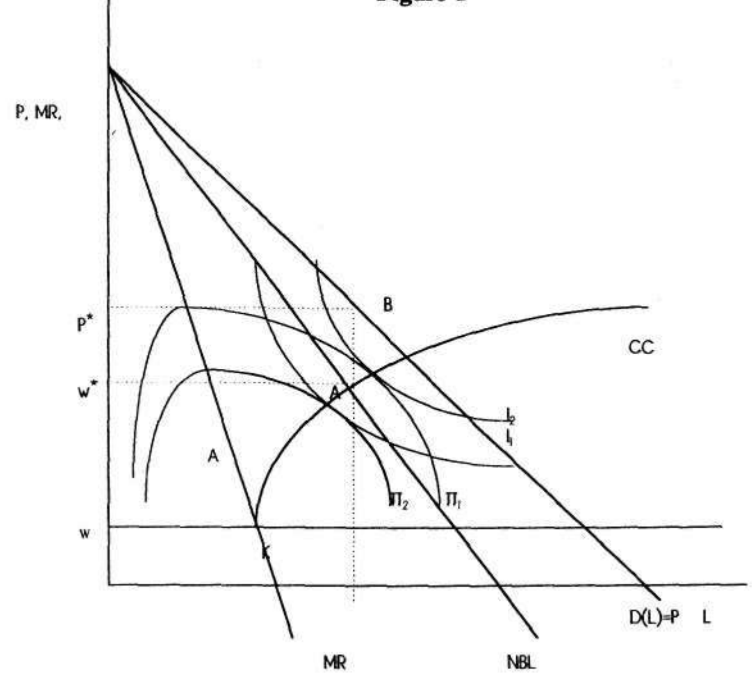
$$\text{where } k = \frac{LD' \lambda [\rho + \beta(1-\rho)] - \lambda(1-\beta)}{\lambda^2 [\rho + \beta(1-\rho)]^2} < 0$$

$$\frac{dL}{d\rho} = \frac{k \frac{\beta - \alpha}{\alpha}}{|J|} > 0, \text{ if } \alpha < \beta \qquad \text{(A6)}$$

and

$$\frac{dL}{d\rho} < 0 \quad \text{if } \alpha > \beta \qquad \text{(A7)}$$

**Figure 1**




# References


*Brander J. A. and Spencer B. J.* (1988), "Unionized Oligopoly and International Trade Policy", Journal of International Economics, 24, pp. 217-234.

*Friedman J. W.* (1989), Game Theory with Applications to Economics, Oxford University Press.

*Hall R. E. and Lilien D. M.* (1979), "Efficient Wage Bargains under Uncertain Supply and Demand", American Economic Review, 69, pp. 868-879.

*McDonald I. M. and Solow R.* (1981), "Wage Bargaining and Employment", American Economic Review, 71, pp. 896-908.

*Mezzetti C. and Dinopoulos E,* (1990), "Domestic Unionization and Import Competition", mimeo, presented at the 1989 Winter Econometric Society Meeting.

*Nash J. F.* (1950), "The Bargaining Problem", Econometrica, 18, pp. 155-62.

*Oswald A. J.* (1982), "The Microeconomic Theory of the Trade Union", Economic Journal, 92, pp. 576-595.

*Oswald A. J.* (1985), "Trade Unions an Introductory Survey" Scandinavian Journal of Economics, 87, pp. 160-193.