THE MARKET POWER VERSUS THE DIFFERENTIAL EFFICIENCY AMBIGUITY
A DISCRIMINATION THEORY*

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Abstract

This paper constructs a framework for distinguishing between the differential efficiency versus the market power hypotheses regardless of whether the relationship between industry and/or firm profitability and concentration and/or market share is positive or negative. This is made possible by the establishing of criteria which express the two hypotheses as mutually exclusive either at the firm level or, if this is not possible, at the industry level. These criteria are based on whether the production is characterised by increasing or decreasing returns to scale and on the magnitudes of the gap in efficiency and the gap in collusion between efficient (innovating) and non efficient (laggard) firms respectively. (JEL L11).

1. Introduction

In the model considered below, the market power versus the differential efficiency hypotheses are addressed within a framework where the benefits of both innovation and collusion are assumed to be private goods.

Innovation refers to the reduction in the production cost of a single, homogeneous product. Assuming product homogeneity should not be viewed as a restricting assumption since the existence of cost differentials is one of the alternative

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ways for modeling differentiation. A similar kind of defence can be found in Schmalensee (1987), who argues on the possibility of superior efficiency in terms of "... the production of the Lancasterian characteristics it [a firm] supplies to an existing market." On the other hand, Waterson (1990) provides a model which combines cost differentials with horizontal product differentiation (locational competition).

Treating innovation as exogenous follows the Demsetzian practice (1973), in which the regime of Schumpeterian competition is reversed by having the causal linkage running from the pace and pattern of innovation to the level of concentration. If successful innovators are rewarded with a relatively higher market share as compared to that of their rivals, then the differential efficiency hypothesis (henceforth, DEH) shall be accepted at the firm level. Note that in our model superior efficiency does not require (as the Demsetz model does) higher profitability and as a side-effect a higher market share. It merely requires that the more efficient firm will have a greater market share than the less efficient firm. Whether this superior efficiency will be rewarded by a relatively higher profitability depends on how the output decisions of the firm will be affected by its conjectures and the scale economies characterising the production. If the more efficient firm has unfavourable conjectures then it may not be able to translate its superior efficiency into higher profitability. Consequently, the differential efficiency hypothesis in this model is an augmented Demsetz theory accommodating both a positive as well as a negative relation between performance and market share. At the industry level the criterion for not rejecting (accepting) or rejecting the DEH is whether an increase in the differential efficiency between two firms will lead \( \text{ceteris paribus} \) to an increase or a decrease in concentration.

There is nothing new in the need for a model capable of explaining not only a positive but also a negative link between market structure and performance; this possibility is demonstrated theoretically in Clarke, Davies and Waterson (1984), Kessides (1989), Waterson (1990), Neumann and Haid (1985) and Schmalensee (1987). In the last two papers this is achieved by allowing the conjectural variation term, \( \lambda \), to be an either positive or negative linear function of the firm's market share, an improvement over the Clarke and Davies (1982) approach in which \( \lambda \) can only be decreasing in market share. This relation is further improved by Machin and Van Reenen (1993).

Viewing not only innovation, but also collusion as a private good, is in direct contradiction with the Demsetzian conviction of collusion benefiting
small and large firms equally. Instead, as Martin (1988a) argues "... concentration benefits primarily the profitability of larger firms..., which suggests that large firms are more profitable in concentrated industries because industries become concentrated when it is efficient to organise production in large units. The efficiency-profitability hypothesis has not been advanced in a way which distinguishes it from the collusion-profitability hypothesis."

Collusion is expressed in terms of what each firm conjectures that the degree of sensitivity of total output with respect to a change to its output is equal to. In other words, the collusion measure is constructed by parameterizing the elasticity of the total industry output variation of each firm (seen in detail later). Consequently, when it is said that a firm is more collusive than another firm this translates as the former firm having a higher elasticity of total industry output variation (henceforth, ETIOV) than the latter.

Consequently, collusion in our model is accounted for by ETIOV and this is shown to be directly related to profitability. To the extent that this happens through an indirect route it means that collusion (ETIOV) affects market structure (creating dominant firms with monopoly power) and as a consequence profitability is affected. If this is true however, then it is not possible to have a negative relation between market share and profitability. As a result, if these two turn out to be negatively related, then this signals that ETIOV affects profitability directly and not through the route of increased monopoly power; a fact that negates the monopoly power hypothesis (henceforth, MPH). Therefore, in our model a greater market share—*not a higher profit rate*—is the crucial measure for rejecting or accepting the market power hypothesis at the firm level. The market power hypothesis is not to be rejected (rejected) at the firm level if the relatively more collusive firm (in terms of ETIOV) has a higher (lower) market share. At the industry level the market power hypothesis is not rejected (rejected) if a further relative increase (decrease) in the ETIOV of the firm which already has the higher (lower) ETIOV between these two firms will lead, *ceteris paribus*, to an increase in concentration.

2. Model

2.1. The Division of an Industry in Technological Terms

Let the firms within an industry be divided into two groups, the leaders' group A, and the followers' group B. The leaders group (innovators) consists of $N_T$ technologically cooperating firms that have simultaneously introduced the
best available, in terms of output productivity, production technique in the
industry. Note that \( \tau \) is a counter for process innovations, \( \tau = 1, 2, \ldots \) rather than
a counter for time, since there is no reason why technological change should
occur every year. If all these innovations are fully protected by patents, then \( N_1 \)
will not change until a new superior process innovation occurs, when it will
automatically become equal to the number of firms in the group that introduced
this new innovation. The followers are the remaining \( N - N_1 \) firms in the
industry (where \( N \) is the total number of firms in the industry).

Working in terms of the Cowling and Waterson model ((1976), henceforth
C & W) of performance and market structure, we determine superior efficiency
in terms of the cost curves each firm faces. It is assumed that there is no
overlapping between any two firms in terms of cost, i.e. the marginal (average)
cost curves of any two firms \( j \) and \( / \) will not intersect at any point. The first order
condition for the profit maximising firm \( j \) in terms of the C & W model is

\[
q_j = \frac{n_p Q}{1 + \lambda_j} \left( 1 - \frac{MC_j}{P} \right)
\]

where \( P \) is the industry price which is a function of the industry output \( Q \) \((P = f(Q))\),
\( q_j \) the firm’s output, \( MC_j \) its marginal cost, \( \lambda_j = \frac{\partial}{\partial q_j} \) the conjectural variation
term, and \( n_p \) the absolute value of the elasticity of demand.

For a homogeneous production function of degree \( \zeta \), the corresponding
cost function is homogeneous of degree \( \frac{1}{\zeta} \) with respect to the output; consequently the marginal cost function is homogeneous of degree \( \frac{1}{\zeta} - 1 \). This
means that \( MC_j \) will be a non-constant function of \( q_j \) unless \( \zeta = 1 \). In the original
C & W model, the problem of marginal cost being a function of \( q_j \) is assumed
away by imposing the restriction that \( \zeta = 1 \). But if \( \zeta \neq 1 \) then the problem with (1)
is that it is no longer the reduced-form equation for \( q_j \).

If both terms in (1) are multiplied by \( \frac{1 + \lambda_j}{Q} \) the following is derived

\[
(1 + \lambda_j) \frac{q_j}{Q} = n_p \left( \frac{P - MC_j}{P} \right)
\]

(2)
where \( q_{jt} \) is the output produced by a firm belonging in group A at time t, \( L_t \) is the units of labour used and \( K_{jt} \) is the quantity of capital measured net of both physical and technical obsolescence, i.e. each unit of capital is a unit of investment which embodies the latest technology available at time t. This means that differences between firms in the rate of qualitative capital improvement are accounted for in the measure of capital \( K \), which is net of obsolescence i.e differences between firms as far as the synthesis of their capital (age distribution, since capital os of more than one vintage) is concerned have been incorporated into the capital measure. Consequently, \( a \) and \( b \) are the static output elasticities with respect to capital and labour respectively. More specifically, they account for how changes in the magnitude of these inputs- but not on how changes in their productivity - will affect output.

The LHS of the above expression is summarised by \( E_j \), where \( E_j = \left( \frac{(1 + \lambda_j) - \frac{q_j}{Q}}{Q} \right) \) is the conjectured elasticity of total industry output variation (henceforth ETIOV), measuring how elastic the firm believes that total industry output is with respect to increases in its own output. Given this formulation, \( E_j \) has a neat interpretation: \( E_j = 0 \) corresponds to perfect competition and \( E_j = 1 \) implies joint profit maximisation, while \( E_j = \frac{MS_j}{Q} \) suggests Cournot behaviour. To put it somewhat differently, the ETIOV is this model’s measure of the degree of implicit collusion. As it is well known, a perfectly collusive outcome is possible without explicit collusion if we perceive this model as a stage game played by the firms an infinite number of times (supergame). Then according to the folk theorem, the combination of ‘grim strategies’ and a discount rate close to one results to a fully collusive outcome in each period which is the unique outcome of a subgame perfect equilibrium. This is tacit collusion "... arising from a self-interested calculation of the benefits and losses that may accrue from ‘polite’ behavior". (Kreps, p. 505). The usual way to escape from this 'too perfect’ equilibrium outcome is to introduce random shocks in demand (Green and Porter, (1984)), so that firms are imperfectly informed on their rival’s output. In our model this is not necessary since unexpected ‘jumps’ in technology occur (on which see later), thus constituting a source of uncertainty.

By parameterising the LHS of (2), \( q_j \) can now be expressed in its reduced form. The first step is to specify \( MC_j \) by assuming a Cobb-Douglas production function which for an adopter firm is of the form

\[
q_j^A = A_j \left( K_j \right)^a \left( L_j \right)^b
\]

where \( q_j \) is the output produced by a firm belonging in group A at time t, \( L_j \) is the units of labour used and \( K_j \) is the quantity of capital measured net of both physical and technical obsolescence, i.e. each unit of capital is a unit of investment which embodies the latest technology available at time t. This means that differences between firms in the rate of qualitative capital improvement are accounted for in the measure of capital \( K \), which is net of obsolescence i.e differences between firms as far as the synthesis of their capital (age distribution, since capital os of more than one vintage) is concerned have been incorporated into the capital measure. Consequently, \( a \) and \( b \) are the static output elasticities with respect to capital and labour respectively. More specifically, they account for how changes in the magnitude of these inputs- but not on how changes in their productivity - will affect output.
2.2. The Generalised Co-operative Model

A_T is an index for the maximum potential productivity of the latest, best available, major process innovation. Unexpected, discontinuous jumps in A_T constitute the source of the disequilibrium force Schumpeter described (creative destruction). \( v_j^A \) reads as a time invariant firm specific effect taking values larger than zero and smaller than one, that exhibits how efficiently each firm individually in the innovators’ group applies the innovation. A_T multiplied by \( v_j^A \) gives A_jT, the realized productivity of firm j, which is always below the currently available maximum potential productivity.

For the non-innovators’ group the Cobb-Douglas production function is respectively

\[
q_j^B = B_T v_J^B (K_{jt})^q (L_{jt})^k \quad I \in B
\]  

(4)

where B_T measures the potential maximum productivity accruing from the most efficient innovation in group B. Obviously for as long as a process innovation reigns A_T>B_T. The gap in potential productivity between the two groups, V_T, is equal to

\[
V_T = \frac{A_T}{B_T} > 1 \quad j \in A, \quad I \in B
\]  

(5)

The role of \( v \)'s in group B is exactly the same as the role of \( v \)'s in group A, i.e. they are time invariant, firm specific effects which take values between zero and one and exhibit how efficiently each firm individually in the followers’ group applies the innovation. Note that differences in productivity among firms in group B where, unlike group A, firms use different innovations, are accounted for in the capital stock measure K_{jt}, not in the \( v \)'s. To avoid overlapping between the two groups, and among firms within the same group, the following restrictions are imposed on the gap in realized productivity

\[
V_T \frac{v_j^A}{v_j^B} \geq 1 \quad j \in A, \quad I \in B
\]  

(5')

\[
0 < V_T \frac{v_j^B}{v_j^A} \neq 1 \quad \forall j, \quad I \in \mu, \quad \mu = A, B
\]  

(5'')

2.2. The Generalised Co-operative Model

The model defined below is a general model incorporating different possible types of behaviour with the economic profit rate being a measure of this behaviour. Moreover, this model implicitly allows for average and marginal cost to be
a non trivial function of output when \(a+b \neq 1\). The latter is in contrast to C&W model which ignored the dependence of average and marginal cost on output by simply restricting their model to \(a+b = 1\).

The key assumption in this model is that the installation of new capital is both costless and instantaneous. Consequently, the long-run profit maximisation is identical to the short-run one. Let firm \(j\) have a profit function of the form (omitting the time subscripts)

\[
\Pi_j = Pq_j - TC_j = Pq_j - wL_j - rK_j
\]

where \(TC_j\) is its total cost function, \(w\) is the price per unit of labour, and \(r\) the user cost of capital. Replacing \(q_j\) in (6) by (3) or (4) (depending on which group the firm belongs) the first order conditions with respect to labour and capital for the profit maximising firm \(j\) are respectively

\[
r = P \alpha \left( 1 - \frac{E_j}{\beta_p} \right) G_j(K_j)^{\alpha-1}(L_j)^{\beta} \tag{7}
\]

\[
w = P \beta \left( 1 - \frac{E_j}{\alpha_p} \right) G_j(K_j)^{\beta-1}(L_j)^{\alpha} \tag{8}
\]

where \(G_j := A_j V_j^a\) for \(j \in A\) is an index of the productivity of a specific process innovation. It can be proved (see the Appendix) that \(MC_j\) is equal to

\[
MC_j = k(G_j)^{\frac{1}{a+b}} (q_j)^{\frac{1-a-b}{a+b}} \tag{9}
\]

where \(k = \left( \frac{1}{b} \right) \left( \left( \frac{a}{b} \right)^{\alpha} (r)^{\alpha} (w)^{\beta} \right)^{\frac{1}{a+b}}.\) Combining (9) with (2) we get

\[
\sigma_j P = k(G_j)^{\frac{1}{a+b}} (q_j)^{\frac{1-a-b}{a+b}} \tag{10}
\]

i.e. the equilibrium condition, setting the perceived (conjectured) marginal revenue equal to the marginal cost, where \(\sigma_j = 1 - \frac{E_j}{\beta_p} > 0\) is an inverse measure of collusion. Solving (10) in terms of output

\[
q_j = \left( \sigma_j P (k)^{\frac{1}{a+b}} G_j \right)^{\frac{a+b}{1-a-b}} \tag{11}
\]
gives the reduced form equation for the output of firm \( j \). Also combining (2) with (9) the profits to sales ratio, \( \pi_i \), is equal to

\[
\pi_i = \frac{Pq_i - TC_j}{Pq_i} = \frac{Pq_i - (a+b) PQ q_j}{Pq_i} = 1 - (a+b) \sigma_j = 1 - a - b + \frac{E_j}{n_p} (a+b)
\]  

(12)

Correspondingly, the price-cost margins, \( PCM_i \), is

\[
PCM_i = 1 - \sigma_j = \frac{E_j}{n_p}
\]

(12'')

The fact that \( \pi_j \) and \( PCM_j \) are direct functions of the collusion parameter \( E_j \) implies that these two are mere proxies for the behaviour of each firm. The higher \( E_j \) is, the higher \( \pi_j \) and \( PCM_j \) are going to be and vice versa. In other words, whether the firm will be in a position to extract the rent that corresponds to the technology it uses solely depends on how collusive (in terms of the ETIOV) this firm is. This explains why the market share rather than the profit rate is used in this model as a criterion for attempting to resolve the ambiguity between the two hypotheses.

Using (12), the weighted average of profits to sales for the industry as a whole can be calculated

\[
\pi = \frac{\sum q_j \pi_j}{Q} = \frac{\sum q_j}{Q} \left[ 1 - \sigma_j(a+b) \right]
\]

(13)

3. The Firm Level Hypotheses Discrimination Analysis

As it was argued in the introduction, at the firm level of analysis the criterion for not rejecting the DEH is whether a difference in the efficiency between two firms has as a consequence a larger market share for the relatively more efficient firm. Analogously, the criterion for not rejecting the MPH is whether the relatively more collusive firm (in terms of the ETIOV) has a relatively larger market share.

For applying these rules we first rewrite relation (11) as

\[
q_j = k \frac{a+b}{1-a-b} C_j \frac{1}{1+a+b} \sigma_j \frac{1}{1-a-b}
\]

(14)

It shall be assumed that within each group firms engage in joint profit maximisation (explicit collusion) and then equally distribute the profits among them. Combining this with (12) or (12'') implies that \( \sigma \)'s (and consequently \( E \)'s)
have to be *group uniform*. Each firm will then produce that quantity of output at which its individual marginal cost is equal to that of the group. Obviously, differences in the output produced by firms within the same group will solely owe their existence to the firm specific u's. The more efficient a firm is (the closer to one its u is) the less (more) it will produce if the production technique is characterised by increasing (decreasing) returns to scale. It is in this way that the condition of equal n's within each group is satisfied. The assumption of group-uniform o's has the advantage of making our model considerably more simple for the task of establishing inter-industry relations while retaining freedom to account for fairly diverse patterns of behaviour since λβ are free to vary among firms.

By dividing relation (14) for firm j with the same relation for firm l, where by definition j is technologically superior to firm l, the following relation is derived

\[
\frac{q_j}{q_l} = \left( \frac{G_j}{G_l} \right)^{1 \over 1+a+b} \left( \frac{\sigma_l}{\sigma_j} \right)^{a+b \over a+b-1}
\]  

(15)

where \( \frac{G_j}{G_l} = \frac{u_l}{u_j} = \frac{A_j}{B_j} \), \( \forall j \in A, l \in B \)

\[
\frac{u_l}{u_j} = \frac{u_j}{u_l} > 1, \quad \forall j, l \in A \text{ or } j, l \in B
\]

The above relation will be analysed for the cases of increasing and decreasing static returns to scale respectively.

We commence with the case of increasing static returns to scale \((a+b)>1\). Equation (15) may be re-written as

\[
\frac{q_j}{q_l} = \left( \frac{G_j}{G_l} \right)^{1 \over 1+a+b-1} \left( \frac{\sigma_l}{\sigma_j} \right)^{a+b \over a+b-1}
\]  

\(\Leftrightarrow\)

\[
\frac{q_j}{q_l} = (V_{ij} \frac{u_j}{u_i})^{1 \over f} = (x_{ij})^{1 \over f}
\]

where \( x_{ij} = \frac{\sigma_l}{\sigma_j}, f = \frac{1}{1-a-b} < 0 \), and \( f_1 = \frac{a+b}{1-a-b} = f - 1 < 0 \). When \( \sigma_l < \sigma_j (E_l > E_j \), i.e. the relatively more efficient firm is relatively more collusive as well), for \( q_j \) to be larger than \( q_l \), or in other words for the DEH to be accepted it is required that

\[
(V_l)^{1 \over f} (u_j)^{1 \over f} > (x_{ij})^{1 \over f}
\]

(16)
However, since the firm with the relatively higher ETIOV (firm j) also has the relatively greater market share, the MPH can not be rejected (is accepted). As a result, in this case neither of the two hypotheses can be rejected in favour of the other. On the other hand, for the more efficient firm to have a smaller market share, which in terms of our theory by definition translates as a rejection of the DEH for this pair of firms, it is required that

\[(\lambda_j)^f (\eta_j)^f < (\lambda_j)^f,\]  \hspace{1cm} (17)

Since firm j, which is relatively more collusive, has a lower market share the MPH is also rejected. The implication in this case is that the combination of both higher collusion and efficiency does not necessarily guarantee a higher market share. The DEH is rejected (and so is the MPH) regardless of the fact that the more efficient firm will have a higher \(\pi\) than the less efficient firm since \(\pi\) is only a proxy for the ETIOV. To summarize, under increasing static returns to scale and for \(x_j > 1\) there are two possibilities: either both hypotheses will be rejected or both hypotheses will be accepted. This means that under these conditions the firm level of analysis is not adequate for resolving the ambiguity between the DEH and the MPH. As it will be shown in the next section, such cases may be dealt with at the industry level, where the criterions used for accepting the DEH and the MPH will prove sufficient for resolving the ambiguity.

When \(a+b > 1\) and \(x_j < 1\), inequality (17) will always hold since by definition \(V\gamma > 1\) and \(f_1 < f < 0\). Therefore, under these circumstances, the less efficient firm will always have a larger market share and therefore the DEH will always be rejected. Moreover, since the firm with the relatively higher collusion has a higher market share the MPH is accepted. Consequently, the ambiguity in this case is resolved with the DEH being rejected in favour of the MPH.

In the case of decreasing static returns to scale \((a+b < 1, f > f_1 > 0)\), if the more efficient firm is relatively more collusive as well \((\Omega j < a_i)\), for \(q_j\) to be larger than \(q_y\) it is required that inequality (16) holds. Both the DEH and the MPH are accepted since the more efficient and more collusive firm has a higher market share than the laggard and less collusive firm. If, on the other hand, \(q\) is smaller than \(q_j\) then both hypotheses are rejected for exactly the opposite reasons. Consequently, for \(a+b<1\) and \(x_j > 1\) either both hypotheses will be accepted or both hypotheses will be rejected.

Finally, when \(a+b<1\) and \(x_j < 1\) the more efficient firm will always have a higher market share (since inequality (16) will always hold) and therefore the
DEH is to be accepted always. Moreover, the firm which is relatively more collusive is the firm with the relatively lower market share. Consequently, under these circumstances the MPH is always rejected in favour of the DEH. The DEH is accepted, this time regardless of the fact that the more efficient firm has a lower π than the less efficient firm (since Ej<Ej). While in such a case the Demsetz hypothesis would have been rejected, our DEH will not be rejected because, as already explained in the introduction, it is more broadly defined and capable of explaining a negative as well as a positive relation between market share and profitability.

To summarize, at the firm level, when the more efficient firms are also the more profitable ones (i.e. the more collusive ones, since x/x >1), the analysis should be performed at the industry level if one wishes to resolve the ambiguity. On the other hand, when the more efficient firms are the less profitable ones (x/x <1) then under decreasing static returns to scale the MPH is rejected in favour of the DEH, while under increasing static returns to scale the DEH is rejected in favour of the MPH. Interpreting this latter result, if the firms that enjoy no cost advantage are relatively more collusive (and thus more profitable) then when a+b<1 these firms do not wish to use their oligopolistic practices for obtaining a relatively higher market share since average costs are an increasing function of size. This is in line with what Waterson (1988, page: 13) argues: "... In the case of decreasing returns, the more monopolistically behaving firms,..., are the smaller ones". At the same time, the firms that enjoy genuine cost advantages can afford to sustain a relatively higher market share since although decreasing static returns characterise the production, their productivity is relatively superior to that of the laggard firms, i.e. in this case innovativeness is the cause of large size as Demsetz argued.

On the other hand, as Martin (1988b, page: 312) writes, with increasing returns to scale it is in the interest of firms to increase output, which will reduce average cost. Consequently, in the case where laggard firms are more collusive while a+b >1, these firms will make sure that they sustain a relatively greater market share since this will have a negative impact on their average costs. Moreover, while the more collusive firms are dominant in terms of size, the engine for innovativeness are the relatively smaller firms. In other words, under increasing static returns to scale and x/x <1 the industry is characterised by small innovating firms, thus rejecting the Schumpeterian assumption of large innovative firms and justifying the need for anti-trust policy.

It is important to note that as long as x/x <1, there is no possibility of mistaking large scale advantages for genuine superior economic efficiency. To
put it in plain words, firms may have a large market share because they are more efficient (in which case the DEH is accepted), but if they have a lower average (and marginal) cost solely because they have a relatively larger market share the DEH is rejected.

4. Industry Level Hypotheses Discrimination Analysis

In this section we shall demonstrate that the cases for which the ambiguity between the MPH and the DEH was not resolved at the firm level, because the MPH and the DEH were either both accepted or both rejected, can be resolved at the industry level. The DEH is to be accepted at the industry level if an increase in the technological gap between the two groups will lead to an increase in the level of concentration. The MPH is accepted if a further relative increase in the ETIOV of the group which already has the higher ETIOV should lead to an increase in the level of concentration.

For applying these rules it is necessary to establish a relationship between concentration on the one side and the gap in technology as well as the gap in ETIOV's between the two groups on the other. By squaring \((14)\) we derive.

\[
(q_i)^2 = (k_i)^{2i} (G_{ij})^{2i} (\sigma_j)^{2i}
\]

Using \((18)\) we can now calculate the Herfindahl index of concentration over the \(N\) firms in the industry

\[
H = \frac{\sum_{i=1}^{N} (q_i)^2}{\left(\sum_{i=1}^{N} q_i\right)^2} = \frac{\sum (G_{ij})^{2i} (\sigma_j)^{2i}}{\left[\sum (G_{ij})^{2i} (\sigma_j)^{2i}\right]^2}
\]

Since it is assumed that the \(\sigma\)’s within each group are uniform then we may set that \(\sigma^B = x\sigma^A\). If we sum over for the firms in each group incorporating the uniformity assumption into \((12)\) and then substitute the result into \((13)\) we get

\[
\pi = \frac{Q^A}{Q} (1 - \sigma^A (a+b)) + \frac{Q^B}{Q} \left(1 - x\sigma^A (a+b)\right)
\]

\[
\pi = 1 - \sigma^A (a+b) \left(\frac{Q^A}{Q} + \frac{Q^B}{Q} x\right)
\]
where \( Q^A = \sum q^A_i \) and \( Q^B = \sum q^B_i \). Also using the group uniformity assumption (19) may be rewritten as

\[
H = \frac{1 + x^{-2f_1 V_T} C_1}{\left[ 1 + x^{-r_f V_T} C_2 \right]^2} C_3
\]  

(21)

where

\[
C_1 = \frac{\sum_{(u_i)^2}^A}{\sum (u_i)^2} > 0 \quad C_2 = \frac{\sum_{(u_i)^2}^A}{\sum (u_i)^2} > 0 \quad C_3 = \frac{\sum_{(u_i)^2}^B}{\sum (u_i)^2} > 0
\]

The signs of the partial derivatives \( \frac{\partial H}{\partial V_T} \) and \( \frac{\partial H}{\partial x} \) can serve as the criteria by which the DEH and the MPH should be rejected or not rejected at the industry level.

It is rather easy to interpret a positive or a negative \( -\frac{\partial H}{\partial V_T} \). A positive (negative) sign denotes that the differential efficiency hypothesis holds (does not hold) since a higher concentration is the result of an increase (decrease) in the gap in efficiency between the two groups.

The interpretation of the sign of the partial derivative \( \frac{\partial H}{\partial x} \) deserves more debate. Since

\[
x = \frac{\sigma^B}{\sigma^A} = \frac{1 - \frac{1}{n_p} E^B}{1 - \frac{1}{n_p} E^A} = \frac{1 - \pi^B}{1 - \pi^A}
\]  

(22)

where \( \pi^A \) and \( \pi^B \) are the profits to revenue ratios for each firm in the innovators' and laggard's group respectively, then if \( x \geq 1 \) \( (E^A \geq E^B) \) this implies that the innovators' group is relatively more profitable and therefore an increase in \( x \) translates as an increase in the divergence between the ETIOV of the two groups in favour of the group which had a higher ETIOV to start with. Consequently, if an increase in the divergence of the ETIOV's brings an increase in \( H \) (in other words if \( \frac{\partial H}{\partial x} \) is positive) then the MPH is to be accepted and if this increase brings a decrease in \( H \) (if \( \frac{\partial H}{\partial x} \) is negative) then the MPH is to be rejected. On
the other hand, when \( x < 1 \) \((E^A < E^B)\) an increase in \( x \) translates as an increase in \( E^A \) and/or a decrease in \( E^B \) which implies a convergence between the ETIOV's of the two groups since there is move in favour of the group with the relatively lower ETIOV. If this decrease in the gap between the ETIOV's of the two groups results to an increase in \( H \) \((-\frac{\partial H}{\partial x} > 0)\) then the MPH is rejected. On the other hand if this decrease brings a decrease in \( H \) as well then the MPH is accepted.

Setting \( C_4 = \frac{C_1}{C_2} \), it can be proved that under increasing static returns to scale and \( x \geq 1 \), if \( C_4 \bar{V}_T \geq \bar{x}^1 \left( C_4 \bar{V}_T \leq \bar{x}^1 \right) \) then \(-\frac{\partial H}{\partial \bar{V}_T} \) is negative (positive) and \(-\frac{\partial H}{\partial x} \) is positive (negative), which implies that the DEH (MPH) should be rejected in favour of the MPH (DEH). On the other hand, when \( a + b > 1 \) and \( x < 1 \), if \( C_4 \bar{V}_T \leq \bar{x}^1 \left( C_4 \bar{V}_T \geq \bar{x}^1 \right) \) then because \(-\frac{\partial H}{\partial \bar{V}_T} \) is positive (negative) and \(-\frac{\partial H}{\partial x} \) is negative (positive) neither (both) the MPH nor (and) the DEH can be rejected. For the decreasing static returns to scale case and \( x \geq 1 \) if \( C_4 \bar{V}_T \geq \bar{x}^1 \left( C_4 \bar{V}_T \leq \bar{x}^1 \right) \) the positively (negatively) signed \(-\frac{\partial H}{\partial \bar{V}_T} \) confirms (rejects) the DEH while MPH is rejected (confirmed) on the basis of a negative (positive) \(-\frac{\partial H}{\partial x} \). On the other hand, when \( a + b < 1 \) and \( x < 1 \) if \( C_4 \bar{V}_T \leq \bar{x}^1 \left( C_4 \bar{V}_T \geq \bar{x}^1 \right) \) both (neither) the DEH and (nor) the MPH can be rejected.

The above results are particularly interesting because they demonstrate that at the industry level the effects of \( \bar{V}_T \) and \( x \) on \( H \) are always of an opposite sign. This suggests that an increase in the gap in efficiency between the two groups, \( \frac{A_T}{B_T} \), will have to be met by a decrease in \( \frac{\sigma^B}{\sigma^A} \) if the distribution of the output shares is to remain unchanged. More importantly, it means that at the industry level when \( x \geq 1 \) the market power hypothesis and the differential efficiency hypothesis are mutually exclusive, while when \( x < 1 \) ambiguity exists since both or neither of the two hypotheses can be rejected against its alternative.

Combining the above with the firm level conclusions of the previous section, for \( a + b < 1 \) ambiguity exists at the firm level when \( x \geq 1 \) which can be resolved at the industry level: if the gap in efficiency between the two groups is sufficiently large and/or the gap between the group \( \sigma^B \left( \frac{\sigma^B}{\sigma^A} \right) \) sufficiently small (i.e. the gap in the differences in conduct between the two groups, \( \frac{E^A}{E^B} \), is sufficiently small) for \( C_4 \bar{V}_T \) to be greater than \( \bar{x}^1 \) then the MPH is rejected in
favour of the DEH. On the other hand, if the gap between the group efficiencies is sufficiently small and/or the gap between the group \( \sigma's \left( \frac{\sigma^B}{\sigma^A} \right) \) sufficiently large for \( C_4V_f \) to be smaller than \( x_1^* \) then the DEH is rejected in favour of the MPH. In the opposite case, i.e. for \( a+b<1 \) if an ambiguity exists at the industry level because \( x<1 \), then this can be resolved by using the results derived at the firm level, where MPH is always rejected in favour of the DEH. For \( a+b>1 \) when ambiguity exists at the firm level because \( x \geq 1 \) this is resolved at the industry level: If the gap in efficiency between the two groups \( \left( \frac{A_T}{B_T} \right) \) is sufficiently small and/or the gap between the group \( \sigma's \) sufficiently large for \( C_4V_f \) to be greater than \( x_1^* \) then the DEH is rejected in favour of the MPH. On the other hand, if the gap in efficiency between the two groups is sufficiently large and/or the gap between the groups \( \sigma's \left( \frac{\sigma^B}{\sigma^A} \right) \) sufficiently small for \( C_4V_f \) to be smaller than \( x_1^* \), then the MPH is rejected in favour of the DEH. When the ambiguity is at the industry level because \( x<1 \), it can be resolved by simply referring to the firm level results which conclude that the DEH is rejected in favour of the MPH.

Consequently, when the innovators enjoy higher profit margins (\( x \geq 1 \)), then the ambiguity is resolved by using the industry level conclusions, while when the non-innovators have higher profit margins (\( x<1 \)), the ambiguity is resolved by using the firm level conclusions. Table 1 provides a summary of the set of criteria established, using the industry level criteria when \( x \geq 1 \) and the firm level criteria when \( x<1 \).

5. Estimation Possibilities

If one wishes to determine which of the four possible conclusions apply for a particular industry, two conditions should be satisfied. First a model at the industry level is required which permits the estimation of \( C_4V_f \) and \( a+b \) as two separate parameters. Second, data should exist that make possible the calculation of \( x \). Knowledge of \( x \) is essential for deciding whether the firm level or the industry level conclusions are going to be used, while the estimates of the two parameters are required so that when \( x \geq 1 \) it shall be possible to say whether \( C_4V_f \) is larger or smaller than \( x_1^* \). Estimates for the two parameters will be derived by running a non linear regression based on relation (21). In particular, one may set as \( \gamma_1 = C_1(V_f)^2 \), \( \gamma_2 = C_3(V_f) \), \( \gamma_3 = C_3 \) (therefore \( \frac{\gamma_1}{\gamma_2} = \frac{C_4(V_f)^2}{C_4(V_f)} \) and estimate from this regression \( \gamma_1 \), \( \gamma_2 \), \( \gamma_3 \) and \( f_i \). All of the above mentioned
parameters will vary between different industries and additionally \( \gamma_1 \) and \( \gamma_2 \) will vary from time to time as discrete jumps in innovations alter the magnitude of the efficiency gap, \( V \). What is therefore required is a set of data that can account for these differences i.e. panel data at the industry level. The regression for industry \( i \) if a multiplicative disturbance term is added will be of the form:

\[
H_{it} = \frac{1 + \gamma_{1it} (x_{it})^{-2f_{it}}}{\left[1 + \gamma_{2it} (x_{it})^{-f_{it}}\right]^2} \gamma_{3it} d_{it} \quad (23)
\]

Since panel data information on \( x = \frac{1 - \pi^*}{1 - \pi} \) is required, it is essential that for each industry data should not only account for innovative successes between the firms so as to successively separate the adopters from the non-adopters in each cross section, but should also provide a continuous tracking of the major innovative activities of each firm through time. Note that no progress will be made in estimating \( \gamma_1, \gamma_3, \gamma_3 \) and \( f_1 \) unless some restrictions are imposed as to how parameters \( \gamma_1 \) and \( \gamma_2 \) vary between industries and time. Having said that, there are two ways of treating \( \gamma_1 \) and \( \gamma_2 \) (Judge et al, 1988). The first is to consider these two parameters as fixed and time invariant for each industry and proceed in estimating \( \gamma' = (\gamma_1', \gamma_2', \gamma_3', \gamma_4') \), where \( \gamma' \) is a \((1 \times 4N)\) vector of unknown fixed parameters to be estimated within the framework of a seemingly unrelated regressions model. Alternatively, one may regard \( \gamma_{1it} \) and \( \gamma_{2it} \) as random parameters with means \( \bar{\gamma}_1 \) and \( \bar{\gamma}_2 \) respectively. If one defines \( \varepsilon_{1it} = \gamma_{1it} - \bar{\gamma}_1, \varepsilon_{2it} = \gamma_{2it} - \bar{\gamma}_2 \), then an alternative to (23) is the model

\[
H_{it} = \frac{1 + \bar{\gamma}_{1it} (x_{it})^{-2f_{it}} + \varepsilon_{1it} (x_{it})^{-2f_{it}}}{\left[1 + \bar{\gamma}_{2it} (x_{it})^{-f_{it}} + \varepsilon_{2it} (x_{it})^{-f_{it}}\right]^2} \gamma_{3it} \quad (24)
\]

In (24) the disturbances \( \varepsilon_{1it} \) and \( \varepsilon_{2it} \) replace the ad hoc disturbance term \( n_{it} \) of the earlier model. Details on how to treat a model of this form can be found in Dassiou (Chapter 3, pages: 83-84).

Once \( \gamma_1, \gamma_2 \) and \( f_1 \) have been successfully estimated it is possible to determine for each industry whether \( a+b \) is larger or smaller than one and whether \( C_a V x^{11} - 1 \) is positive or negative and consequently conclude for each industry with \( x \geq 1 \), whether the MPH or the DEH is the prevailing hypothesis using the
industry level conclusions. If $x < 1$ then the firm level conclusions should be used. *It is still not necessary to actually work at the firm level since one can identify in which case the industry belongs by simply looking at the estimates derived at the industry level.* If the estimates reveal that $a + b < 1$ and $x < 1$, the MPH is rejected in favour of the DEH and if they reveal that $a + b > 1$ and $x < 1$ then the DEH is rejected in favour of the MPH.

6. Conclusions

This paper establishes a set of testable criteria for distinguishing between the DEH and the MPH by expressing the two hypotheses as *mutually exclusive* either at the firm level or, if this is not possible, at the industry level. These criteria depend on scale economies and on the magnitudes of the gap in efficiency and the gap in collusion between efficient (innovating) and non efficient (laggard) firms respectively. We have modeled efficiency in terms of the impact of major process innovations on productivity, while for collusion we use as its parameter the elasticity of total industry output variation (ETIOV). Using ETIOV as a measure of collusion implies that this can be proxied by the profits to revenue ratio, an observable variable. This in turn implies that hypothesis testing in our model can not be based on (is independent of) the sign of the relationship between market share and firm profitability since the latter is tautological with firm collusion.

The usefulness of distinguishing between the MPH and the DEH hypotheses lies in the justification of a hands-off policy in the cases where an efficient market structure is diagnosed (or perhaps even an active pro-merger policy to reward and/or further enhance innovativeness), and equivalently the application of anti-monopolistic policies in industries where market power abuse is identified.

**Appendix**

Dividing (8) by (7) gives

$$\frac{w}{r} = \frac{b}{a} \frac{K_j}{L_j} \Leftrightarrow K_j = \frac{w}{r} \frac{a}{b} L_j \tag{A1}$$

Substituting (A1) into either (3) or (4) gives the following expressions for firm’s j profit maximising level of output and the demand for labour respectively

$$q_j = G_j (L_j)^b (L_j)^a \left( \frac{w}{r} \frac{a}{b} \right)^a \tag{A2}$$
\[ L_j = \left( (G_j)^{-1} (q_j) \left( \frac{w}{r} - \frac{a}{b} \right)^a \right)^{\frac{1}{a+b}} \]  

(A3)

Substituting (A3) for \( L_j \) into (A1) gives the demand function for capital

\[ K_j = \left( (G_j)^{-1} (q_j) \left( \frac{w}{r} - \frac{a}{b} \right)^b \right)^{\frac{1}{a+b}} \]  

(A4)

Consequently, the total cost of producing the profit maximising level of output is equal to

\[ TC_j = w \left( (G_j)^{-1} (q_j) \left( \frac{w}{r} - \frac{a}{b} \right)^a \right)^{\frac{1}{a+b}} + r \left( (G_j)^{-1} (q_j) \left( \frac{w}{r} - \frac{a}{b} \right)^b \right)^{\frac{1}{a+b}} \]

\[ TC_j = (a+b) k (G_j) \left( \frac{1}{a+b} (q_j) \right)^{\frac{1}{a+b}} \]  

(A5)

where \( k = \left( \frac{1}{b} \right) \left( \left( \frac{a}{b} \right)^a (r)^b (w)^b \right)^{\frac{1}{a+b}} \). By differentiating total cost with respect to the profit maximising level of output relation (9) is deduced.

### TABLE I

Set of Criteria for Resolving the Ambiguity Between the MPH and the DEH

<table>
<thead>
<tr>
<th>Condition</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a+b&gt;1 ), ( C_{iV_{i}}&gt;x^{1} )</td>
<td>MPH accepted - DEH rejected.</td>
</tr>
<tr>
<td>( a+b&gt;1 ), ( C_{iV_{i}}&lt;x^{1} )</td>
<td>DEH accepted - MPH rejected.</td>
</tr>
<tr>
<td>( x \leq 1 ) , ILC</td>
<td>( a+b&lt;1 ), ( C_{iV_{i}}&gt;x^{1} ) : MPH rejected - DEH accepted.</td>
</tr>
<tr>
<td></td>
<td>( a+b&lt;1 ), ( C_{iV_{i}}&gt;x^{1} ) : DEH rejected - MPH accepted.</td>
</tr>
<tr>
<td>( x&lt;1 ) , FLC</td>
<td>( a+b&gt;1 ) : MPH accepted - DEH rejected.</td>
</tr>
<tr>
<td></td>
<td>( a+b&lt;1 ) : DEH accepted - MPH rejected.</td>
</tr>
</tbody>
</table>

FLC = Firm-level criteria.  
ILC = Industry-level criteria.
Footnotes

1. For a similar treatment, see Appelbaum, (1982), where he uses ETIOV as a measure of the degree of competition.

2. In the first period of the game all players choose the collusive outcome, and in any subsequent period they continue to do so if all players were loyal. If however one firm 'cheats' then all firms retaliate by playing the noncooperative strategy thereafter (Kreps, 1990, Rasmussen, 1989).

3. In the case of constant returns to scale while assuming group uniform ETIOV's, (14) becomes $MC_j = k(G_j)^{-1}$, and for the equilibrium condition (10) to hold for firms within the same group, since perceived marginal revenues in the group are equal (since $\sigma$ is group uniform), then marginal costs will also have to be equal. This in turn implies, that only the most efficient firm within each group will produce (the firm with the closest to one value of $\omega$).

4. A thorough diagrammatic exposition of this, as well as the rest of the cases to be described in this paper, can be found in a paper by Dassiou currently under submission (July 1992, available from the author on request).

5. The case of constant static returns to scale is analyzed in Dassiou, section 4.3.

6. Proposition: the partial effects of $V_T$ and $x$ on $H$ are always of an opposite sign.

Proof:

We can examine by partial differentiation the effect on $H$ from a change in the technological gap between the two groups and the effect of a change in the gap between $\sigma$'s of the two groups the signs of which are the criteria in this model for accepting or rejecting the DEH and the MPH respectively.

$$\frac{\partial H}{\partial V_T} = \frac{(2fx^{-2}V_T^{-1}C_2 + \omega)C_1 (1 + x^{-1}V_T^{-1}C_2)^2}{[1 + x^{-1}V_T^{-1}C_2]^3}$$

$$-2 \left[ 1 + x^{-1}V_T^{-1}C_2 \right] f^{-1} C_1 V_T^{-1} C_2 \left[ 1 + x^{-1}V_T^{-1}C_2 \right] C_1$$

$$\left[ 1 + x^{-1}V_T^{-1}C_2 \right]^2$$

Consequently, for $\frac{\partial H}{\partial V_T}$ to be positive when $a + b < \lambda (f, f_i > 0)$ it must be the case that

$$2f C_1 C_2 x^{-2} V_T^{-1} + 2 f C_1 C_2 x^{-3} V_T^{2-1}$$

$$-2 f C_2 C_3 x^{-1} V_T^{2-1} - 2 f C_1 C_2 C_3 x^{-3} V_T^{2-1} > 0 \text{ since } \lambda, f > 0$$

$$C_1 C_2 x^{-1} V_T^{2-1} - C_2 C_3 x^{-1} V_T^{2-1} > 0$$

$$C_1 V_T > x^{(f)}$$
Therefore, for $a+b<1$ when $C_4 V_T < x^f$, $\frac{\partial H}{\partial V_T}$ is negative and when $C_4 V_T > x^f$, $\frac{\partial H}{\partial V_T}$ is positive.

For $\frac{\partial H}{\partial x}$ to be positive when $a+b<1$ it is required that:

$$-2f_1 C_1 C_3 x^{2f_1-1} V_T^{2f_1} - 2f_1 C_1 C_2 C_3 x^{2f_1-1} V_T^{2f_1}$$

$$+ 2f_1 C_1 C_3 x^{2f_1-1} V_T^{2f_1} + 2f_1 C_1 C_2 C_3 x^{2f_1-1} V_T^{2f_1} \leq 0 \text{ since } f>0$$

$\Rightarrow$

$$- C_1 x^{2f_1-1} V_T^{2f_1} + C_2 x^{2f_1-1} V_T^{2f_1} > 0$$

$\Rightarrow$

$$C_4 V_T < x^f$$

Consequently, for $a+b<1$ when $C_4 V_T > x^f$, $\frac{\partial H}{\partial x}$ is negative and when $C_4 V_T < x^f$, $\frac{\partial H}{\partial x}$ is positive.

For $\frac{\partial H}{\partial V_T}$ to be positive when $a+b>1$ (i.e., $f>0$) it is required that:

$$2f_1 C_1 C_3 x^{2f_1-1} V_T^{2f_1} - 2f_1 C_1 C_2 C_3 x^{2f_1-1} V_T^{2f_1} > 0 \text{ since } f>0$$

$\Rightarrow$

$$C_1 x^{2f_1-1} V_T^{2f_1} - C_2 x^{2f_1-1} V_T^{2f_1} < 0$$

$\Rightarrow$

$$C_4 V_T < x^f$$

Therefore, for $a+b>1$ when $C_4 V_T < x^f$, $\frac{\partial H}{\partial V_T}$ is negative and when $C_4 V_T > x^f$, $\frac{\partial H}{\partial V_T}$ is positive.

For $\frac{\partial H}{\partial x}$ to be positive when $a+b>1$ (i.e., $f<0$) it is required that:

$$-2f_1 C_1 C_3 x^{2f_1-1} V_T^{2f_1} + 2f_1 C_1 C_2 C_3 x^{2f_1-1} V_T^{2f_1} > 0 \text{ since } f>0$$

$\Rightarrow$

$$C_4 V_T > x^f$$

Consequently, for $a+b>1$ when $C_4 V_T < x^f$, $\frac{\partial H}{\partial x}$ is negative and when $C_4 V_T > x^f$, $\frac{\partial H}{\partial x}$ is positive.
References


