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ON THE TRUE COST OF LIVING INDEX

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Abstract

The purpose of this paper is to present true cost of living indices derived from specific expenditure systems, and specifically from the LES, GLES and DGLES systems. According to Frisch [2] index numbers are classified into two categories: "statistical" and "functional". The former are purely descriptive statistics without any direct theoretical underpinning, while the latter are based on a formal theory and a precise theoretical interpretation. Here we present the advantage of the true cost of living index compared to the Laspeyre (statistical) index. Going a step further we show the advantage of using "dynamic" true cost of living index instead of using a "static" one. Finally by using the DGLES model to construct a dynamic true cost of living index we introduce habit formation hypothesis and changes in relative prices into these indices.

1. Introduction

Index numbers, according to Frisch [2] are classified into two categories: "statistical" index numbers and "functional" index numbers. The former are purely descriptive statistics without any direct theoretical underpinning, while the latter are based on a formal theory and a precise theoretical interpretation.

The official cost of living indices published by the National Statistical Institute in Greece (and in most other countries) belong to the first category.

The purpose of this paper is to present true cost of living indices derived from specific expenditure systems and to compare these with the official (statistical) cost of living index published by the Greek Statistical Service.

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2. The True Cost of Living Index

In other works [3-11] we have introduced the utility function as a guide to the specification of complete sets of consumer demand functions. It is tempting to consider its implications for normative economics, that is, its use as a guide to the construction of welfare indicators.

A comparison of welfare across two income-price situations should in principle be based on the computation of the (maximum attainable) utility in the two situations. That is to say, the indirect utility function would itself serve as a welfare indicator. A variant of this takes real income as a welfare indicator: The true cost of living index, constructed with one situation taken as a reference base, is used to deflate nominal income in the second situation, yielding a real income measure, which is directly comparable with income in the reference situation.

This amounts to employing a monotonic transformation of the indirect utility function, $v=v(u)$, in which $v^*(y^0, p^0)=y^0$ and $v^*(y^1, p^1)=y^1/(y^*/y^0)$, where the superscripts 0 and 1 denote the reference and second situation respectively, and (y^*/y^0) is the true cost of living index.

The true cost of living index is defined by Pollak [17-19] as the ratio of the minimum cost of attaining a reference indifference curve under comparison prices to that of attaining it under base prices.

3. The True Cost of Living Index in the Linear Expenditure System (LES)

The true index can be computed if one is willing to specify the utility function algebraically. An exact expression for this index may be found in using the Klein-Rubin utility function(1).

Since the Linear Expenditure System (LES) receives some support as a descriptive model(2) we can compute the true cost of living index based on this model.

Suppose that in an initial situation, period 0, prices are $p_1^0,\dots,p_n^0$, and that in a second situation, period 1, they are $p_1^1,\dots,p_n^1$, for 1,.,.,n commodities.

The true index in period 1 (relative to period 0) is

$$C\ (1,\ 0) = \frac{y^{*}}{y^{0}} \tag{1}$$

where y^0 is income in the initial situation and y^* is the income which affords the same utility in the price situation of period 1, as y^0 did in the price situation of period 0.

As a rule the utility function is unknown, so that this index is not operational. However, estimation of a specific expenditure system, like the LES, provides estimates of the Klein-Rubin utility function which underlies it.

The indirect Klein-Rubin utility function in an initial income-price situation with $y^0, \mathbf{p}^0 = (p_1{}^0, \ldots, p_n{}^0)'$ is of the form

$$u^0 = u^*(y^0, \mathbf{p}^0) \tag{2}$$

$$= \sum_{i=1}^{n} \beta_i \log\beta_i + \log(y^0 - \sum_{i=1}^{n} p_i^0 \gamma_i) - \sum_{i=1}^{n} \beta_i \ \log p_i^0.$$

If price changes to $\mathbf{p}^1 = (p_1{}^1, \ldots, p_n{}^1)'$ then the resulting change in utility as a function of income is

$$\Delta u = \log(y^1 - \mathbf{p}^{1\prime}\gamma) - \log(y^0 - \mathbf{p}^{0\prime}\gamma) - \boldsymbol{\beta}'(\log\mathbf{p}^1 - \log\mathbf{p}^0), \tag{3}$$

so that y^*, the income required to maintain utility (obtained by setting $\Delta u = 0$) satisfies

$$\log[(y^* - \mathbf{p}^{1\prime}\gamma)/(y^0 - \mathbf{p}^{0\prime}\gamma)] = \sum_{i=1}^{n} \beta_i \log(p_i^1 / p_i^0). \tag{4}$$

Taking antilogs and then rearranging we have

$$y^* = \mathbf{p}^{1\prime}\gamma + (y^0 - \mathbf{p}^{0\prime}\gamma) \prod_{i=1}^{n} (p_i^1 / p_i^0)^{\beta i} \tag{5}$$

$$= y^0 \left\{ \left(\frac{\mathbf{p}^{0\prime}\gamma}{y^0} \right) \left(\frac{\mathbf{p}^{1\prime}\gamma}{\mathbf{p}^{0\prime}\gamma} \right) + \left(\frac{y^0 - \mathbf{p}^{0\prime}\gamma}{y^0} \right) \left[\prod_{i=1}^{n} (p_i^1 / p_i^0)^{\beta i} \right] \right\}.$$

The expression in curly brackets, $y^*/y^0=C(1,0)$ say, is therefore the true cost of living index, which may be written out as

$$C(1,0)=[1-|\varphi(0)|]\sum_{i=1}^{n} w_i^*(0)\mathbf{p}_i(1,0)+|\varphi(0)|\prod_{i=1}^{n}[\mathbf{p}_i(1,0)]^{\beta i} \qquad (6)$$

where

$$-\varphi(0)=(y^0-\mathbf{p}^{0\prime}\gamma)/y^0 \qquad (?)$$

is the "supernumerary ratio" in the initial situation,

$$w^*_i(0)=p^0_i\gamma_i \Big/ \sum_{i=1}^{n} p^0_i\gamma_i \quad (i=1,\dots,n) \qquad (8)$$

are the "subsistence budget shares" in the initial situation, and

$$w_i(0)=[1-|\varphi(0)|]w_i^*(0)+|\varphi(0)|\beta_i \ ^{(2a)} \qquad (8a)$$

is the average budget share $w_i(0)=e_i(0)/y(0)$ in initial situation, and

$$P_i(1,0)= p_i^1/p_i^0 \quad (i=1,\dots,n) \qquad (9)$$

are the relative prices, and $\prod_{i=1}^{n}$ is the product operator denoting multiplication over i=l,...,n. This result, due to Geary [12], is discussed in Goldberger and Gamaletsos [13] and Phlips [16]. There it is compared with the conventional Laspeyre price index

$$L(1,0)=\sum_{i=1}^{n} w_i(0)P_i(1,0) \qquad (10)$$

It is shown that if the utility function is of the Klein-Rubin form, then L(1,0) can be written as

$$L(1,0)=\left[1-|\varphi(0)|\right]\sum_{i=1}^{n} w^*_i(0)P_i(1,0)+|\varphi(0)|\sum_{i=1}^{n}\beta_i P_i(1,0). \qquad (11)$$

A comparison of (11) and (6) shows that L(1,0) differs from C(1,0) only in that the second of its two terms is a weighted arithmetic, rather than geometric mean of the relative prices. Since geometric means cannot exceed the corresponding arithmetic means, this provides an example for the classical theorem that the Laspeyre index constitutes an upper bound to the true index, i.e.

$$L(1,0) \geq C(1,0). \tag{11a}$$

In contrast to Laspeyre index, the true index takes correct account of the substitution effect. Equation (6) also illustrates one of the points made from Frisch([3]); The true index is a function (of initial income) rather than a scalar. For consumers close to subsistence the first term on the right of equation (6) dominates: heavy weight is given to changes in the prices of goods which have high subsistence budget shares. For consumers far from subsistence the second term on the right dominates: heavy weight is given to changes in the prices of goods which have high marginal budget shares. The true cost of living index may well differ between workers and millioners![(4)]

4. The True Cost of Living Index in the Generalized Linear Expenditure System (GLES)

Another specific expression for the true cost of living index may be found in using the Gamaletsos utility function[(5)], which is of the form

$$u = \sum_{i=1}^{n} \delta_i^{(1-\varrho)} (q_i - \gamma_i)^{\varrho} \tag{12}$$

where δ's, γ's and ρ are parameters with

$0<\delta_i<1$ $(i=1,\ldots,n)$, $0<\rho<1$, $\Sigma^n_{i=1}\delta_i=1$, the function being defined only for $(q_i-\gamma_i)>0$ $(i=1,\ldots,n)$. The utility function (12) is therefore directly additive. With $0<\varrho<1$, $0<u_i$ and $(q_i-\gamma_i)>0$ all i, diminishing marginal utility prevails for each commodity. The GLES model which results from maximizing u subject to the budget constraint $y=\sum_{i=1}^{n} p_i q_i$,

with y and p's given is the form

$$e_i = p_i\gamma_i + \delta_i p_i^{\tau} \left[\sum_{i=1}^{n} \delta_i p_i^{\tau}\right]^{-1} \left(y - \sum_{i=1}^{n} p_i\gamma_i\right) \quad (i=1,\ldots,n) \tag{12a}$$

where $\tau=\varrho/(\varrho-1)$. With $0<\varrho<1$ we have $-\infty<\tau<0$.

We like to remind the main advantages of the GLES model compared to the LES model: first, the β_i's, which are the "marginal budget shares" in the GLES do depend on prices while in the LES model β_i's are constant. This means that in the GLES changes in the relative prices affects the β_i's and so it affects consumer choices. Second, in the GLES model the partial elasticity of substitution σ_{ij} (6) between the commodities q_i and q_j is not equal to 1, as it is in the LES model, but it is a parameter, which is between 0 and $+\infty$. These means that the GLES model can produce indifference curves of any shape in spite of the LES model.

The indirect utility function of (12) is

$$u^* = \sum_{i=1}^{n} \delta_i^{(1-\varrho)}[\beta_i p_i^{-1}(y - \sum_{i=1}^{n} p_i \gamma_i)]^{\varrho}$$
$$= (y - \sum_{i=1}^{n} p_i \gamma_i)^{\varrho} \sum_{i=1}^{n} \delta_i^{(i-\varrho)} \beta_i^{\varrho} p_i^{-\varrho} \tag{13}$$

where

$$\beta_i = \delta_i p_i^{\tau} / (\sum_{i=1}^{n} \delta_i p_i^{\tau}) \quad (i = 1,\ldots,n) \tag{14}$$

and $\tau=\varrho/(\varrho-1)$. With $0<\rho<1$ we have $-\infty<\tau<0$, while with $1<\varrho<\infty$ we have $1<\tau<\infty$. For $\rho \to 0$ this utility function becomes the Stone-Geary one, or for $\tau=0$ the GLES becomes the LES model.

In an initial income-price situation with y^0, $\mathbf{p}^0=(p_1^0,\ldots,p_n^0)'$ the indirect utility function (13) becomes

$$u^0 = u^* (y^0, \mathbf{p}^0) = (y^0 - \sum_{i=1}^{n} p_i^0 \gamma_i)^{\varrho} \sum_{i=1}^{n} \delta_i^{(1-\varrho)} (\beta_i^0 / p_i^0)^{\varrho}. \tag{15}$$

Now if price changes to $\mathbf{p}^1= (p_1^1,\ldots,p_n^1)'$ the change in utility is

$$\Delta u = (y^1 - \sum_{i=1}^{n} p_i^1 \gamma_i)^{\varrho} \left[\sum_{i=1}^{n} \delta_i^{(1-\varrho)} (\beta_i^1 / p_i^1)^{\varrho} \right] - (y^0 - \sum_{i=1}^{n} p_i^0 \gamma_i)^{\varrho} \left[\sum_{i=1}^{n} \delta_i^{(1-\varrho)} (\beta_i^0 / p_i^0)^{\varrho} \right] \quad (16)$$

and because $\Delta u = 0$ we have

$$\left[\frac{y^* - \sum_{i=1}^{n} p_i^1 \gamma_i}{y^0 - \sum_{i=1}^{n} p_i^0 \gamma_i} \right]^{\varrho} = \frac{\sum_{i=1}^{n} \delta_i^{(1-\varrho)} (\beta_i^0 / p_i^0)^{\varrho}}{\sum_{i=1}^{n} \delta_i^{(1-\varrho)} (\beta_i^1 / p_i^1)^{\varrho}} \quad (17)$$

Using equation

$$\frac{y^* - \sum_{i=1}^{n} p_i^1 \gamma_i}{y^0 - \sum_{i=1}^{n} p_i^0 \gamma_i} = \frac{(y^* - \sum_{i=1}^{n} p_i^1 \gamma_i)/y^*}{(y^0 - \sum_{i=1}^{n} p_i^0 \gamma_i)/y^0} \cdot \frac{y^*}{y^0} = \left[\frac{|\varphi(1)|}{|\varphi(0)|} \right] . T(1,0) \quad (18)$$

we obtain for the Generalized Linear Expenditure System (GLES) the true cost of living index, which is of the form

$$T(1,0) = \left[\frac{|\varphi(0)|}{|\varphi(1)|} \right] \left[\frac{\sum_{i=1}^{n} \delta_i^{(1-\varrho)} (\beta_i^0 / p_i^0)^{\varrho}}{\sum_{i=1}^{n} \delta_i^{(1-\varrho)} (\beta_i^1 / p_i^1)^{\varrho}} \right]^{1/\varrho} \quad (19)$$

where $\varphi(0) = \frac{y^0 - \sum_{i=1}^{n} p_i^0 \gamma_i}{y^0}$ and $\varphi(1) = \frac{y^* - \sum_{i=1}^{n} p_i^1 \gamma_i}{y^1}$ are the supernumerary ratios in an initial situation, period 0, and in a second situation, period 1, respectively.

5. The True Cost of Living Index in the Dynamic Linear Expenditure System (DLES)

The LES model, upon which is based the previous true cost of living index, has been dynamized to take into account changes in tastes. It is worthwhile, I think, to try to construct truly dynamic indices, such they are affected by taste changes even if all prices remain constant.

The "cardinal" index belongs to the class of "temporal" indices. In this traditional "cardinal" viewpoint we compare incomes in two different periods assuming that $\Delta u=0$

The Dynamic Linear Expenditure System (DLES)

$$e_{it}=p_{it}\gamma_i^*+\beta_i(y_t-\sum_{i=1}^{n}p_{it}\gamma_i^*)+\alpha_i p_{it-1}q_{it-1}-\beta_i\sum_{i=1}^{n}\alpha_i p_{it-1}q_{it-1} \quad (20)$$

$(i=1,...,n;t=1,...,T)$

comes from the (static) Linear Expenditure System (LES)

$$e_{it}=p_{it}\gamma_i+\beta_i(y-\sum_{i=1}^{n}p_{it}\gamma_i) \quad (21)$$

$(i=1,...,n;t=1,...,T)$

by assuming that the "minimum required expenditures" $p_{it}\gamma_i$ are functions of last period expenditures, that is

$$\tilde{e}_{it}=p_{it}\gamma_i=p_{it}\gamma_i^*+\alpha_i e_{it-1} \quad (22)$$

$(i=1,...,n;t=1,...,T)$

where $\tilde{e}_{it}$ is the minimum required expenditure for commodity i, e_{it-1} is the last period expenditure for commodity i and γ_i^*'s and α_i's are parameters[7].

This dynamic specification of the LES model is based on a "habit formation hypothesis" adjusted for the rate of inflation. If there is no "habit formation hypothesis" then α_i's in equation (22) will all be zero, so $\gamma_i=\gamma_i^*$ and $\tilde{e}_{it}=p_i\gamma_i$.

To see more explicitly how inflation affects habits we divide (22) by p_{it}, which becomes

$$\tilde{q}_{it} = \gamma_i = \gamma_i^* + \alpha_i \left(\frac{p_{it-1}}{p_{it}}\right) q_{it-1} = \gamma_i^* + \alpha_i \left(\frac{1}{r_i}\right) q_{it-1} \tag{22a}$$

where $r_i = p_{it}/p_{it-1}$ is the rate of inflation of i commodity.

Now if r=1 i.e. no inflation in i commodity, then we have only habit formation hypothesis [8]. But if r is different than 1 which is the most usual case (and usually r>1) then an increase of inflation (of price) of the i commodity, i.e. an increase of r it affects inversely the habit formation hypothesis.

In other words, as r increases the second terms on the right of the equation (22a) decreases. An increase of the price of i commodity it will make the consumer to change habit, and instead of buying i commodity (which he used to consume) he will buy a substitute of it, say j commodity.

The indirect Klein-Rubin utility upon which is based the DLES model is of the form

$$u_t^* = \sum_{i=1}^{n} \beta_i \log\left[(\beta_i / p_{it})(y_t - \sum_{i=1}^{n} p_{it}\gamma_i^* - \sum_{i=1}^{n} \alpha_i e_{it-1})\right] = \sum_{i=1}^{n} \beta_i \log\beta_i - \sum_{i=1}^{n} \beta_i \log p_{it} + \log\left(y_t - \sum_{i=1}^{n} p_{it}\gamma_i^* - \sum_{i=1}^{n} \alpha_i e_{it-1}\right) \tag{23}$$

$(i=1,\dots,n;\ t=1,\dots,T)$[9].

This utility function in an initial income-price situation with y^1 $\mathbf{p}^1=(p_1^1,\dots,p_n^1)'$ is of the form

$$u_t^1 = u_t^*(y^1, \mathbf{p}^1) = \sum_{i-1}^{n} \beta_i \log\beta_i + \log(y^1 - \sum_{i-1}^{n} p_i^1\gamma_i^* - \sum_{i=1}^{n} \alpha_i p_i^0 q_i^0) - \sum_{i=1}^{n} \beta_i \log p_i^1. \tag{24}$$

If price changes to $\mathbf{p}^2=(p_1^2,\dots,p_n^2)'$ then the resulting change in utility is

$$\Delta u = \log\,(y^2 - \sum_{i=1}^{n} p_i^2 \gamma_i^* - \sum_{i=1}^{n} \alpha_i p_i^1 q_i^1) - \log\,(y^1 - \sum_{i=1}^{n} p_i^1 \gamma_i^* - \sum_{i=1}^{n} \alpha_i p_i^0 q_i^0) - \sum_{i=1}^{n} \beta_i \log(p_i^2 / p_i^1). \tag{25}$$

Now the cardinal true index based on DLES model is equal to the ratio y^*/y^1, where y^* is the income required to maintain utility unchanged.

Assuming $\Delta u = 0$ we obtain from (25) the equation

$$\log\left[(y^* - \sum_{i=1}^{n} p_i^2 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^1) / (y^1 - \sum_{i=1}^{n} p_i^1 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^0)\right] = = \sum_{i=1}^{n} \beta_i \log(p_i^2 / p_i^1). \tag{26}$$

Taking antilogs we have

$$\frac{y^* - \sum_{i=1}^{n} p_i^2 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^1}{y^1 - \sum_{i=1}^{n} p_i^1 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^0} = \prod_{i=1}^{n} \left(\frac{p_i^2}{p_i^1}\right)^{\beta i}. \tag{27}$$

From the above equation (27) we obtain the equation

$$y^* = (y^1 - \sum_{i=1}^{n} p_i^1 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^*) \prod_{i=1}^{n} (\frac{p_i^2}{p_i^1})^{\beta i} + \sum_{i=1}^{n} p_i^2 \gamma_i^* + \sum_{i=1}^{n} \alpha_i . e_i^1$$
$$= y^1 \left\{ \prod_{i=1}^{n} \left(\frac{p_i^2}{p_i^1}\right)^{\beta i} - \frac{\sum_{i=1}^{n} p_i^1 \gamma_i^*}{y^1} \prod_{i=1}^{n} \left(\frac{p_i^2}{p_i^1}\right)^{\beta i} - \frac{\sum_{i=1}^{n} \alpha_i e_i^0}{y^1} \prod_{i=1}^{n} \left(\frac{p_i^2}{p_i^1}\right)^{\beta i} + \frac{\sum_{i=1}^{n} p_i^2 \gamma_i^*}{y^1} + \frac{\sum_{i=1}^{n} \alpha_i e_i^1}{y^1} \right\}. \tag{28}$$

The (dynamic) true cost of living index based on DLES model is therefore given by the form

$$C^{*}(2,1)=\left|\varphi^{1}\right|\prod_{i=1}^{n} p_{i}^{\beta i}+\left(1-\left|\varphi^{1}\right|\right)\left(\frac{\sum_{i=1}^{n} p_{i}^{2}\gamma_{i}^{*}+\sum_{i=1}^{n}\alpha_{i}e_{i}^{1}}{\sum_{i=1}^{n} p_{i}^{1}\gamma_{i}^{*}+\sum_{i=1}^{n}\alpha_{i}e_{i}^{0}}\right) \qquad (29)$$

where

$$\varphi^{1}=\frac{y^{1}-\sum_{i=1}^{n} p_{i}^{1}\gamma_{i}^{*}-\sum_{i=1}^{n}\alpha_{i}e_{i}^{0}}{y^{1}} \qquad \text{and} \qquad P_{i}=\frac{p_{i}^{2}}{p_{i}^{1}}$$

In static theory, as we proved before, the Laspeyre index is an upper bound for the true index and biased upwards whenever all prices do not change proportionally.

In dynamic theory, however, a Laspeyre index loses much of its meaning, and is no longer an upper bound of the true cost of living index.

6. The True Cost of Living Index in the Dynamic Generalized Linear Expenditure System GLES

The GLES model, upon which is based the true cost of living index $T(1,0)$, has been dynamized, like the LES model, to take into account changes in tastes [10]

The Dynamic Generalized Linear Expenditure System (DGLES) is of the form

$$e_{it}=p_{it}\gamma_{i}^{*}+\beta_{i}\left(y_{t}-\sum_{i=1}^{n} p_{it}\gamma_{i}^{*}-\sum_{i=1}^{n}\alpha_{i}p_{it-1}\,q_{it-1}\right)+\alpha_{i}p_{it-1}q_{it-1} \qquad \textbf{(30)}$$

$(i=1,\ldots,n;\ t=1,\ldots,T)$

where γ_i*'s and α_i's are parameters and β_i is specified by equation (14). The dynamic form of this model is based on a habit formation hypothesis adjusted for the rate of inflation, like in the DLES model.

The indirect utility function upon which is based the (DGLES) model is of the form

$$u_t^* = \sum_{i=1}^{n} \delta_i^{(1-\varrho)} \left[\beta_i p_{it}^{-1} (y_t - \sum_{i=1}^{n} p_{it} \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_{it-1} \right]^{\varrho}$$

$$= (y_t - \sum_{i=1}^{n} p_{it} \gamma_t^* - \sum_{i=1}^{n} \alpha_i e_{it-1})^{\varrho} \sum_{i=1}^{n} \delta_i^{(1-\varrho)} \left(\frac{\beta_i}{p_i} \right)^{\varrho} . \quad (31)$$

In an initial income-price situation with $y^1, \mathbf{p}^1 = (p_1{}^1, \ldots, p_n{}^1)'$ the above equation becomes

$$u_t' = u_t^*(y^1, \mathbf{p}^1) = (y^1 - \sum_{i=1}^{n} p_i^1 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^0)^{\varrho} \sum_{i=1}^{n} \delta_i^{(1-\varrho)} \left(\frac{\beta_i^1}{p_i^1} \right)^{\varrho} . \quad \textbf{(32)}$$

Now if price changes to $\mathbf{p}^2 = (p_1{}^2, \ldots, p_n{}^2)'$ the change in utility is

$$\Delta u = \left(y^2 - \sum_{i=1}^{n} p_i^2 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^1 \right)^{p} \sum_{i=1}^{n} \delta_i^{(1-\varrho)} \left(\frac{\beta_i^2}{p_i^2} \right)^{\varrho}$$

$$- \left(y^1 - \sum_{i=1}^{n} p_i^1 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^0 \right)^{\varrho} \sum_{i=1}^{n} \delta_i^{(1-\varrho)} \left(\frac{\beta_i^1}{p_i^1} \right)^{\varrho} \quad \textbf{(33)}$$

from which we obtain equation

$$\left[\frac{y^* - \sum_{i=1}^{n} p_i^2 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^1}{y^1 - \sum_{i=1}^{n} p_i^1 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^0} \right]^{\varrho} = \frac{\sum_{i=1}^{n} \delta_i^{(1-\varrho)} \left(\frac{\beta_i^1}{p_i^1} \right)^{\varrho}}{\sum_{i=1}^{n} \delta_i^{(1-\varrho)} \left(\frac{\beta_i^2}{p_i^2} \right)^{\varrho}} \quad \textbf{(34)}$$

assuming that $\Delta u = 0$

Finally using equation

$$\frac{\left(y^* - \sum_{i=1}^{n} p_i^2 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^1\right) / y^*}{\left(y^1 - \sum_{i=1}^{n} p_i^1 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^0\right) / y^1} \cdot \left(\frac{y^*}{y^1}\right) = \frac{\varphi^2}{\varphi^1} \cdot T^*(2,1) \tag{35}$$

we obtain for **the DGLES** model **the** (dynamic) true cost of **living** index, which is of the form

$$T^*(2,1) = \left(\frac{\varphi^1}{\varphi^2}\right) \left[\frac{\sum_{i=1}^{n} \delta_i^{(1-\varrho)} (\beta_i^1 / p_i^1)^{\varrho}}{\sum_{i=1}^{n} \delta_i^{(1-\varrho)} (\beta_i^2 / p_i^2)^{\varrho}}\right]^{1/\varrho} \tag{36}$$

where

$$\varphi^1 = \frac{y^1 - \sum_{i=1}^{n} p_i^1 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^0}{y^1} \quad \text{and} \quad \varphi^2 = \frac{y^2 - \sum_{i=1}^{n} p_i^2 \gamma_i^* - \sum_{i=1}^{n} \alpha_i e_i^1}{y^2}$$

are the dynamic supernumerary ratios in an initial situation, period 1, and in a second situation, period 2, respectively. For $\rho \to 0$ the term in brackets on the right of the equation (36), tends to 1. So equation (36) becomes $T^*(2,1)=\varphi^1/\varphi^2$[11]

7. Final Comments

The criterion which leads to the true cost of living index is taken from the theory of consumer demand. Once we know the utility function, upon which is based a complete demand system (like the LES and the GLES models), we can construct true cost of living indices, as we presented.

However, the problem of these (LES and GLES) models is that they are static. In these static models we do not take into account the phenomenon of changing tastes. To do this is to dynamize the utility function rather than the demand systems, as we did in this work.

A preliminary question is why we prefer to do this approach. The answer to this question is that the maximization of a utility function (in a static or in a dynamic form) will provide a complete system of demand equations, which will have the property of being theoretically plausible. In other words, all "classical" restrictions will be automatically satisfied. If one starts to dynamize a complete demand system (and not a utility function) then he has to impose these restrictions. Even in a static world, this is a very difficult task. In a dynamic setting, it seems hard to know how to proceed(12)

The advantage of the true cost of living index compared to the Laspeyre index is given previously. Going a step farther, we show the advantage of using "dynamic" true cost of living index instead of using "static" true cost of living index. This advantage is to take into account changes in consumer tastes. By using DLES and better by using DGLES model, which are coming from a dynamic utility function, to construct a dynamic true cost of living index we introduce habit formation hypothesis and changes in relative prices (inflation) into these indices.

This means that even when there is no change in the general price index, when we have changes in consumer tastes (either by changes in habits or/and by changes in relative prices) then we will have changes in "dynamic" true cost of living index(13)

Notes

(1) See Klein and Rubin [15], Geary [12], Goldberger and Gamaletsos [13,14]. Professor L.R. Klein has a Nobel Prize in Economics 1980.

(2) See Stone [22,23] Nobel Prize in Economics 1984, Pollak and Wales [21], Phlips [16], Gamaletsos [3].

(2a) As we see $w_i^*(0)$ is positive and less than unity; also $-\varphi(0)$ is positive and less than unity when the domain of the LES is confined to points where $(y-\sum_{i=1}^{n} p_i\gamma_i)\geq 0$. When equality holds i.e. when $\varphi(0)=0$ t h $w_i=w_i^*$; that is there no "supernumerary income" left to be allocated. As the supernumerary ratio increases either by an increase in income or by a decrease in prices the consumer allocates more of his income according to marginal (β;) rather than subsistence w_i^* budget shares.

(3) R. Frisch has a Nobel Prize in Economics 1969.

(4) In other words, for poor people the Laspeyre index is the same with the true cost of living index.

(5) This utility function as well as the GLES model has been presented by the author of this paper (with professor A.S. Goldberger) in the meetings of the Econometric Society at Washington D.C., in December 1967 and at Amsterdam in September 1968. This work contains the most important part of the author's doctoral dissertation [3].

(6) The σ_{ij} is related to the parameter ρ or τ by the form $\sigma_{ij}=(\frac{1}{\varrho-1})=\tau-1$. For $\propto>\varrho>1$ we have $1<\tau<\infty$ and $1<\sigma_{ij}<\propto$. In the LES model σ_{ij} is equal to 1.

(7) For a different dynamic form of this model see Pollak [20] and Phlips [10]

(8) This is actually Pollak' s dynamic specification of the LES model. See Pollak [20].

(9) The difference between u* and u_t* is that consumer preferences in u* are constant, while in u_t* these preferences do changes if habits changes, and prices changes.

(10) See Gamaletsos [10,11]

(11)If we assume for example t h $y^2=100$, $y^1=99$, $\Sigma p^2\gamma^*=90$ and $\Sigma p^1\gamma^*=80$, we have $(y^1-\Sigma p^1\gamma^*)=19$, $(y^2-\Sigma p^2\gamma^*)=10$ (assuming no changes in habits), then $T^*(2,1)=\varphi^1/\varphi^2=$ (19/10) (100/99) = (1,9)x(1,01) = 1,919 (assuming that ρ->0). In this example $\varrho\ y^2/y^1=1{,}01$ while $T^*=1{,}919$! So we should give an increase in income 1,919 instead of 1,01 to keep consumers even.

(12) A good example is to compare the "Rotterdam School" model or the "Almost Ideal Demand System" to the GLES model. The advantages of the GLES model compared to those two models are given in my work [8].

(13) In this work purely econometric considerations have been avoided. I did this in purpose, because I would like to pay attention to the advantages of using the dynamic cost of living index based on the DLES or my DGLES model. It remains to the future researcher to estimate those indices.

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