TRANSACTION COSTS AND
THE NATURE OF INFORMATION:
AN APPLICATION TO THE CREDIT MARKET

By

Maria Psillaki*, Gerard Mondello**
LATAPSES-Idefi-CNRS, University of Nice

Abstract

In this paper we suggest a framework to interpret the transaction costs concept. We consider
transaction costs as the resource losses due to imperfect information. We relate the concept of
transaction costs to the nature of information. According to the information set it belongs to,
an information may be redundant or complementary. As a consequence, transaction costs will
be different. We have applied this notion of transaction costs to the credit market taking into
account the nature of information. We consider a large population of borrowers and lenders
who, matching randomly, may or may not incur information costs. These costs may have a
positive effect on the probability of success of the project. We emphasize that upon the nature
of information, the lenders as the borrowers may behave strategically in order to minimize the
transaction cost losses.

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1. Introduction

In the literature¹, transaction costs are understood as an easy explanatory
concept to market failures such as imperfect competition, agents' unequal
access to information, transporation and commision costs, etc. They are
viewed as the costs of "using market mechanism", Coase (1937). However,

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their application field appears so huge that a precise definition may be difficult to grasp. Dalhman's (1979) contribution helps narrowing a little bit this conception considering them as the price to pay to reduce uncertainty. They become the "costs" induced by imperfect information. It is noteworthy that, following this line of argument, gathering information (whatever it may be) guarantees efficiency because it reduces uncertainty features. This could give the false idea that agents improve their situation accumulating new information whatever its nature may be.

This paper focuses on the nature of information. Indeed, the kind of information agents can obtain determines their behavior and consequently, transaction costs incurred. For instance, agents may adopt either some co-operative attitude when they need to complete their information set, or to defect when they think that information may be acquired and transmitted by someone else without cost. In our framework, transaction costs are resources losses due to imperfect information.

Our analysis will take the banking sector as an application field. In this sector, transaction costs are generally used to partly justify the existence of financial intermediaries. More precisely, their existence could lie on the economies of scale and/or of scope to transform the financial assets issued by the borrowers. In this theory, transaction costs are exogeneously given [see e.g. Hellwig (1991), Pyle (1971)]. Other intermediaries approaches insist on the role of informational asymmetries. The transaction costs are linked to ex-ante (adverse selection), interim (moral hazard) or ex-post (costly state verification) informational asymmetries. For the literature, these asymmetries generate market imperfections implying transaction costs. So the financial intermediaries may be seen as information sharing coalitions (Rochet and Freixas 1998). However, none of the above studies has looked at the motive for lenders and/or borrowers to incur transaction costs applied to the credit market as it is studied in this paper. More precisely, we analyse information cost expenses that could increase the probability of success of the project the borrowers will undertake.

Consequently, the goal of this paper is twofold: first it analyzes the notion of transaction costs understood as the losses due to lack of information and, secondly, it studies the nature of information which would differentiate transaction costs and motivate a strategic behavior. In section two, we will refer to some aspects of information theory. Section three sets out a model in which the lenders and the borrowers may or may not carry out transaction
costs. Section four of the paper derives comparative static results, showing that the information set the agent faces may induce specific behavior. Section five of the paper concludes.

2. Some elements on Information Theory

It is known that, because information is costly, agents may be induced either to spend money to become much more informed or to keep on with a low level of information. This is depending upon their own guess about the quality of information they really need or they can dispose of when making effort to acquire it. Hence, the relevance of information may be as important as its asymmetrical distribution. Information theory has recognized for a long time that the information content is related to its asymmetrical distribution. Information theory has recognized for a long time that the information content is related to its predictability.

Our starting point is Claude Shanon 1948's information theory. We focus particularly on the quality of the transmission and the quality/nature of information. The quality of information means two things: first, the quality of the transmitted signal, secondly, the information set completeness i.e. how "informative" an information is liable to be. In other words, this "informative"ness corresponds to the level of surprise it brings to the agent or more precisely what we call the "surprise caused by an information". This latter is closely related to the notion of information entropy (Gray 1990). The more certain some event is, the less surprising an information confirming the event will be. In other words, let $\chi$ be a random variable, the information entropy is the average amount of surprise received when the value of $\chi$ is observed. Thus to measure the degree of uncertainty of a system or of a variant $\chi$ information theory uses the notion of entropy. It is obvious that we can measure the quantity of information by the reduction of the system's entropy. We have to note that the quality of information is linked to the relevancy of the message it contains. How to be sure about relevancy? Either the receiver is absolutely certain about the information source, or he prefers to have the information confirmed by, for example, repetition or another source transmission.

In fact, for information theory, repeating information helps in reducing the probability of error in a noisy channel. Two sources sending the same information to the same receiver imply several things. Hence, if both channels are good, the receiver gets redundant information - i.e. one channel among
two is sending a useless information. Notice that the timing of reception may be important. However, this redundancy may confirm the previous information: the agent has to be sure that the channel is not noisy, so the information is not distorted.

Redundancy may involve costs when information has a price and the agent is sure about the channels (the agent does not need the confirmation from the second channel). In the opposite case, when independent information is supplied by different sources, the information is complementary.

Consequently, we are facing some kind of dilemma. Hence, either the agent knows with certainty that the second channel informs redundantly or he discovers that this supplementary information is useless and therefore redundant. In the first case, the agent will avoid to pay twice some costly information, in the second case (when he discovers), the agent incurs some unexpected cost.

The problem is becoming more complex whether some channels are liable to transmit information with errors and some other ones are not. Indeed, the receiver has to choose the right channel among several -i.e. between a costly channel and a free one. Consequently, the Grossman and Stiglitz (1980) paradox on information is likely to be met. In this context, we recall that on a given market (the asset market for example), two information sources supply the same information (asset's prices). However, one source is costly because privately shared (informed agents), while the other one is free, agents have only to observe the market tendency. Under the no noise assumption, the rational agent will prefer to wait for the market information rather than buy the private one. Here the information is typically redundant and the source choice is not neutral.

How to relate the above considerations to the transaction costs concept? Our starting point is both Dahlman's (1979) and Cheung's (1990) transaction costs analysis, where these costs result from the search for information. Warneryd (1994) gave an operational meaning to the transaction costs concept. He considers transaction costs as the expected value of information in games played by individuals randomly matched from a large population. Transaction costs correspond to the discrepancy between the losses of the informed choice and the uninformed one. More precisely, transaction costs are deduced from the perfect knowledge of the mutual strategies played simultaneously by the agents and the actual strategies played by the same
agents with an imperfect knowledge. In literal terms, since it is not useful here to begin with a general formalism, the transaction costs are understood as:

\[ \text{Transaction costs} = \text{payoff with complete information (minus) expected payoff under incomplete information} \]

3. The model

In our model, agents meet randomly and act pairwise; they choose an action before their matching round and the payoffs are assessed then. Two types of agents are considered here: lenders and borrowers. In our model we don't have a continuum of identical individuals but two different populations: the borrowers and the lenders who may or may not carry out information costs. So we have to deal with the problem of interpersonal comparisons of utility. Using transaction costs in Dhalman's (1979) sense, we model them on the expected functions of the agents' payoffs. Here, the sorting and incentive effects of loan contracts - permitting the bank to identify the borrowers - are not analyzed. Therefore, contrary to the theory of credit rationing (Stiglitz and Weiss 1981, 1986, 1992) we consider only the financed borrowers in the credit market. Our scope is not an investigation of borrowers' discrimination. Hence, they are not taking into account those borrowers to whom credit has been denied.

Specifically, the problem is related to whether the borrowers and/or the lenders will incur information costs. These costs may have a positive effect on the probability of success of the project. As a consequence, a priori, both type of agents are motivated to incur information costs. In the model, two informational sources are available: borrowers and lenders. Hence, each of them is able to acquire some information from his own and this impacts positively on the success probability of the project. We denote this probability by \( \rho (\cdot) \). This latter has two arguments, the first one refers respectively to the borrower (B) and the second to the lender (L) information set. Let be \( \phi(B) \) and \( \phi(L) \) their respective information set and \( \rho(\phi(B), \phi(L)) \) be the associated probability of success. As a simplification we will consider the information cost associated to each information set. We denote by \( c_B \) and \( c_L \) these information costs. Consequently, \( \rho(C_B, C_L) \) is the probability of success depending on information costs. A further simplification discretizes \( C_B \) and \( C_L \) to the values \{0,1\} where 0 is a null cost of information and 1 the maximum of information expenses. For example \( p(0,1) \) denotes the probability of success of the project when the borrower does not incur any
information cost (i.e. 0) but the lender does (i.e. 1). These elements lead us to the following definition.

**Definition**

i) When \( \rho(1,1) \rho(0,1) \rho(1,0) > \rho(0,0) \) the information is considered as redundant.

ii) \( \rho(1,1) > \rho(0,1) \rho(1,0) > \rho(0,0) \) or \( \rho(1,1) > \rho(1,0) > \rho(0,1) > \rho(0,0) \), the information is considered as complementary.

From the previous analysis under a redundant information it is easy to check that when both agents incur information costs and are plugging in the same information set as shown in appendix 1, the project's probability of success is not necessarily increasing. In the alternative case of complementary information this probability is increasing.

### 3.1. Basic assumptions

As noted before two populations are considered: the borrowers and the lenders. Each of them belongs respectively to the sets \( J \) and \( I \), with, \( j \in J \) and \( i \in I \). \( J \) and \( I \) are a continuum of identical agents defined respectively on \([0,1]\). The borrowers and the lenders meet randomly and each agent has to choose an action before meeting his partner. Each borrower chooses a pure strategy among a set \( \mathcal{M} \) of potential strategies and each lender chooses a pure strategy among a set \( \mathcal{N} \). Knowing that an action is associated to each individual, this may be described formally as:

**Borrowers:** \( s: I \rightarrow \mathcal{M} \), where \( s \) stands for an action,

**Lenders:** \( s: J \rightarrow \mathcal{N} \), where \( s \) stands for an action.

We denote their respective von Neumann - Morgenstern payoff function by \( (u) \) for the borrowers and \( (v) \) for the lenders. The payoff functions may be described as:

\[ p: \mathcal{M} \times \mathcal{N} \rightarrow \mathbb{R}, \]

is the payoff to an individual playing action \( a \in \mathcal{M} \) and \( b \in \mathcal{N} \), i.e \( p=(u,v) \). As a simplification, for further developments, let the generic letters \( k \) and \( k' \) standing for \( a \) or \( b \); \( p_{kk} \) is the payoff of an individual playing \( k \) when his partner has played \( k' \). The payment of the game may be represented by a bi-matrix \([A, B]\) with elements \( u_{ab}, v_{ba} \).
Let \( x_a \) and \( y_b \) be respectively the proportion of borrowers and lenders playing action \( a \) and \( b \), where, \( \mu \) is the Lebesgue measure on \( I \) and \( J \), then \( \chi_a = \mu[I^2 (a)] \) and \( \chi_b = \mu[I^2 (b)] \). The proportion of each population taking action \( a \) or \( b \) have to sum to 1 i.e. \( \sum_{x \in M} x_a = 1 \) and \( \sum_{y \in N} y_b = 1 \), (generically for \( x \) and \( y \) the proportion will be indexed by \( n \), \( \sum_{k \in K} n_k = 1 \)).

In our study, the costly improvement in the informational structure is a possible strategy. Hence, they have the choice to improve their information level by buying new information or to keep on with the same level of information.

Agents have the opportunity to become more and more informed after they meet their partner and every new information is transmitted to the partner in order to improve the project’s probability of success of the project. This point is very important. This is the consequence of our above development about the improvement of the information’s quality and the relationship between borrowers and lenders. Hence, to improve the position of each party, agents choose to support information cost expenses. Once they have acquired it, they transmit it freely.

After having defined the agents’ payoffs we can precise now the value of transaction costs. The maximum expected payoff of the agent when he is correctly informed is \( E(\Gamma) = \sum_{k \in K} n_k \Gamma_k \) where \( \Gamma_k = \text{Max}_k \ p_{kh} \), then the transaction cost is the difference between the maximum value the agent can expect if the strategy of his opponent is known and the true strategy he will play under incomplete information. That is:

\[
\tau_k(n) = \sum_{k \in K} n_k \Gamma_k - \sum_{k \in K} n_k \cdot p_{kh} \tag{A}
\]

Summing up on the whole strategies gives the average transaction cost in the population:

\[
\bar{\tau}(n) = \sum_{k \in K} n_k \tau_k(n) = \sum_{k \in K} n_k \Gamma_k - \sum_{k \in K} \sum_{k \in K} n_k n_k \cdot p_{kh} \tag{B}
\]
From this definition it is easy to check that transaction cost is always non-negative. It is important to mention that transaction costs are considered as opportunity costs.

### 3.2. The structure of the game

**The borrowers**

We consider a large population of borrowers on the set $J$, \( \{j = 1, 2, ..., J\} \). For simplicity we assume that each firm borrows the same amount $B$. We also assume that the project is not divisible (as the cost of the project is fixed if the borrower is unable to have the amount desired, the project will not be undertaken). If the firm borrows the amount $B$, and the interest rate is $r$, then we say the firm defaults on his loan if the return $R$ of the project is insufficient to pay back the promised amount $B(1+r)$, i.e., if $R < B(1+r)$ and in that case the firm gives to the bank the collateral $C$. Different firms have different probability distributions of returns. The bank cannot ascertain the riskness of a project. Finally we assume that the borrowers are risk neutral. The expected utility of a borrower is

$$
E(u) = u(X_1)p(.) + u(X_0)(1-.p) - \rho(C) = u[X_1 - X_0]p(.) + u[X_0 - C]
$$

where, if the project is succesful, $X_1$ is the end-of-period return to the borrower which is equal to $R - B(1+r)$, while if the project is unsuccessful, the end-of-period wealth is $X_0 = -C$. Where $c$ (c is a constant and can take two values 0 or 1) stands for information cost expenses which the borrower may or may not incur. Finally recall that $p(.)$ denotes the probability of project's success. From [1], the associated payoff matrix is:

$$
\begin{bmatrix}
q(0,0)(X_1 - X_0) + X_0 & q(0,1)(X_1 - X_0) + X_0 \\
q(1,0)(X_1 - X_0) + X_0 - 1 & q(1,1)(X_1 - X_0) + X_0 - 1
\end{bmatrix}
$$

or

$$
A = u = \begin{bmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} - 1 \end{bmatrix}
$$

This matrix expresses the payoffs of the borrowers according to the assumption that both type of agents incur or not information costs.
The lenders

In a similar way, we consider a large population of identical lenders on the set I (i.e 1, 2, ..., I). To simplify, we assume that lenders are risk neutral too. As before we consider two cases: either the borrower defaults on his loan if the investment’s return R is insufficient to pay back the promised amount B(1+r) i.e. R < B(1+r) and in that case he gives to the bank the collateral C, either the borrower does not default and he pays back the promised amount. The expected utility to the bank, v, (given a particular type of borrower) is:

\[ E(Y) = v(Y_1)\rho(.) + v(Y_0)[1 - \rho(.)] - v(c) = v(Y_1 - Y_0)\rho(.) + v[Y_0 - c] \tag{3} \]

where \( Y_1 \) is the return if the project is successful = B(1+r), \( Y_0 \) is the return if the project is unsuccessful = C, and where c are the information costs expenses which the lender may or may not incur. As before \( \rho(.) \) denotes the probability of project’s success.

By symmetry with the previous analysis, the associated payoffs’ matrix is:

\[
\begin{bmatrix}
q(0,0)(Y_1 - Y_0) + Y_0 & q(0,1)(Y_1 - Y_0) + Y_0 - 1 \\
q(1,0)(Y_1 - Y_0) + Y_0 - 1 & q(1,1)(Y_1 - Y_0) + Y_0 - 1
\end{bmatrix}
\]

or

\[
B = \begin{bmatrix}
v_{00} & v_{01} - 1 \\
v_{10} & v_{11} - 1
\end{bmatrix} \tag{4}
\]

We are defining a bi-matrix game the solution of which have to be identified.

3.3 Solutions of the game

Let A and B be the bi-matrices of the game, with respectively A the payoff matrices for the borrowers and B for the lenders:

\[
A = \begin{bmatrix}
u_{00} & u_{01} \\
u_{10} - 1 & u_{11} - 1
\end{bmatrix}, \quad B = \begin{bmatrix}
v_{00} & v_{01} - 1 \\
v_{10} & v_{11} - 1
\end{bmatrix}
\]
We denote $X$ with $X = (x_0, x_1)$, and $Y$ with $Y = (y_0, y_1)$, the borrowers' population and the lenders' population. The first term in each bracket represents the proportion of each population that does not support information expenses, and the second term the opposite case - i.e. information costs are incurred. These strategies may be viewed as determined by the probabilities $x_i$ and $y_i$ with which each proportion chooses their first pure strategy (the second pure strategy is then automatically chosen with probabilities $1-x_i$ and $1-y_i$, respectively). So we have $X = (x_1, 1-x_1)$, and $Y = (y_1, 1-y_1)$. As $X$, $Y$, are vectors representing mixed strategies for borrowers and lenders it is easy to verify that:

$$H_1(x_1, y_1) = XAY^T = \begin{bmatrix} u_{00} - u_{10} & u_{10} - u_{00} + 1 \\ u_{01} - u_{11} & u_{11} - u_{01} + 1 \end{bmatrix} x_1 y_1 + u_{11} - 1$$ \[5\]

$$H_2(x_1, y_1) = XBY^T = \begin{bmatrix} v_{00} - v_{10} & v_{10} - v_{00} + 1 \\ v_{01} - v_{11} & v_{11} - v_{01} \end{bmatrix} x_1 y_1 + v_{11}$$ \[6\]

By $x_i$, $y_i$ are both in the closed interval between 0 and 1 (including the end points), any situation in a bi-matrix 2x2 game is uniquely determined by a point $(x_i, y_i)$ of the unit square. Consequently, the payoffs for the borrowers and the lenders will be denoted for convenience respectively by $H_1(x_1, y_1)$, $H_2(x_1, y_1)$. To derive the equilibrium situation in a non-co-operative game we have to describe admissible situations for each of the players separately.

**The borrowers**

In order to have these situations for the borrowers it is necessary and sufficient that

$$H_1(1, y_1) = A_1 Y^T \quad XAY^T = H_1(x_1, y_1)$$ \[7\]

$$H_1(0, y_1) = A_2 Y^T \quad XAY^T = H_1(x_1, y_1)$$ \[8\]

[7] and [8] be satisfied. If we write the payoffs explicitly we have

$$(1-x_1)y_1[u_{00} - u_{10} + u_{11}] + (1-x_1)[u_{01} - u_{11}] \leq 0$$ \[9\]

$$x_1 y_1[u_{00} - u_{10} + u_{11}] + x_1[u_{01} - u_{11}] \geq 0$$ \[10\]

To simplify the notation we denote:
\[ G = [U_{U_0} - U_{U_1}, U_{U_0} + U_{U_1}] \quad \text{and} \quad H = [u_{x_0}, -1] \]

with this notation, inequalities [9] and [10] may be written as

\[ G(1-x_1)y_1 - H(1-x_1) \leq 0 \quad [11] \]

\[ Gx_0y_1 - Hx_1 \geq 0 \quad [12] \]

The structure of the solution set depends on the values of the invariants \( G \) and \( H \). \( G \) may be either positive or negative. Consequently, we can assess them knowing the nature of information. Hence, we consider the two identified cases in the following propositions

**Proposition 1:** Under the assumption of redundant information (definition), the game admits a set of solutions such as

The set of all solutions of the above system (i.e. [11, 12]) in the strip \([0,1]x(\infty, +\infty)\) consists of

(a) for \((0, y_1)\): \( y_1 \geq \frac{H}{Gr} = \lambda_1 \) because \( Gr < 0 \);

(b) for \((1, y_1)\): \( y_1 \leq \lambda_0 \) because \( Gr < 0 \);

(c) and for \((x_0, y_1)\) where \(0 < x_1 < 1\) entails that \( y_1 = \lambda_1 \) with \( \lambda_1 = \frac{1}{u_{x_0} - u_{x_1}} < 1 \), if \( 1 < u_{x_1} + u_{x_0} \) (proof in appendix 2).

When the information is complementary we can write the following proposition.

**Proposition 2:** Under the assumption of complementary information the solution set is

(a') for \((0, y_1)\): \( y_1 \geq \frac{H}{Gr} = \lambda_1 \) when \( Ge < 0 \);

(b') for \((1, y_1)\): \( y_1 \leq \lambda_0 \) because \( Ge < 0 \);
and

\((a'')\) for \((0, y_1)\): \(y_1 \leq \frac{H}{Gc} = \lambda_1\) when \(Gc > 0\);

\((b'')\) for \((1, y_1)\): \(y_1 \geq \lambda_b\) because \(Gc > 0\);

\((c')\) and for \((x_b, y_1)\) where \(0 < x_1 < 1\) then \(y_1 = \lambda_1\) with \(u_{11} - u_{00} < 1\).

According to the sign of \(Gc\) i.e. \(u_{11} - u_{00} > 1\) or \(u_{11} - u_{00} < 1\) (Proof in appendix 2).

The lenders

Taking into account the lenders' matrix payoff \(B = \begin{bmatrix} v_{00} & v_{01} - 1 \\ v_{10} & v_{11} - 1 \end{bmatrix}\) and with a similar analysis as previously we can determine the equilibrium conditions. The structure of the set of all solutions depends on the value of the invariants \(G'\) and \(H'\). Where as before

\[ G' = v_{00} - v_{10} + v_{11} \quad \text{and} \quad H' = v_{11} - v_{10} - 1, \]

with this notation we obtain the system

\[ G'(1-x_1)y_1 - H'(1-x_1) \leq 0 \quad [13] \]
\[ G'x_1 - Hx_1 \geq 0 \quad [14] \]

Two propositions are deduced from these results

\textbf{Proposition 3:} Under the assumption of redundant information, then the game admits a set of solutions such as

\(x_1 = n_1\) and \(n_1 = \frac{1}{v_{10} - v_{00}}\) is a mixed strategy if \(v_{10}v_{11} > 1\) (proof in appendix 2).

\textbf{Proposition 4:} Under the assumption of complementary information the solution sets is

\((a')\) for \((x_b, 0)\): \(x_1 \geq \frac{H'}{G'} = n_1\) when \(G' < 0\);
The model has been voluntarily restricted to a simple game between two populations with two values for information costs, are equal to 1 or 0. As a conclusion, the equilibrium of the game is depending upon the nature of the information (complementary or redundant) the agents are liable to reach.

In this type of game, the sets of all admissible situations for borrowers (respectively for lenders) depend only on the parameters G and H (respectively G' and H') of their payoff matrix A (respectively B). Thus, H for borrowers represents the probability with which lenders choose their first pure strategy in a certain mixed strategy game for these agents. In other words if H falls into the interval (0,1) it may be interpreted as a mixed strategy for the other agent (i.e. lenders). More explicitly, it appears that the equilibrium behavior of the borrowers/lenders is directed towards the minimization of the payoff for the opponent than towards the maximization of their own payoff\(^1\).

We will now analyse how these differences influence the level of transaction cost. Upon the nature of information, lenders and borrowers alike may behave strategically in order to minimize the transaction cost losses.

### 3.4 Transaction costs applied to the credit market

Using the formula given previously we can calculate the transaction costs for each population and for each strategy (to incur or not information costs). The transaction costs can be determined for each population in terms of proportion of agents, (this proportion for lenders is equal to \(\lambda\)) and for
borrowers to \( n_i \) who choose to incur such costs. So, we have two different cases, more precisely two different values of transaction costs corresponding to the two strategies.

**The borrowers**

Taking into account the proportion of the lenders who may or may not incur information costs we can calculate the borrower's transaction costs. We consider the different natures of information according to the information set. Hence, we begin first by a complementary information, then we study the consequences of a redundant information.

**i) Complementary information**

When the information is complementary this means that either \( \varrho(1,1) > \varrho(0,1) \) or that \( \varrho(1,1) > \varrho(1,0) \). We have to distinguish two cases.

ia) First when the borrower incur information costs. In that case we denote by \( \tau_1 \) the transaction costs. From formula (A) they are equal to:

\[
\tau_1 = u_{11} - \lambda_0(u_0 + (1 - u_0)) \lambda_1 = (1 - \lambda_1)(u_{11} - u_{10}) = \lambda_0(u_{11} - u_{10}) = \lambda_0 D_1 \tag{15}
\]

To simplify the notation we denote \( (u_{11} - u_{10}) = D_1 \)

ib) In the alternative case (no information costs) we denote transaction costs by \( \tau_0 \)

\[
\tau_0 = u_{11} - (u_0 \lambda_0 + u_0 \lambda_1) = (u_{11} - u_{10}) - \lambda_0(u_{01} - u_{00}) = D_2 \lambda_1 D_3 \tag{16}
\]

by simplification \( D_2 = (u_{11} - u_{10}) \) and \( D_3 = (u_{01} - u_{00}) \)

In that case these costs are positive even without information costs expenses. From formula (B), the average transaction cost for the borrowers' population in the presence of a complementary information is

\[
\bar{\tau} = n \tau_1 + n_0 \tau_0 = n \tau_1 + (1 - n_1) \tau_0 \Rightarrow \bar{\tau} = n_1 (1 - \lambda_1) D_1 + n_0 D_2 \lambda_1 D_3 + D_2 \tag{17}
\]

From these results it is easy to see that transaction costs are considered as opportunity costs
ii) Redundant information

On the opposite case [i.e. when the information is redundant \( \rho(1,1) \sim \rho(0,1) \sim \rho(1,0) \)] and when the borrower decides to incur transaction costs we can calculate these costs as before.

iia) If \( \tau_{iR} \) are the transaction costs incurred by the borrower when the information is redundant, then

\[
\tau_{iR} = u_{i,-}\lambda_i(u_{i,-} - 1) + \lambda_i(u_{i,-} - \lambda_i) = u_{i,-} - \lambda_i - u_{i,-} + \lambda_i \Rightarrow \tau_{iR} = 1
\]

iib) when such costs are not incurred

\[
\tau_{0R} = u_{i,-} - \lambda_i = U_{01} - (1-\Lambda)U_{00} + \Lambda U_{01} = U_{01} - \Lambda U_{00} \Rightarrow [18]
\]

We then calculate the average transaction costs of the borrowers

\[
\bar{\tau}_R = n_1\tau_{1R} + n_0\tau_{0R} \Rightarrow \bar{\tau}_R = n_1 + n_0\tau_0(U_{01} - U_{00}) \quad [19]
\]

The lenders

In a similar way we can calculate the transaction costs of the lenders when two strategies are available: may incur or may not incur transaction costs.

i) Complementary information

ia) Let consider the case when the lenders decide to incur such costs and denote these costs by \( \tau'_1 \)

\[
\tau'_1 = (v_{11} - v_{00})n_0 + v_{11}n_1 = (1-n_1)(v_{11} - v_{01}) = n_0(v_{11} - v_{01}) = n_0Z_1 \quad [20]
\]

To simplify the notation we denote \( (v_{11} - v_{01}) = Z_1 \)

ib) if the lenders do not incur transaction costs \( \tau'_0 \)

\[
\tau'_0 = v_{11} - (v_{01}n_0 + v_{10}n_1) = v_{11} - v_{00}(1-n_1) + v_{11}n_1 \Rightarrow \tau'_0 = (v_{11} - v_{00})n_1 = Z_2n_1Z_3 \quad [21]
\]
4. Transaction costs, nature of information and strategic behavior

Transaction costs are highly dependent on the nature of the information the agents are faced with and the strategies they adopt. It has been shown that reducing uncertainty by gathering information does not exclude strategic behavior about the way to become informed. This involves that transaction costs could arise not only because information is lacking, but equally agents are looking for how to be informed without information expenses. To show this point, let us analyze the different strategic behaviors.

In table 1 and 2 below are gathered results for the specific case of agents using pure strategies: $\lambda_1$ or $\lambda_0$ and $n_1$, $n_0$. We recall that they represent the proportions of the lenders/borrowers who may incur or may not information costs. At the top of the tables is presented the transaction cost born by borrowers and lenders under redundant information (table 1 and 2) and at the bottom the transaction costs under a complementary information (table 1 and 2). These matrices may be considered as some bi-matrices payoff. The analysis of these polar cases is interesting because it makes appear specific strategic behaviors and consequently, potential equilibrium.
a) Redundant information

When both players decide not to incur information cost, they bear together opportunity costs corresponding to $v_{10} - v_{00}$ for the lenders, and $u_{10} - u_{00}$ for the borrowers. The most favorable case for each one is the conflict case, in which the opponent bears the information cost. More precisely for the borrowers the most favorable situation is when we have $\lambda$ and $\nu$. The reverse is true for the lenders. In such a context the population of agents (namely of the borrowers) who does not incur information costs, transaction costs or opportunity costs are equal to zero. We are clearly facing a prisoner dilemma conjecture. This case may appear comparing the opportunity cost of the non-cooperative solution and the co-operative one.

Hence, when the agents are liable to dispose the same kind of information, a conflicting situation appears and the probabilities of a prisoner dilemma are not zero. So, agents may adopt a free rider behavior.

b) Complementary information

When agents know the nature of information, their self-interest is to co-operate and hence to spend money for information. In such a context, they are certain not to incur transaction costs, i.e. for $\lambda$ and $\eta$, transaction costs are zero. More precisely, in a context of complementary information agents best response is to incur information costs and consequently transaction costs or opportunity costs for both populations of agents are equal to zero. It is obvious in all other situations they incur opportunity costs.

5. Conclusion

The aim of our paper was to provide a theoretical framework for transaction costs linked to the nature of information in the credit market. More precisely, this paper has examined how the quality/nature of information (redundant or complementary) influences the strategic behavior of agents and on transaction costs. It is useful to remind that transaction costs are considered as resource losses. It has been sketched that under a complementary information assumption, economic efficiency increases when every type of agents cooperates and that tends to induce transaction costs equal to zero. In the opposite, i.e. under redundant information assumption, each agent has interest not to incur information costs, leaving his opponent seek
for costly information. In order to minimize individual resource losses, a free rider behavior issues on some prisoner dilemma.

It's important to underline that in this paper the role of institutions is not analyzed. In literature institutions and transaction cost are linked: institutions emerge to economize on transaction costs. In our framework the existence of institutions is not necessary in order to minimize transaction costs.

It might still appear that the analytical framework of this paper is too narrow to capture all of the various situations that have been ascribed in the literature of information theory. We are more precisely referring to the notion which was first noted by Blackweel in 1953 "more informative versus less informative message". To interpret the "informativeness" property we have to introduce in the model prior beliefs and some possible belief revisions using the Bayes Theorem. In order of to complete the study we have to analyze the individual's choice in a dynamic context: instead of taking immediate terminal action, the agent prefers first to acquire better information, with the aim of improving the ultimate terminal decision to be made. We leave the development of this generalization to future research.
Appendix 1

In order to clarify the concept of the nature of information, let us examine a simple example. We start from the notion of standard information set. So, considering the outcomes of three tosses (H for Head and T for Tail), the outcome set is

\[ \Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\} \]

Some \( \sigma \)-algebra from \( \Omega \) may be distinguished, and particularly the following filtration such as

\[ H_0 = \{ \emptyset, \Omega \} , \]

\[ H_1 = \{ \emptyset, \Omega, B_H, B_T \} \]

where \( B_H, B_T \) stand respectively for “the first toss is Head” and “the first toss is Tail”

\[ H_2 = \{ \emptyset, \Omega, B_H, B_T, B_{HH}, B_{HT}, B_{HT}, B_{HH} \cup B_{TH}, B_{HH} \cup B_{TT}, B_{HT} \cup B_{TT}, \} \]

\[ B_{kk}, \text{ stands for the first toss is } k \text{ and the second is } k', \]

\( B_{kk}' \) is the complementary of \( B_{kk} \)

\[ H_3 = \text{All the subsets from } \Omega \]

Supposing that a player’s activity is to guess the true outcome of the three tosses and that its reward depends on the result. A right guess gives a payoff of \( G \) (\( G > 0 \)), while a wrong one is 0 rewarded. Before giving the definitive answer the player has the possibility to get informed partially about the results by buying some piece of information. This means that he has the opportunity to look at the \( \sigma \)-algebra corresponding to the filtration \( H_1, H_2, H_3 \). Clearly, \( H_0 \) may be excluded from the analysis because the information is trivial. \( H_3 \) means that the agent could buy full information, by assumption that cost exceeds the payoff \( G \) and is impossible. Hence, in this particular example, the remaining possible \( \sigma \)-algebra are \( H_1, H_2 \). The agent will buy either a rough level of information (being informed of the
outcome of the first toss with \( H_1 \), or a thinner one with \( H_2 \), (the two first level). With \( H_1 \) the probability to be wrong is 3/4, while with \( H_2 \), the probability is 1/2. The choices between the information level are depending on the expected payoff.

Here the information set is quite narrow. Because of this, we can give a good illustration of what the notion of redundancy of information looks like. Now we suppose that, two independent agents are interested in the outcome \( G \) (because, for example, the success of the first one depends on the success of the second one as in a borrower/lender relationship). Supposing, furthermore, for simplicity that they have the same level of information \( H_1 \). They communicate to each other the further information they may obtain. If both of them buy information from \( H_2 \) they will be informed together of the outcome of the first and second outcome. They will dispose of the same redundant information.

In the case where \( \Omega \) is larger, then it is possible to conceive a thinner \( \sigma \)-algebra and consequently higher level of possible information. In such a context, and because of the level of the agents’ resources, the information set may be complementary: they may obtain different information allowing to increase the probability of success of the project.
Appendix 2

The set of all admissible solutions for borrowers is the intersection of the set of solutions of the system [11], [12] with the unit square $[0,1] \times [0,1]$. Before describing all the solutions of this system we can enumerate separately solutions of this system with $x=0$, $x=1$ and $0<x<1$:

- if $x=0$, then $Gy-H \leq 0$ and [12] is automatically satisfied,
- if $x=1$, [11] is an identity and $Gy-H \geq 0$.

Now let $0<x<1$, then $Gy-H=0$.

**Proof of proposition 1**

i) We have to establish first that redundancy of information [proposition 1 i.e. $\rho(1,1) \rho(0,1) \rho(1,0) > \rho(0,0)$] induces $u_{iy} = u_{xj} = u_{xj} > u_{xy}$. This may be checked immediately from matrix [2]. Hence, under the assumption of redundant information we have

$$G = Gr = [u_{00} - u_{10}] < 0 \quad \text{or} \quad G = GR = [u_{00} - u_{01}] < 0, \quad \text{and} \quad H = [u_{11} - u_{00} - 1], \quad \text{then i) of Proposition 1 ensues.}$$

The ii) of proposition 2 may be obtain explicitly, considering the values of $G$ and $H$, we have first $y_i = \lambda_i$, then $\lambda_i = \frac{[u_{11} - u_{01} - 1]}{u_{00} - u_{01}}$.

$\lambda_i$ may be considered as a mixed strategy if the only if: $0 < \lambda_i < 1$. Hence, we have to note that because $u_{10} = u_{01} = u_{11} > u_{00}$ that $\lambda_i = 1/[u_{01} - u_{00}]$, with $0 < \lambda_i$, because $G = Gr = [u_{00} - u_{01}] < 0$, and for $\lambda_i$, then $1 < u_{01} - u_{00}$.

**Proof of proposition 2**

As previously - for proposition 2 - the complementary information case - we have to show that $\rho(1,1) > \rho(0,1), \rho(1,0) > \rho(0,0) \Rightarrow u_{11} > u_{01}, \quad u_{11} > u_{10}, \quad u_{11} > u_{01}, \quad u_{01}, \quad u_{11} > u_{00}$. This may be done following the same line of argument with matrix [2]. Consequently $G = Ge = [u_{00} - u_{10} - u_{01} + u_{11}]$, then two cases are possible

- i) either $Ge = [u_{00} - u_{10} - u_{01} + u_{11}] < 0$, $[u_{00} + u_{11}] < [u_{10} + u_{01}]$, and, conversely
ii) or $G_c = |u_{00} - u_{10} + u_{11}| > 0, \ [u_{00} + u_{11}] > [u_{00} + u_{01}]$.

The solutions for both cases are depending on $H = [u_{11} - u_{01} - 1]$ in which $H > 0$ if $=[u_{11} - u_{01} > 1]$ and $H < 0$ if $=[u_{11} - u_{01} < 1]$.

i) $\frac{H}{G_c} > 0$, if $H > 0$ and $G_c > 0$ or $H < 0$ and $G_c < 0$;

ii) $\frac{H}{G_c} < 0$, if $H > 0$ and $G_c < 0$ or $H < 0$ and $G_c > 0$.

Considering the fact that only i) issues on acceptable mixed strategies, it appears that $G$ and $H$ must have the same sign. This has consequences on the mixed strategy solutions.

a) The result comes from the i) and ii) considerations, the result is immediate.

b) The explicit solution is such that $H/G < 1$ hence $H < G$ and the result ensues (i.e. $u_{11} - u_{00} < 1$). The same argument is used for the case $H/G < 0$ taking into account that $H = [u_{11} - u_{01} - 1] < 0$ or $H = [u_{11} - u_{01} - 1] > 0$

**Proof of proposition 3**

The demonstration follows the same argument than proposition 1.

**Proof of proposition 4**

Similar to proposition 2.
TABLES

Table 1
Borrowers payoff in the case of redundant information (top) and complementary information (bottom).

<table>
<thead>
<tr>
<th>$\lambda_1$ (redundancy)</th>
<th>$\lambda_1$ (complement)</th>
<th>$\lambda_0$</th>
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<td>$n_0-1$</td>
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<td>$u_1+u_0$</td>
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<td>0</td>
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<td>$u_1+u_0-1$</td>
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Table 2
Lender's payoff in the case of redundant information (top) and complementary information (bottom).

<table>
<thead>
<tr>
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<th>$\lambda_1$ (complement)</th>
<th>$\lambda_0$</th>
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<td>$\nu_1+\nu_0-1$</td>
<td>$\nu_1+\nu_0-1$</td>
</tr>
</tbody>
</table>
Notes


2. Considered te banking firm theories.


4. See for example, Marschak (1968), Marschak and Migasava C. (1968), Marschak, and Radner, R. (1972), etc.

5. For example, if on Monday you tell somebody "I guess that tomorrow is Tuesday", you do not communicate him a very useful information because evidently the probability of the event "after Monday is Tuesday" is one.

6. The pioneering work of this author has highly inspired the information theory applied to economics. Originally his theoretical developments have been applied to the statistical characteristic of data and communication systems and coding theory.

7. His main problem was to efficiently transmit information over a noisy communication channel and he dealt with this problem by creating the first mathematical theory of entropy.

8. Formally if we consider a random variable X which must take on one of the values $X_1, \ldots, X_n$ with respective probabilities $P_1, \ldots, P_n$. Then, the expected amount of surprise we shall receive upon learning the value of X is $H(X) = - \sum p_i \log p_i$.

9. In the literature this treatment is far from being new, we can refer for instance, to Rubinstein and Wolinsky (1987), Kandori (1992), Wärneryd (1994), etc.

10. The probability $q(c_l, c_t)$ is asymptotically increasing in each argument towards 1 without reaching it: that means that full information cannot guarantee full success.

11. For an exposition of some separately solutions of this system see appendix 2.

12. Similarly the same reasoning stands for lenders.

Bibliography


