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FISCAL POLICY IN AN ENDOGENOUS GROWTH MODEL WITH HORIZONTALLY DIFFERENTIATED INTERMEDIATE GOODS

By

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Theoretical and empirical work in endogenous growth theory suggests that market economics underinvest in scientific and technical research and achieve growth rates that are lower than the socially optimal. Among the suggested explanations are the positive productivity spillovers associated with the nature of knowledge as a nonrival and only partially excludable good, the monopolistic pricing of the product of R&D, and the related issue of imperfect patent protection and imitation. This paper studies the above distortions in a model where technological progress is the product of research activity and takes the form of expansions in the variety of intermediate goods. The market and the socially optimal solutions are presented and the first-best fiscal policy is considered for each case of market failure (JEL: Classification H32, 033, 038, O40, 041).

1. Introduction

Long-run economic growth is characterized by continuing increases in per capita output (GDP). Neoclassical growth theory, introduced by Solow (1956), and Swan (1956), and later elaborated by Uzawa (1964), Cass (1965), and Koopmans (1965), has made clear that an ever increasing per capita output is possible only as the result of sustained increases in factor productivity. However, with only few exceptions such as Arrow (1962), Uzawa (1965), Shell (1966), and Sheshinski (1967), no systematic attempt was made to study changes in factor productivity as an integral part of the process of economic growth. Neoclassical growth models focused on the accumulation of factors of production, and productivity gains were treated as an exogenous, unexplained "residual".

Since the mid-eighties concern about the slowing productivity growth in virtually all industrialized economies has sparked off substantial interest in the forces that influence the level and the rate of change of factor productivity. Starting with Romer (1986, 1987, 1990), Lucas (1988), Barro (1990), Segerstrom, Anant, and Dinopoulos (1990), Segerstrom (1991), Rebelo (1991), Grossman and Helpman (1991), and Aghion and Howitt (1992), the result has been an extensive literature collectively referred to as *endogenous growth theory*. To uncover the main determinants of productivity growth, endogenous growth theory focuses on the accumulation of human capital and investment in infrastructure and research and development (R&D).

Investment in R&D is the driving force of technological progress. *Technology* refers to a society's accumulated knowledge on how to produce goods and services. Technological progress is the advancement of this knowledge. It is reflected in the introduction of new or impoved productive inputs and of new ways to combine existing ones. The effects of technological progress is broadly consistent with the experience of all advanced economies over long periods of time: per capita output is steadily increasing and the process shows no clear tendency to peter out.

Endogenous growth models typically treat the advancement of technological knowledge as an economic good that is the outcome of purposive R&D activities. Private firms have an incentive to invest in R&D because successful innovations, being protected by patents, bear potentially significant monopoly rents. However, empirical evidence demonstrates that the social returns to R&D considerably exceed the respective private returns (Griliches 1992, Keely and Quah 1998). As a result, market economies underinvest in scientific and technical research and achieve growth rates that are lower than the socially optimal.

Several features have a share in explaining this difference. Important among them are the positive productivity spillovers associated with the nature of knowledge as a nonrival and only partially excludable good (Mansfield, 1985, Caballero and Jaffe, 1993), the monopolistic (markup) pricing of the product of R&D, and the related issue of imperfect patent protection and imitation (Mansfield, Schwartz, and Wagner, 1981, Mansfield, 1986, and Levin, Klevorick, Nelson, and Winter 1987).

The present paper studies the above market failures and introduces fiscal policy in a model where technological progress takes the form of expansions in the variety of intermediate goods. This specification, based on Romer (1990), perceives technological progress as a fundamental innovation akin to the generation of a new industry. A description of the model follows in the next section. In section 3 the growth equilibrium of the decentralized economy is found, while section 4 looks at the case of a planned economy. The main result is that the growth rate of the decentralized economy falls short of the socially optimal rate. In section 5 the distortions responsible for the less-than-optimal growth performance of the decentralized economy are examined, and the first-best fiscal policy is considered for each case. Section 6 summarizes and addresses a number of related issues and extensions of this study.

2. The Model

The economy consists of two sectors. In the first, a large number of identical price-taking firms produce a homogeneous final (consumption) good. The aggregate production function is

$$Y = L_Y^{\alpha} \int_{j=0}^{A} X_j^{1-\alpha} \mathrm{d}j \qquad 0 < \alpha < 1 \qquad (1)$$

where Y is total output, L_Y is the amount of labor allocated to final good production, and X_j are the quantities used from a continuum of intermediate (producer) goods, X_j, $j \in [0, A]$.[1] An important feature of equation (1) is that the marginal product of an intermediate good, $\frac{\partial Y}{\partial X_j} = L_Y^{\alpha}(1-\alpha)\, X_j^{-\alpha}$, is independent of the quantity used of any other. This implies that no substitutability or complementarity exists between intermediate goods. Further, the marginal product is monotonically decreasing with X_j, starting at infinity for $X_j = 0$. Then, if all intermediate goods are available at finite prices, final good producers will use all A types of them. The available variety of intermediate goods indentifies the level of technology. Specificall, if $\{X_1, X_2, ..., X_A\}$ is the set of currently existing intermediate goods, them A can be thought of as an index that represents the present level of "know-how".

Intermediate goods are invented and manufactured in the second sector of the economy. Entry in this sector is free. However, each firm j must first engage in R&D and incur an overhead research cost C_R, to invent its own intermediate good, X_j. Once the design of a new good has been developed, the firm can manufacture the good and sell it to final good producers. Intermediate goods are manufactured with the same production technology as the consumption good. Thus, one unit of the consumption good must be given up to free the resources that will be instead allocated to producing one unit of some intermediate good.

New designs are being developed according to

$$\frac{dA}{dt} \equiv \dot{A} = \delta A L_A \qquad \delta > 0 \qquad (2)$$

where $\dot{A}$ is the number of new designs per unit of time, L_A is the amount of labor employed in research, and δ is a research productivity parameter. According to (2), the number of inventions at any instant increases with the stock of existing intermediate goods and the amount of labor allocated to research. Observe that only the technological knowledge associated with the variety of intermediate goods, A, enters the above equation, and not intermediate goods themselves. This is an assumption that approximates the idea that R&D is more labor intensive than the manufacture of consumables and intermediate goods.

Inventions are protected by patents that provide the owner with exclusive production and distribution rights over the respective capital good. To allow for imperfect patent protection it will be assumed here that intermediate goods transform from monopolized to competitive according to a *Poisson process* with *intensity* $\mu > 0$. Hence, if a good is monopolized at time t, then the probability of it still being monopolized up to and including some later time $s = t + h$, $h \geq 0$, is equal to $e^{-\mu.(s-t)} < 1$.[2]

People in this economy are each endowed with one unit of labor which they supply inelastically to either the final good or the intermediate good sector. Further, they face the usual income and wealth constraints and choose a consumption path so as to maximize their total discounted utility over an infinite time horizon. The labor force, L, coincides with the economy's population and is fixed with

$$L = L_Y + L_A \qquad (3)$$

The representative consumer's preferences are described by the *constant elasticity of intertemporal substitution* instantaneous utility function

$$U(c)=\begin{cases}\frac{c^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma>0,\ \sigma\neq 1\\ \ln c & \text{if } \sigma=1\end{cases} \tag{4}$$

where c is per capita consumption, and parameter σ characterizes consumer's willingness to shift consumption between different points in time.

Finally, all prices are measured in units of the consumption good.

3. Growth in the Decentralized Economy

3.1. The Final Good Sector

Each firm in the final good sector determines its demand for factors of production in every period by maximizing the profit function

$$\Pi^F(L_Y, X_j)=L_Y^{\alpha}\int_{j=0}^{A} X_j^{1-\alpha}\,\mathrm{d}_j - W_{LY}L_Y - \int_{j=0}^{A} P_{Xj}X_j\,\mathrm{d}_j \tag{5}$$

where W_{LY} is the wage rate of labor employed in the final good sector, and P_{xj} is the price of intermediate good X_j. According to (5) all intermediate goods are non-durable and, therefore, new units of these goods must be purchased by final good producers in each period.

The necessary conditions yield the demand functions for labor and intermediate goods as

$$L_Y=\frac{\alpha Y}{W_{LY}} \tag{6}$$

and

$$X_j=\left(\frac{1-\alpha}{P_{xj}}\right)^{1/a} L_Y \tag{7}$$

3.2. The Intermediate Good Sector

In the intermediate good sector, after inventing and manufacturing intermediate good X_j, firm j faces a demand for its product given in equation

(7), and its objective in every period will be to maximize total operating profit, $P^j(P_{xj})$, where

$$P^j(P_{xj})=(P_{xj}-1)\left(\frac{1-a}{P_{xj}}\right)^{1/a}L_Y \tag{8}$$

This is a classic monopoly pricing problem of a firm that produces at constant marginal cost and faces a demand curve with constant price elasticity.[4] The solution yields the profit-maximizing price as

$$P_{Xj}=P_X=\frac{1}{1-\alpha}>1 \tag{9}$$

Notice that P_{xj} is a constant markup on the marginal (and average) cost of production. This cost is 1, and the markup is $M=\frac{1}{1-a}=\frac{1}{1-\frac{1}{/\varepsilon/}}$, where $\varepsilon=-\frac{1}{\alpha}$ is the price elasticity of demand. From equations (7) and (9) the quantity demanded X_j, of intermediate good X_j, is

$$X_j = X = (1-\alpha)^{2/\alpha}LY \tag{10}$$

Equations (9) and (10) show that the equilibrium price and quantity are the same for all intermediate goods. The reason is that all intermediate goods are produced at the same cost and are used symmetrically in the production of the consumption good. Given this, the aggregate production function (1) may be written as

$$Y = A^{\alpha}L^{a}_{Y}(AX)^{1-\alpha} \tag{11}$$

where AX is the total quantity of intermediate inputs. Since labor and technology share the same output elasticity, technological progress is *Harrod-neutral.* For given A, production exhibits constant returns to scale in labor, L_Y, and the aggregate quantity of intermediate inputs, AX. For given L_Y and AX, the term A^a indicates that output increases with the level of technology. Consequently, production is characterized by increasing returns to scale in technology, labor and intermediate goods taken together. This is what necessitates imperfect competition in the model. In the presence of increasing returns, average cost is always above marginal cost. Then, a competitive market with marginal cost pricing would lead to losses, and no firm would be willing to invest in R&D, develop a design and manufacture the respective intermediate good. Firms will enter the market only if they can charge a price higher than marginal cost that will allow them to recoup the research cost of inventing a capital good.

Equations (10) and (11) can be combined further to yield

$$Y = AL_Y\ (1-\alpha)^{2(1-\alpha)/\alpha} \tag{12}$$

If L_Y is constant, as it will be shown to be true in long-run equilibrium, the above equation implies that output is proportional to the level of technology. Consequently, it is

$$\gamma_{[Y]} = \gamma_{[A]} \tag{13}$$

where $\gamma_{[Z]}$ denotes the growth rate of any variable Z, that is, $\gamma_{[Z]} \equiv \frac{\dot{Z}}{Z}$. Further, let $C = Lc$ denote aggregate consumption, and notice from (11) that AX units of final output must be allocated to the production of intermediate goods. Then, the economy's resource constraint is

$$Y = C + AX \tag{14}$$

and implies

$$C = AL_Y\ [(1-\alpha)^{2(1-\alpha)/\alpha} - (1-\alpha)^{2/\alpha}] \tag{15}$$

Since it is $0 < \alpha < 1$, the expression in equation (15) always results in a positive consumption level. In addition, with a constant L_Y, aggregate consumption is also proportional to the level of technology and equation (13) extends to

$$\gamma_{[Y]} = \gamma_{[C]} = \gamma_{[A]} = \gamma^{(de)} \tag{16}$$

where $\gamma^{(de)}$ denotes the rate of growth of the decentralized economy.

Returning to equations (8) and (9), one may write the (maximum) per period operating profit of the inventor / manufacturer of a capital good as

$$P = \alpha(1\text{-}\alpha)^{(2\text{-}\alpha)/\alpha}L_Y \tag{17}$$

Then, the expected at time t present value of the flow of all per period future profits is

$$E(V)=\frac{a(1-a)^{(2-a)/a}L_Y}{r+\mu} \tag{18}$$

where the interest rate, r, measured in units of the consumption good, and the instantaneous probability of losing monopoly power, μ, are used as discount factors.[5]

On the other hand, the number of researchers required for an invention is

$$\frac{L_A}{\dot{A}}=\frac{1}{\delta A} \tag{19}$$

resulting in a research cost per invention equal to

$$C_R=\frac{W_{LA}}{\delta A} \tag{20}$$

where W_{LA} is the wage rate for labor employed in R&D. Under the assumption that labor can move costlessly between sectors of employment, the wage rate of researchers must equal the wage rate received in final good production. Then, it is

$$W_{LA} = W_{LY} = W = \alpha A(1\text{-}\alpha)^{2(1\text{-}\alpha)/\alpha} \tag{21}$$

where the last part of (21) results from equations (6), and (12). Based on the above, the research cost of inventing a new intermediate good is

$$C_R=\frac{a(1-a)^{2(1-a)/a}}{\delta} \tag{22}$$

Long run equilibrium with technological progress and positive growth dictates that

$$E(V) = C_R \tag{23}$$

If $E(V) < C_R$, no firm would be willing to invest in the development of a new capital good. As a result, no resources would be devoted to R&D, and A would be constant. Similarly, continuous entry in the intermediate good sector would exist, and an ever increasing amount of resources would be channeled into R&D, for as long as $E(V) > C_R$[6]. Clearly, the monopolistic operating profit $P > 0$ is necessary for technological progress as firms need to recoup the sunk research cost *CR*. However, free entry in the intermediate good sector implies a long run equilibrium with no economic profit: $\Pi^1 = E(V)\ C_R = 0$. Following this, equations (2), (3), (16), (18) and (22) can be combined to yield the real interest rate as

$$r = \delta(1 - a)\ L - \mu - (1 - \alpha)\gamma^{(de)} \tag{24}$$

Equation (24) depicts an important aspect of the productiion side of the model: there is a negative association between the interest rate and the rate of growth of output. The link between these two variables is investment in the development of new capital goods. From (18), a higher interest rate reduces the present value of the flow of monopoly profits earned by intermediate good firms. This lowers the private rate of return to R&D and weakens the incentive to invest in research. Then, employment in the intermediate good sector will decline, and so will the rate of technological progress and the rate of growth of output and consumption.[7]

3.3. Consumers and Utility Maximization

Turning to consumers, dynamically optimal behavior with interest rate r requires that an extra unit of consumption at time t yield the same utility as the discounted equivalent of e^{rh} additional units of consumption at time $t + h$. For an algebraic statement it is, $U'[c(t)] = e^{-ph}e^{rh}U'[c(t+h)]$, where $U'[c(r)] \equiv \dfrac{dU}{dc(\tau)}$ denotes marginal utility of consumption at time τ, and ϱ is the rate of time preference that is used to discount future flows of utility. It preferences are described by equation (4) with $\sigma \neq 1$, the above condition becomes $c(t + h) = e^{1/\sigma(r-\rho)h}c(t)$. The implid optimal rate of consumption growth is expressed in the familiar Euler *equation*

$$\gamma_{[C]} = \frac{1}{\sigma}(r-\varrho) \tag{25}$$

whose satisfaction is necessary for intertemporal utility maximization.[8] From (25) and (16), the interest rate is found to be

$$r = \sigma\gamma^{(de)} + \varrho \tag{26}$$

Equation (26) is the direct analogue of equation (24) for the consumption side of the model. It can be thought of as a dynamic saving function that establishes a positive association between the interest rate and the rate of growth of output. A higher interest rate makes future consumption relatively cheaper and, thus, leads consumers to cut back on current consumption in favor of more consumption in the future. This will release resources from production in the final good sector. The allocation of these resources to research activities will result in an increase in the rate of technological progress and, consequently, in the rate of growth of output and consumption.

3.4. Growth Characteristics of the Decentralized Economy

From the above analysis it is evident that when the interest rate is low, R&D investment is attractive but saving is not. Of course, the opposite is true at high levels of the interest rate. Saving and investment decisions will be compatible only at the interest rate at which both equations, (24) and (26), are satisfied. Together, these equations yield the equilibrium rate of growth of the market economy

$$\gamma^{(de)} = \frac{\delta L - M(\mu+\varrho)}{1+\sigma M} \tag{27}$$

and the equilibrium real interest rate

$$r^{(de)} = \frac{\sigma(\delta L - \mu M)+\varrho}{1+\sigma M} \tag{28}$$

According to (27), the growth rate increases with population, L, and research productivity, δ. It decreases with the monopolistic markup M in the price of intermediate goods, the probability of loss of monopoly power, μ, and the "degree of people's impatience", ρ.

Finally, from equations (2), (16), and (27), the amount of labor allocated to R&D is

$$L_A^{(de)} = \frac{\delta L - M(\mu + \varrho)}{\delta(1 + \sigma M)} \tag{29}$$

4. Growth in the Planned Economy

Here the economy is run by a social planner whose aim is to maximize the representative consumer's total discounted utility over an infinite time horizon. From a social perspective the planner makes all the allocative decisions optimally. Thus, at the microeconomic level, the planner imposes perfect competition throughout the economy, satisfies the conditions for efficient production, and internalizes all externalities. The macroeconomic decisions must satisfy the economy-wide technological and resource constraints given in equations (2), (3), (11), and (14). Assuming the instantaneous utility function given in (4) with o σ 1, the planning problem is to

$$\max_{c,X,L_A} \int_{t=0}^{\infty} e^{-pt} \frac{c^{1-\sigma} - 1}{1-\sigma} \mathrm{d}t \tag{30}$$

subject to

$$(L - L_A)^{a} A X^{1-\alpha} - AX - Lc \geq 0 \tag{31}$$

and

$$A = \delta A L_{\mathrm{A}} \tag{2}$$

Of course, the population size, L, and the initial level of technology, A, are taken as given.

This is a typical dynamic optimization problem with three control *variables*, c, X, and L_A, and one *state variable*, A. The solution is reached by setting up the Lagrangian

$$L=\frac{c^{1-\sigma}-1}{1-\sigma}+\lambda_A\delta AL_A+\lambda_Y[(L-L_A)^a AX^{1-a}-AX-L_c] \tag{32}$$

where λ_A and λ_Y are the current value Lagrange multipliers. They measure the change in maximum attainable utility, in values of period t, that is brought about by a unit increase in A or Y, also in period t.

The necessary conditions and some algebra yield the welfare maximizing choices as[9]

$$\gamma^{(op)} = \frac{\delta L-\varrho}{\sigma} \tag{33}$$

$$X^{(op)} = (1\text{-}\alpha)^{1/\alpha}L_Y \tag{34}$$

and

$$L_A^{(op)}=\frac{\delta L-\varrho}{\sigma\delta} \tag{35}$$

Again, the growth rate increases with L and δ. However, the terms M and μ do not appear in equation (33) as perfect competition throughtout the economy is necessary for welfare maximization.

5. Market Inefficiencies and Fiscal Policy

Taking into account that $M = \dfrac{1}{1-a}>1$, and $\mu>0$, it is $\delta L\text{-}\varrho>\delta L\text{-}M(\mu+\varrho)$, and $\sigma<1+\sigma$M. Then, comparison of (27) with (33) and (29) with (35) yields

$$\gamma^{(op)} > \gamma^{(de)} \text{ and } L_A^{(op)} > L_A^{(de)} \tag{36}$$

In the steady state the planned economy allocates a larger fraction of the labor force to research and, as a result, it attains a higher rate of growth than the decentralized economy. It then appears that government policy could be used to improve upon market outcomes by providing market participants with incentives to follow a socially optimal course of action. In the following analysis of such policy it is assumed that the government budget is always balanced, and that only non-distortionary taxes are used to finance all expenditures.

One reason for the sub-optimal allocation of labor between final-good production and research is the probability of loss of monopoly power for intermediate good firms. Indeed, it is clear from (18) that the expected present value of future profits decreases with μ, for any $\mu > 0$. Consequently, as (24) reveals, the growth rate $\gamma^{(de)}$ associated with any level of the real interest rate, r, decreases with μ.

Equivalently, equations (17), (18), and (23), imply that

$$r = \frac{P}{C_R} - \mu \tag{37}$$

The interest rate in (37), being the difference between the profit-cost ratio and the probability of loss of monopoly rents, represents the *risk-adjusted* rate of return to entering the intermediate goods sector. Clearly, for any $\mu > 0$ this return is lower compared to the case $\mu = 0$, where monopoly power is certain to last for ever.This is one case where the private rate of return to research is less than its social counterpart. While for society the gains from an invention are permanent, from a private firm's perspective they only last for as long as the discovery earns monopoly rents.

Interpretive variations notwithstanding, the uncertainty regarding the future flow of monopoly profits reduces the market value of research and innovation. The end result is lower investment in the development of new intermediate goods, lower rate of technological progress, and lower rate of growth of output. This is shown by the presence of μ in the denominator of (18) and by the term - $M\mu$ in the numerators of (27) and (29).

In terms of policy, the government needs to compensate firms in the intermediate good sector for the probable loss of monopoly power. The compensation should make firms indifferent with respect to that loss. Then, if S_μ is the required subsidy, it must be

$$\frac{P}{r+\mu} + S_\mu = \frac{P}{r} \tag{38}$$

which implies that

$$S_\mu = \frac{\mu}{r}\left(\frac{P}{r+\mu}\right) \tag{39}$$

In order to neutralize the negative effect of the probability of losing monopoly power on the level of investment in R&D, the government must subsidize the expected profits of intermediate good firms at a rate equal to μ/r. Not surprisingly, the required subsidy rate increases with the intensity, μ, with which capital goods become competitive.

With this subsidy in effect, μ is eliminated from equations (18) and (24). The market solution now yiels

$$\gamma^* = \frac{\delta L - Mp}{1+\sigma M} \tag{40}$$

$$r^* = \frac{\sigma\delta L + p}{1+\sigma M} \tag{41}$$

and

$$L_A^* = \frac{\delta L - Mp}{\delta(1+\sigma M)} \tag{42}$$

where * signifies values after subsidy S_μ has been enacted. Comparison of equations (40) nad (42) with equation (27) and (29) shows that as S_μ has offset the negative impact of a probable loss of monopoly position, the expected profitability of research increases. As a consequence, employment in the intermediate good sector rises and so does the rate of growth of the economy.

The *markup pricing* of intermediate goods is a second distortion responsible for the sub-optimal growth rate of the decentralized economy. The higher than marginal cost price leads to under-utilization of intermediate goods. Indeed, from equations (10) and (34) one can find that the optimal quantity of intermediate goods is a multiple, by a factor $\eta \equiv \left(\frac{1}{1-\alpha}\right)^{1/\alpha} > 1$, of the profit-maximizing quantity. The result is a static and a dynamic inefficiency.

The static inefficiency relates to the fact that the market economy produces less output than the social planner at the same level of technology. In particular, equations (11), (12), and (34), yield

$$Y^{(op)} = AL_Y(1\text{-}\alpha)^{(1-\alpha)/\alpha} > AL_Y(1\text{-}\alpha)^{2(1-\alpha)/\alpha} = Y^{(de)} \quad (43)$$

The dynamic inefficiency appears in connection with the profitability of investment in research. Given that the R&D cost *CR* is a sunk cost, markup pricing is necessary *ex ante* for intermediate good firms to invest in R&D, but it is inefficient *ex post* as it results in lower sales of intermediate goods to the final good sector. This translates directly into smaller expected profits for intermediate good firms, less R&D investment, slower technological progress, and a lower rate of growth of output. Comparing equations (27) or (40) with (33), the described negative effect is evidenced by the presence of the markup coefficient $M = \frac{1}{1-a} > 1$ in the growth rate of the decentralized economy.

The government can neutralize the effects of monopoly pricing by subsidizing the purchase of intermediate goods by final good producers at rate α. In particular, the optimal subsidy, S_M, is given as

$$S_M = \alpha P_x \quad (44)$$

and effectively equates the user price of intermediate goods to the marginal cost of their production. Then, final good producers will have to pay only the fraction $(1\text{-}\alpha)P_x = 1 = MC$ of the price of any intermediate good, whereas the government will cover the excess of price over marginal cost,

$$SM = \frac{1}{1-a} - 1.$$

The subsidy will directly offset the static inefficiency as the *effective* price $(1\text{-}\alpha)P_x$ will lead final good producers to use the optimal quantity of intermediate goods. Efficient use of intermediate goods will yield the optimal level of output shown in (43) from which the wage rate is now found as

$$W = A(1\text{-}\alpha)^{(1-\alpha)/\alpha} \quad (45)$$

Based on (45), (20), and (21), the research cost becomes

$$\mathrm{CR} = \frac{a(1-a)^{(1-a)/a}}{\delta} \tag{46}$$

The subsidy will also remedy dynamic inefficiency. As the demand for intermediate goods is multiplied by η, with the price received by intermediate good producers still equal to P_X, the operating profits of intermediate good firms rise by the same factor. Then, the expected at time t present value of all future profits will become

$$E(V) = \frac{\alpha(1-\alpha)^{(1-\alpha)/\alpha} L_Y}{r} \tag{47}$$

It can now be shown that equatiions (46) and (47) along with the condition $E(V) = C_R$ lead to

$$r = \delta L - \gamma^{**} \tag{48}$$

Equations (48) and (26) yield the growth rate of output

$$\gamma^{**} = \frac{\delta L - \varrho}{1+\sigma} \tag{49}$$

and the real interest rate

$$r^{**} = \frac{\sigma\delta L + \varrho}{1+\sigma} \tag{50}$$

Further, from (2), (16), and (49), the amount of labor allocated to R&D is

$$L_A^{**} = \frac{\delta L - \varrho}{\delta(1+\sigma)} \tag{51}$$

where in all of the above, "**" signifies values after subsidies S_μ, and S_M have been put into effect.

Comparison of (49) and (51) with (40) and (42) reveals that $\gamma^{**} > \gamma^*$ and $L_A^{**} > L_A^*$. Absorbing the burden of monopoly pricing, the planner induces final good firms to increase their use of intermediate goods. This raises the present value of profits of intermediate good producers, and, therefore, the "private value of investment in R&D". The end result is an increase of employment in the intermediate good sector and a higher rate of growth.

However, after comparing (49) and (51) with (33) and (35), one finds that $\gamma^{**} < \gamma^{(op)}$ and $L^{**}_A < L_A^{(op)}$. Thus, even after the subsidization of the purchase of intermediate goods, the decentralized economy allocates a smaller fraction of labor to research and achieves a lower steady state rate of growth compared to the planned economy. This indicates that the private rate of return to research is still short of the respective social rate of return.

Another distortion, in the form of *intertemporal knowledge spillovers,* is responsible. To clarify the nature of the problem, notice from equation (19) that the number of researchers required for the development of a new intermediate good is inversely proportional to the existing stock of knowledge, A. The message of this observation is clear: Expansions in the current stock of knowledge increase the productivity of research and lower the resource cost of inventing new capital goods in the future. Nevertheless, the market has no mechanism to compensate researchers for this positive externality of knowledge. As a result, private firms do not internalize the "standing on shoulders" effect of their research, and R&D investment is less than optimal.

Clearly the policy maker subsidize research investments so that the market takes account of the benefits that current research brings to research in the future. Based on (46), if the government subsidizes research at the rate ∅, the *effective* research cost of inventing a new intermediate good will become

$$C_R = (1-\varphi)\frac{\alpha(1-\alpha)^{(1-\alpha)/\alpha}}{\delta} \tag{52}$$

At the same time, the present value of profits for intermediate good firms is still given by (47). Then, the condition $E(V) = C_R$ combined with equations (47) and (52) now implies

$$r = \frac{\delta L - \gamma^{\circ}}{1-\varphi} \tag{53}$$

Proceeding as before, equations (53) and (26) yield the growth rate of output

$$\gamma^{\circ}=\frac{\delta L-(1-\varnothing)\varrho}{1+(1-\varnothing)\sigma} \tag{54}$$

and the real interest rate

$$r^{\circ}=\frac{\sigma\delta L+\varrho}{1+(1-\varnothing)\sigma} \tag{55}$$

where o signifies values after all three subsidies have been put into effect. Further, from (2), (16), and (54), the amount of labor allocated to R&D is

$$L_A^{\circ}=\frac{\delta L-(1-\varnothing)\varrho}{\delta[1+(1-\varnothing)\sigma]} \tag{56}$$

Of course, the optimal value of $\varnothing$ must equate the growth rate in (54) with the socially optimal rate $\gamma^{(op)}$ given in (33). Hence,

$$\varnothing=\frac{\delta L-\varrho}{\sigma\delta L} \tag{57}$$

which results in the optimal subsidy being

$$S_R=\left(\frac{\delta L-\varrho}{\sigma\delta L}\right)C_R \tag{58}$$

Equation (58) implies that the required subsidy is increasing with the labor force, L. This is reasonable as larger L implies larger L_A in equilibrium. In turn, this results in a higher research output and, thus, a larger amount of knowledge spillovers to be internalized with the subsidy. For analogous reasons, the need for a higher research subsidy will also arise following an increase in research productivity, δ, and a decrease in the rate of time preference, p.

As expected, substitution of equation (57) for into $\varnothing$ equations (54) and (56) results in the socially optimal values

$$\gamma=\frac{\delta L-p}{\sigma} \tag{59}$$

and

$$L_A=\frac{\delta L-p}{\sigma\delta} \tag{60}$$

The reduction of research costs effected by the subsidy through internalization of the positive knowledge spillovers, further increases employment in the intermediate good sector. This raises the rate of technological progress and, accordingly, the growth rate of the economy. Finally, equations (55) and (57) yield the real interest rate as $r = \delta L$. From (25) and (33) this value can be seen as the rate of return implicitly used by the social planner.

6. Concluding Comments and Extensions

In a model where the creation and use of new capital goods is the driving force of increases in factor productivity it is found that the decentralized allocation of resources achieves a sub-optimal steady state rate of growth of output. Three distortions are responsible for this result:

(a). The probability of loss of monopoly rents that reward successful innovators. The result is a weakening of the incentive to invest in R&D.

(b). The monopolistic pricing of capital goods. The result is an inefficient allocation of these goods in the production of the final good.

(c). The inability of the market to reward researchers for the reduction in the cost of future technological advancements that follows the expansion of current knowledge. The result is a lower than optimal investment in research activities.

The first-best government policy that would induce the private economy to attain the socially optimal outcomes consists of three corresponding subsidies:

(a). A subsidy that will bring the present value of profits of intermediate good firms up to the level that prevails when the monopoly power of intermediate good firms lasts for ever. This will increase the market value of research thereby leading to an increase in employment in R&D and in the growth rate of the economy.

(b). A subsidy on the purchase of capital goods by final good producers. This will correct the inefficiency associated with monopoly pricing of intermediate goods, and will equate the privately chosen quantity of intermediate goods to the socially optimal one. Of course, the subsidy on the purchase of intermediate goods must be limited to the purchase of monopolized intermediate goods.[10]

(c). A subsidy directly on R&D spending. This subsidy will internalize the positive spillover of current expansions in knowledge on future research productivity.

The analysis in this study is based on the assumption that the government has recourse to lump-sum household taxes and raises tax revenue equal to the total value of 11 subsidies in every period. In such a setting the first-best policy can be pursued in all cases. However, if lump-sum taxes are absent or insufficient to fully finance the subsidies, distortionary taxes must be considered. Such taxes may include the tax on capital and labor income, the tax treatment of depreciation and capital gains, and probably special taxes on investment expenditures. With distortionary finance, an R&D subsidy raises equilibrium R&D investment, but at the cost of increasing distortions in other activities. Given that this may lead to a reduction in equilibrium R&D expenditures, the best policy in this case is not obvious in advance.

Generally, in the absence of lump-sum taxes it is optimal for the policy maker to initially tax existing assets at very high rates, taking advantage of their inelastic supply. The revenue can then be used to provide required subsidies or finance future government expenditures with less dependence on the use of distortionary taxes.[11] In the present model, if the government decides to follow this policy pattern it must make all existing intermediate goods available at a competitive price. At the same time, it should promise protection of property rights on, and "ensure" sufficient returns to, future inventions. This situation involves a clear *time-inconsistency problem* as in all following periods the government will repeat the same action.

The problem is centered in a well-known tradeoff: the static gain from increased competition and more efficient use of the existing intermediate goods, versus the dynamic loss from too low an incentive to invest in research and the consequent low rates of technological progress, productivity enhancement, and growth. If the government does not possess a mechanism to commit itself to a given path of future policies, the reward for research will be monopoly rents that tend to disappear in a short period of time. In such an envirnoment agents will undertake little research and the resulting equilibrium is bound to generate low growth and welfare.

Further, in a more general model where price over marginal cost markups vary over time and types of intermediate goods, the optimal subsidy rate for each good depends on the markup on the particular good. Then, everything else being the same, equation of the decentralized and optimal allocations requires a subsidy to the purchase of intermediate goods that differentiates between monopolized and competitive goods. Given the difficulty of implementing a fully differentiated subsidization scheme, the welfare-maximizing policy must be derived under the additional constraint that the intermediate good subsidies are sub-optimally differentiated.

In addition note that in the present model technological progress is formulated as an expansion in the variety of intrmediate goods. Consequently, new intermediate goods add up to the older ones and faster rates of growth are always welfare improving. In other endogenous growth models technological progress takes the form of successive improvements in the quality of a fixed number of types of intermediate goods.[12] There, the introduction of an improved version of some intermediate good renders the previous generation of the same good obsolete. In such models, there is an excessive private incentive to conduct research due to the transfer of rents from an old inventor to a new one.[13] Due to this *creative destruction* effect, faster rates of innovation are not necessarily Pareto-improving in terms of welfare.

Last but not least, the analysis of the effects of policy must be extended beyond steady state comparisons. Jones (1995a,b) presents empirical evidence that supports the presence of diminishing returns to R&D. This implies a long run rate of factor productivity growth independent of policy. However, it is important to note that although the steady state rate of growth may be invariant to policy, this rate is sensitive to changes in policy regimes during the transition between steady states. If the transition lasts for a long period of time, exclusive steady state comparisons regarding the effectiveness of policy may be misleading. Then, an analysis of how policy may alter the transition path towards a new long-run equilibrium becomes a necessity. After all, to paraphrase Lucas (1988, p. 5), the consequences for human welfare involved in these issues are simply too serious to make light of.

Notes

1. Spence (1976), and Dixit and Stiglitz (1977) used a form similar to (1) to express consumer preferences over collections of final goods. Ethier (1982) was the first to use it in the context of production with several intermediate goods.

2. By definition of a Poisson process with intensity μ, the probability that a monopolized good turn competitive within a time interval of length $h > 0$, is approximately equal to μh, for $h \to 0$. Now, call the transformation of a monopolized good to competitive a *Poisson event*, and let T be the time that goes by between two successive events. Then, T is a random variable that follows the *exponential distribution* with parameter μ, and $Pr\ (T > t + h | T > t) = G(h) = e^{-\mu h}$. The functiion $G(h)$ is often called the *survival function*. For details see J. Pitman (1993), S. Ghahramani (1996), and R. Durrent (1999).

3. The elasticity of intertemporal substitution is equal to $e \equiv \frac{U'(c)}{cU''(c)} = \frac{1}{\sigma}$. Then, a higher value of σ implies that people are less willing to allow their consumption to vary over time. Further, in a model with stochastic returns to saving, σ is also the *coefficient of relative risk aversion* defined as $\sigma \equiv - \frac{cU''(c)}{U'(c)}$.

4. At this point all intermediate good firms are assumed to be monopolies. This way the effect of imperfect patent protection, and of the resulting possibility of loss of monopoly power, will appear more clearly in the following.

5. Equation (18) holds for all t if r and L_Y are constant. Again, it turns out that this is indeed the case in long-run equilibrium.

6. It is assumed that potential investors are interested only in the mean return $E(V)$.

7. However, since labor is supplied inelastically, the implied increase of employment in the final good sector will increase the current level of output.

8. For an interior solution with positive steady-state growth, it must be $r > \varrho$. Otherwise, a corner solution arises with $\gamma_{[Y]} = \gamma_{[C]} = \gamma_{[A]} = 0$. Also note that no distinction is made between the growth rate of per capita consumption, $\gamma_{[c]}$, and the growth rate of aggregate consumption, $\gamma_{[C]}$, as with a constant population it is $\gamma_{[c]}=\gamma_{[C]}$.

9. Optimality requires that there is no output waste. Then, in the problem's solution, expression (31) is satisfied as equality.

10. Alternatively, the government could stimulate the demand for intermediate goods by subsidizing production of the final good. Under this scheme, final good producers would receive $\frac{1}{1-a}$ units of revenue for each unit of final good produced.

11. Jones, Manuelli, and Rossi (1993), discuss optimal time-varying fiscal policies in an endogenous growth model.

12. See, for example, Segerstrom, Anant, and Dinopoulos (1990), Grossman and Helpman (1991, Ch. 4), and Aghion and Howitt (1992).

13. Reinganum (1989) surveys models where patent-races result in too high, from a social perspective, levels of R&D.

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