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POISSON RANDOM SUMS IN MODELLING OPERATIONS FOR THE TREATMENT OF ONGOING RISK OCCURRENCES

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Abstract

The paper makes use of a very important result from stochastic theory of service systems in order to formulate a Poisson random sum of nonnegative random variables. Applications of the proposed Poisson random sum in stochastic modelling of risk financing operations for ongoing risk occurrences at given time point are provided (JEL, C51).

Keywords: random sum, risk, modelling.

1. Introduction

Risk management is one of the disciplines within the domain of management. As such, risk management shares many of the characteristics of management, and yet is unique in several important respects. Risk management is devoted to reducing the adverse effects which risks may have on an organization. In order to achieve its more ambitious objectives of profit, growth, or public service, an organization must attain survival in the face of risks. Beyond mere survival, the top management of an organization may also want to prevent lowering its profits by more than a specified, tolerable amount. Given this focus on risks, risk management may be defined as the process of planning, organizing, directing and controlling the resources and activities of an organization in order to reduce the adverse effects of risks on that organization at the least possible cost. This broad definition makes quite clear that neither the risk manager of an organization nor any other single executive acting alone can fully accomplish the whole risk management process within any organization. The participation of all other managers and personell is required. The function of the executive charged with responsibility for risk management is not to personally reduce the adverse effects of all risks but, instead, to coordinate the efforts of all managers in reducing risks for which each of them has some responsibility and, therefore, control, Head (1978).

Depending on perspective, risk management means different things to different people. Generally, the aim of risk management is to reduce the possibility of future events harming an organization or to control the probability that results will deviate from the expected. The goal is to avoid risks and to balance positive and negative results over time. This task becomes increasingly difficult as organizations perform in global markets and face constant structural modifications through changes in their business environment. Besides global risk, such as climate changes or changes in the legal environment over which an organization has only limited control, the organization also faces the traditional risks. Whatever the definition, risk management, including proactive financing, should provide a competitive advantage and allow organizations to experiment and take calculated chances in ways that were not possible before.

Risk profile represents the entire portfolio of risks that constitute an organization. At the foundation of risk management is the integration of the three ways that an organization can alter its risk profile. These basic ways that an organization can implement risk management objectives, by modifying its operations, by employing targeted financial instruments, and by adjusting its capital structure, interact to form its management strategy. Managers must weigh the advantages and disadvantages of any particular approach in order to find an optimal mixture of the three. Traditional insurance, then, a type of targeted financial instrument, is but one tool available to the organization, Meulbroek (2002).

Risk managers today are faced with an ever greater range of risks, which demand new innovative solutions. Risk management literature in the last two decades appears to have concentrated on developing stochastic models for measurement, assessment, evaluation and treatment of risks faced by modern complex organizations. These models give risk managers a chance to isolate and investigate the various thought processes involved.

The rapidly expanding universe of stochastic models for risk measurement, risk assessment, risk evaluation and risk treatment offers risk managers significant opportunities for value creation, but this growth also creates new responsibilities. Risk managers must understand how to use these stochastic models and actively decide on their selective application.

The main part of the paper is organized in the following way. The second section is devoted to the role of stochastic models in the description, analysis and control of risk management operations. The application in risk management of a result from stochastic theory of service systems is established by the third section. The fourth section concentrates on the formulation of a Poisson random sum for stochastic modelling of risk financing operations applied to ongoing risk occurrences at a given time point.

2. Risk Management Thinking and Stochastic Models

The application of mathematics in solving practical problems has become broad in the last seven decades. This is partly due to the application of systems thinking to problem solving, and to the increasing computational power of computers and computing methodology, both of which have made many more problems responsive to mathematical solution. In systems thinking the real world associated with the problem is viewed as a system, and the solution to the problem as an investigation of the system with a defined goal.

The most decisive and significant step in using mathematics to solve practical problems is the satisfactory translation of the problem from the real world into a mathematical description. Once this is done, standard methods of mathematical analysis can be applied to find a solution to the problem. The validity of the solution depends on how well the mathematical description represents the real world. The mathematical description is called a mathematical model, and the process of developing it is called mathematical modelling. More precisely, a mathematical model of a system is a symbolic representation incorporating an abstract mathematical formulation. Such a formulation is not a mathematical model by itself. It is only by relating the mathematical formulation to a system partial description that the mathematical formulation becomes a mathematical model. A satisfactory mathematical model is a mathematical model, which is satisfactory for the purpose in the mind of the modeller. Such a mathematical model can be achieved by relating, on a one-to-one basis, the features of a satisfactory system partial description with the variables of a suitable mathematical formulation, Artikis and Artikis (2004).

Mathematical modelling is an art as well as a science. The science side deals with the topics needed to execute the various steps in the mathematical modelling process. Because no two problems are the same in the real world, features such as creativity, intuition and foresight also play a very important role in the mathematical modelling problems. The science side can be appreciated in a passive learning mode, whereas, the art side can be appreciated by constructing mathematical models and learning from experience. In general, one seldom obtains a satisfactory mathematical model at the very first attempt. This is because either the system partial description is not satisfactory or the mathematical formulation is not appropriate. This implies than an iterative procedure is required, where at each stage an improved mathematical model is built and tested for adequacy. If it passes the test the model can then be said to be a satisfactory mathematical model, if not the iterative process is continued.

There is no theory of mathematical modelling, and it is difficult to envisage the evolution of a theory in the near future. Each new mathematical modelling exercise poses new challenges. This does not mean that there is no procedure for mathematical modelling. In fact there is a basic methodology, which is common to all mathematical modelling exercises.

A mathematical model is said to be static if it does not contain time dependent behavior. If a time dependent behavior is included in a mathematical model then the model is said to be dynamic. A mathematical model is said to be deterministic if it always gives the same output when a specific input is applied. Finally, a mathematical model is said to be stochastic if the model includes a chance mechanism. From these definitions it follows that the mathematical models can be classified into four categories based on the mathematical structure of the underlying formulation. The four categories of mathematical models are: deterministic static, deterministic dynamic, stochastic static, and stochastic dynamic.

In real life there is always uncertainty. If the uncertainty is insignificant, then it can be ignored and the system can be described by making use of a deterministic static or deterministic dynamic mathematical model. This is a process of simplification or idealization. If the uncertainty is significant then it can not be ignored and must be taken into account. In this case the system can be described by making use of a stochastic static or stochastic dynamic mathematical model. Therefore, when uncertainty is a significant feature of the system, then the system partial description must be done in a stochastic framework. As a consequence the mathematical formulations, which can be used in mathematical modelling must allow the variables to change in an unpredictable fashion, so that they adequately represent the uncertainty of the system when the formulation becomes a model. Such formulations belong to classical probability theory and dynamic probability theory. More precisely, when the formulation is static then it belongs to classical probability theory, and when the formulation is dynamic then it belongs to dynamic probability theory.

It is quite clear that stochastic models are more realistic than deterministic models in many situations of particular practical interest. Even then, very often a much better representation is given by considering a collection or a family of random variables instead of a single random variable. Collections or families of random variables that are indexed by a parameter such as time and space are known as stochastic processes. It is also quite clear that the mathematical analysis of stochastic dynamic models is much more difficult than the mathematical analysis of stochastic static models. Stochastic differential equations, stochastic difference equations and stochastic integrals are very powerful analytical tools of stochastic calculus which can substantially contribute to a deep mathematical analysis of very complex stochastic dynamic models.

Many real world phenomena require the description and analysis of a system in stochastic rather a deterministic setting. Stochastic models are becoming increasingly useful for understanding or making performance evaluation of complex systems in a broad spectrum of disciplines such as operational research, computer science, telecommunication, engineering, economics and many areas of management.

Planning and scheduling have been long standing goals of organizations in the private and public sector. Organizations, however, face uncertainty, typically in demands, resource availability and yields. Thus, within the stochastic environment, the goals of maximum return in the private sector or best service level in the public sector are not always achievable as predicted by deterministic mathematical models. In practice this translates directly into the concept of risk. Thus, risk may appear in many forms involving profit, liquidity, market share, service level and can be attributed to changes in economic conditions, the environment, accidents and disasters. It is therefore necessary to develop and apply stochastic models in the fundamental risk management operations, Artikis and Artikis (2005). Stochastic modelling in the discipline of risk management is one of the areas of academic research that is in a constantly active interaction with present developments in its domain of application. Indeed, it both draws from and has direct implications upon every-day practice in organizations. The research in this area aims at a better understanding of the stochastic evolution of systems through the formulation of appropriate stochastic models, as well as at the development of efficient new methodologies for fundamental risk management operations. The principal mathematical tools in risk management derive from classical and dynamic probability theory. Indeed, it is becoming increasingly essential for risk managers and risk experts that they should have a mastery of a variety of probabilistic techniques along with knowledge of the theory of Brownian motion, martingales, and stochastic differential equations.

Analysing market conditions, industry needs and the emerging regulatory requirements, shows that there is a fast growing global requirement for professionals with skills in formulating stochastic models for risk management operations. In particular, in the finance industry, banks, insurance companies and pension fund managers amongst others, have to comply with the requirements to quantify, control and report enterprise risk. The term enterprise risk management is a relatively new one that is quickly becoming viewed as the ultimate approach to risk management. Enterprise risk management is, in essence, the latest name for an overall risk management approach to business risks. Precursors to this term include corporate risk management, business risk management, holistic risk management, strategic risk management and integrated risk management. Although each of these terms has a slightly different focus, in part fostered by the risk factors that were of primary concern to organizations when each term first emerged, the general concepts are quite similar. Enterprise risk management is defined as the discipline by which an organization in any industry assesses, controls, exploits, finances and monitors risks from all sources for the purpose of increasing the organization's short and long term value to its stakeholders, Dickinson (2001). Implicit in this definition is the recognition of enterprise risk management as a strategic decision support framework for management. It improves decision making at all levels of the organization. The new focus on the concept of enterprise risk management provides an opportunity for risk experts to develop stochastic models for describing the entire portfolio of risks faced by an organization.

In some areas of risk management, a gap exists between theoretical development and practice. Naturally, such gaps are to be expected. It takes time for the practical development of risk management theories into a working state. The existence of a wide gap, however may be indicative that theoretical development in risk management is not fruitful. This may be a sign that a change of emphasis is required in risk management theoretical efforts. A rather wide gap exists in risk treatment, which is the most dynamic risk management operation. In this the contrast between the theoretical and practical is often distinct. Risk treatment implies the reduction of event probability and/or its impact, as well as the mitigation of impact via various risk transfer mechanisms. While the concept is simple, application is often complex. In risk management areas where the gap between theory and practice seems a lot smaller, or at least where we get a sense that the gap has become manageable, the overriding consideration seems to be the ability to develop practical stochastic models. The decision sciences, including risk management, however, have been slow to invoke a similar "cut and try" philosophy. Ideas in risk management often remain page-bound. That is, they are developed on paper and rarely leave it. Clearly, the decision maker in the discipline of risk management does not think in terms of steel, wood and other tangible physical objects, yet his results do affect the physical world. The key is moving stochastic models from paper to practice.

The tools for effective stochastic modelling in risk management are certainly available. To make stochastic models a larger part of risk management thinking, however, we need to actively promote their usage. Otherwise, the development of theory without practical support becomes a risk-free exercise. Rewarding practical success, not just formal rigor, would go a long way to help assure that proper attention is paid to the development of "application-friendly" theory. The corresponding threat is that without practical justification, or at least its promise, the theory will not be taken seriously. Where risk management theories are allowed to breed, unchecked by the test of applicability, they tend to proliferate. In turn, the proportion of theories to practical applications grows and the gap between theory and practice widens. On the other hand, there is not a commensurate increase in what we could reasonably qualify as real knowledge about risk management.

Some positive steps could act to enhance the development of more applicable risk management theories. Promotion of risk management theories through research publications, funds for development and other incentives for builders of such theories must be more strongly linked to applicability. The main purpose here is not to intimidate risk management theorists into the development of practical stochastic models. Rather, we need to counterbalance the effects of a system that rewards theorizing for theorizing's sake.

Conciliation between risk management theorists and risk management practitioners can come with wider development and application of effective stochastic models. Such models give the two groups the opportunity to talk about. At the same time, risk management theorists are grounded by a healthy dose of reality, and risk management practitioners are elevated from the mundane. The overall result is a genuine improvement in the knowledge base of risk management community, with the resulting benefits incurring to society at large, Kloman (1992).

3. Number of Ongoing Risk Occurrences

Stochastic theory of service systems was developed to formulate stochastic models to predict behavior of systems that attempt to provide service for randomly arising demands, and not unnaturally, then, the earliest problems studied were those of telephone traffic congestion. The early work in stochastic theory of service systems picked up momentum rather slowly, but the trend changed and there has been a flood of papers in the area. Stochastic theory of service systems originated as a very practical subject, but regrettably most of the recent literature has been of little practical value. It is now imperative for researchers working in this area to once again become concerned about the application of the sophisticated theory that has mostly arisen in the past five decades. It is clear that the emphasis in the literature on the exact solution of problems arising in service systems with clever mathematical ideas must become secondary to stochastic modelling and the direct use of these techniques in management decision making. Most real problems do not correspond exactly to a stochastic model, but very little of the literature deals with approximate solutions, sensitivity analysis and the like. The development of the practice of stochastic theory of service systems must not be restricted by a lack of closed-form solutions. A problem solver cannot become frustrated with transform solutions, and we must learn to put to work the developed theory.

There are many well-known, common but nontrivial, areas of application for the stochastic theory of service systems. The first to be mentioned is, in fact, the first area of application, that is, telephone conversations. A second interesting application is the landing of aircraft. This does by no means imply that this application was chronologically second to telephony with no others in between. Rather, aircraft-landing problems have many of the ingredients necessary for the appropriate application of service systems techniques. There are many other important applications of the stochastic theory of service systems, most of which have been well-documented in the literature of probability theory, operations research, and management science. Some of the more prominent of these are machine repair, tool booths, taxi stands, inventory control, the loading and unloading of ships, scheduling patients in hospital clinics, production flow, and applications in the computer field with respect to program-scheduling, time sharing, and systems design. The purpose of the present section is to establish an application in risk management of a result from stochastic theory of service systems.

The establishment of such an application needs the concepts of risk cause, risk occurrence, risk frequency, duration of a risk occurrence and number of ongoing risk occurrences, which are basic components of risk. The term risk used in the present papers means pure risk.

Risk cause is an event which can give rise to a loss of random size at a random time point in the future.

Risk occurrence is a realization of a risk cause. This component of risk is extremely useful in risk management practices. More precisely, risk occurrence is a fundamental element of risk identification, which is the most important operation of the risk management process.

Risk frequency is the number of occurrences of a risk in a given time interval. Risk frequency is particularly useful for the selection, description, analysis and application of risk measurement, risk assessment, risk evaluation and risk treatment operations. Discrete random variables taking values in the set of nonnegative integers are considered by risk analysts, and risk modellers as very strong analytical tools for stochastic modelling of risk frequency in practical situations. These discrete random variables are more precisely described as random counters, for which a probability distribution is considered as the most suitable means for obtaining information concerning risk frequency, that will help the risk manager to evaluate the relative importance of an exposure to risk. For several decades, the risk management community has recognized probability theory as one of the most important representations of uncertainty. This is a very good reason for undertaking research activities in the area of probability distributions, corresponding to discrete random variables taking values in the set of nonnegative integers, applied to practical problems incorporating the concept of risk frequency.

Duration of a risk occurrence is the length of the time interval into which a risk cause is active. This component of risk plays a fundamental role in determining the severity of risk, or equivalently the size of the loss due to a realization of a risk cause. Random variables taking values in the set of nonnegative real numbers are also considered by risk experts as very strong analytical tools for stochastic modelling of the duration of a risk occurrence in practical situations. Moreover, such random variables can be used in stochastic multiplicative models for investigating the severity of a risk. This is also a very good reason for undertaking research activities in the area of probability distributions corresponding to products of random variables taking values in the set of nonnegative real numbers.

Finally, if a realization of a risk cause is active at a given time point then such a realization is an ongoing risk occurrence at that given time point. The number of ongoing risk occurrences at a given time point is a component of risk with very important applications in investigating the risk profile of an organization.

Now, we suppose that the number of customers arriving at a service system in a given time interval is modelled by a homogeneous Poisson process. Upon arrival, a customer is immediately served by one of an infinite number of servers, and the service times are assumed to be independent and identically distributed random variables. The contribution of the present section is the interpretation of the random variable denoting the number of customers in the service system at a given time point as the random variable denoting the number of ongoing occurrences of a risk at a given time point, when the frequency of that risk in a given time interval is modelled by a homogenous Poisson process.

Let

$\{N(t), t > 0\}$

be a homogenous Poisson process with intensity . We suppose that the random variable

$N(t)$

denotes the frequency of a risk in the time interval

[0,/]

Let

$\{X_n, \quad n=l,2,..\}$

be a sequence of nonnegative, independent random variables distributed as the random variable

X

with distribution function

$Fx(x)$

We suppose that the random variable

X_n

denotes the duration of the *nth* risk occurrence. We set

$$=\int_0^t (1 - F_X(x))dx$$

If the random variable

$S(t)$

denotes the number of ongoing occurrences of that risk at the time point t, then $S(t)$ follows the Poisson distribution with parameter $\lambda=v\theta$, Ross (1970).

4. Capital Employed for Treatment of Ongoing Risk Occurrences

Random sums are very powerful tools to quite a wide range of fundamental branches of probability theory and statistics. It is generally accepted that theory of risk and insurance, reliability, stochastic theory of service systems, stochastic processes, mixtures of probability distributions, stochastic integrals, random sampling and statistical inference are the most celebrated branches of probability theory and statistics which make extensive use of random sums. Moreover, random sums have very important applications in a variety of significant practical disciplines. Stochastic modelling activities arising in physics, chemistry, informatics, biology, medicine, neurophysiology, psychology, economics, logistics, management, engineering and other practical disciplines make substantial use of random sums. Such practical disciplines take advantage of theoretical results, numerical methods and computer simulation techniques related to random sums, and provide analysts, modellers and decision makers with more detailed and more realistic criteria upon which to base analysis, predictions and decision making under conditions of uncertainly. It is of some particular importance to note that Poisson random sums are the most significant cases of random sums from a theoretical and practical point of view.

The purpose of the present section is to establish applications in risk management of a Poisson random sum. More precisely, this section makes use of the practical result established by the previous section, in order to introduce a Poisson random sum for stochastic modelling of operations applied to treatment of ongoing risk occurrences at a given time point. Considering the practical contribution to risk management of the present section of the paper from the point of view of stochastic modelling, it can be said that the proposed Poisson random sum can be a very useful analytical tool for making decisions in the area of risk treatment, which is the most dynamic operation of the risk management process. It is quite clear that the formulation, investigation and application of efficient stochastic models for describing and analyzing practical situations related to risk avoidance, risk frequency reduction, risk severity reduction, risk combination, risk separation and risk finance have as direct consequence the improvement of decision making in the area of risk treatment.

Let

$$\{C_s,\ s = 1,2,...\}$$

be a sequence of nonnegative, independent and identically distributed random variables. We suppose that the random variable

$$C_s$$

denotes the capital employed for treating the *sth* ongoing risk occurrence at the time point *t*. If the random variable

$$S(t),$$

denoting the number of ongoing risk occurrences at the time point *t*, is independent of the sequence

$$\{C_s,\ s = 1,2...\}$$

then the random sum

$$L = C_1 + C_2 + ... + C_{S(t)}$$

denotes the total capital employed for treating all the ongoing risk occurrences at the time moment t.

It is easily seen that the above Poisson random sum of nonnegative random variables provides risk analysts, risk managers and other risk experts with valuable information for describing, analyzing and implementing operations related to treatment of ongoing risk occurrences at a given time point. The mathematical structure of such a Poisson random sum seems to be suitable for the investigation of the risk profile of an organization at any point of a time interval. Moreover, the formulation of stochastic discounting models based on the Poisson random sum L can substantially extend the usefulness of research activities undertaken by risk experts for the description of the risk profile of an organization in a time interval. In consequence, the probabilistic information provided by stochastic discounting models incorporating the Poisson random sum L can be very useful for constructing and implementing proactive risk management programs. It is generally recognized by the risk management community that the contribution of proactive risk management as a significant organizational discipline has been proved very important. This means that the consideration of the Poisson random sum L as an analytical tool for constructing proactive risk management programs can offer new opportunities to risk managers for extending the applicability of proactive risk management methodologies.

It is of some particular practical importance to mention risks which can be described by the proposed Poisson random sum L. Risks such as fire, flood, windstorm, change in temperature and humidity, electrical and magnetic disturbance, hardware and software failure, human error, criminal action, strike, war, riot, economic recession, management error, rise of interest rate that can reduce the market value of an investment, liquidation of a position during a down period due to general market pressures, and unability of a borrower to make interest or principal payments in a timely manner seem to have total capital, employed for treating all the ongoing risk occurrences at a given time point, modelled by a Poisson random sum of nonnegative random variables.

The derivation and investigation of the distribution function of the random sum L can significantly extend the applicability in risk management practices of this random sum. Since an explicit analytical form of the distribution function of L is not possible, it is appropriate to consider the corresponding characteristic function.

If the random variables of the sequence

$\{Q_{s,} s=1,2...\}$

are distributed as the random variable

C

with characteristic function

$\varphi_c(u)$

then it is readily verified that the characteristic function of the random sum *L* is given by

$\varphi_L(U)=e^{\lambda(\varphi c(u)-1)}$

$$\varphi_L(u) = e^{\lambda(\varphi c(u)-1)}$$

The characteristic function of the random sum *L* provides a powerful tool for calculating the corresponding distribution function

FL(1)

The method makes use of Fourier inversion of the characteristic function of the random sum *L*. By first calculating and then inverting the characteristic function of *L* one can calculate the corresponding distribution function. Moreover, by making use of a computational algorithm known as the Fast Fourier Transform makes this process more manageable by providing substantial reduction of computational time, Paulson and Dixit (1989). Fourier analysis is an extremely useful analytical tool that could eventually revolutionize the theoretical investigation of distribution functions corresponding to Poisson random sums of random variables taking values in an interval of the set of real numbers. Acceptance of Fourier analysis has been slowed by the relatively advanced nature of the level of mathematics it entails and by the absence of computer programs that have been adapted for convenient use in practical applications. As these barriers are overcome, use of Fourier methodology to analyze problems of distribution functions in finance, risk management, operational research and other important practical disciplines will become more common.

The characteristic function of the random sum *L* is called compound Poisson characteristic function. The class of compound Poisson characteristic functions plays a very significant role in the study of the family of infinitely divisible characteristic functions. The limit of a sequence of finite products of compound Poisson characteristic functions is infinite divisible. The converse is also true. Every infinitely divisible characteristic function can be written as the limit of a sequence of finite products of compound Poisson characteristic functions. The family of infinitely divisible characteristic functions is quite broad. It includes the class of stable characteristic functions, as well as the class of self-decomposable characteristic functions and the class of infinitely divisible characteristic functions with Levy spectral function having a unique mode at the point zero.

The concept of infinite divisibility is very important in probability theory, particularly in the study of limit theorems. The most important practical occurrence of infinite divisibility is in stochastic modelling activities. Some stochastic models can only be defined in terms of infinitely divisible distributions. A less basic reason for interest in the concept of infinite divisibility is the curious phenomenon that infinite divisibility of distributions in statistics and other areas of applied probability seems to be the rule than the exception. In these cases it may be useful to know that a given distribution is infinitely divisible, because this implies special properties of this distribution, such as a typical tail behavior, Steutel (1979).

It is quite obvious that the characteristic function of the Poisson random sum L is, in general, very complicated and, hence, the establishment of conditions for embedding this characteristic function into well known classes of characteristic functions is a powerful analytical method for investigating theoretical properties of the distribution function corresponding to such a random sum. It is clear that the Levy canonical representation and the corresponding Levy spectral function of the characteristic function of the Poisson random L are the main mathematical tools for embedding this characteristic function into well known classes of characteristic functions. From the fact that, the theoretical properties of a distribution function are extremely important for the applications in different practical disciplines of the corresponding random variable, it easily follows that the establishment of the above mentioned conditions substantially extends the practical applicability in risk management of the Poisson random sum L.

In conclusion, it can be easily said that the main result of the present section of the paper is the establishment of a new stochastic derivation in the discipline of risk management for a wide class of Poisson random sums of nonnegative random variables. It seems that this result can be of some practical importance for the description, analysis and treatment of ongoing risk occurrences at a given time point.

5. Concluding Remarks

It is generally recognized by the risk management community that random sums are powerful analytical tools for describing practical situations related to risk measurement, risk assessment, risk evaluation and risk treatment. The Poisson random sum of nonnegative random variables, proposed by the present paper, can be of some particular practical importance for stochastic modelling of operations applied to treatment of ongoing risk occurrences at a given time point. The mathematical structure of such a Poisson random sum seems to be suitable for investigating the risk profile of an organization at any point of a time interval. Moreover, the incorporation of the proposed Poisson random sum in stochastic discounting models can contribute to the development and application of proactive risk management programs. It can be said that the results of the paper constitute a new stochastic derivation in the discipline of risk management for a wide class of Poisson random sums of nonnegative random variables.

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