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A MULTI-DYNAMIC GENERALIZED EXPENDITURE SYSTEM OF THE DEMAND FOR CONSUMER GOODS

By

Theodore Gamaletsos
Emeritus Professor, University of Piraeus, Greece

Abstract

The dynamic form of the GLES model, the DGLES model, has been presented by the author at the International Econometic Society European Meeting in 1979 and is published in the Proceedings of the Econometric Society European Meeting 1979 (Selected Econometric Papers in Memory of Stefan Valavanis) by North-Holland, chapter 17 pp. 379-389.

In this article I will present a more dynamic form of the DGLES model which I call Multi-Dynamic Generalized Linear Expenditure System, in short MDGLES model, in which will not destroy the basic (classical) properties of the previous GLES and DGLES models. For better understanding of the presentation of the MDGLES model I will first present the DGLES upon which is based the MDGLES model.

1. The DGLES model

The GLES model in Gamaletsos (1970, 1973) permints the marginal budget shares to depend on prices. In this respect it is more general than Stone's LES model. The GLES model based on an additive utility function, which is a monotonic transformation of the CES-type utility function, is of the form

$$e_i = p_i\gamma_i + \beta_i(y - \sum_{j=1}^{n} p_j\gamma_j) \quad (i=1,...,n) \tag{1}$$

where ei is the expenditure on commodity i, p_i is the price of commodity i, $y = \sum_{i=1}^{n} e_i$ is "income" (total consumer expenditures in current prices) and $\beta_i = \delta_i p_i^{\tau}(\Sigma_{j=1}^{n}\delta_j p_j^{\tau})^{-1}$, $\tau = p/(p-1)$, and δ's, γ's and τ are parameters with $0<\delta_i<1$, $\Sigma_{i=1}^{n}\delta_i = 1, -\infty < \tau < +\infty$.(1)

The β_i's are identified as "marginal budget shares", they give the proportions in which incremental income is allocated. As in the LES model, the β_i's do not depend in income, so that the Engel curves are still linear. But in the GLES model the marginal budget shares do depend on prices. However, in the GLES model the γ_i's which are identified as "minimum required quantities", are constant over time. This defect of the GLES model can be removed by making the $p_i \gamma_i$'s, the "minimum required expenditures", functions of last period expenditures. Specifically we assume that

$$\tilde{e}_{it} = p_{it}\gamma_i = p_{it}\gamma_i^* + \alpha_i e_{it-1} \quad (i=1,...,n)\ (t=1,...,T) \tag{2}$$

where $\tilde{e}_{it}$ is the "minimum required expenditure" for commodity i, e_{it-1} is the last period expenditure for commodity i, and γ_i^*'s α_i's are parameters.

Equation (2) introduces a habit formation hypothesis adjusted for the rate of inflation. We can see this if we divide eq. (2) by pit, which becomes

$$\gamma_i = \gamma_i^* + \alpha_i q_{it-1}\left(\frac{p_{it-1}}{p_{it}}\right) \quad (i=1,...,n)\ (t=1,...,T) \tag{2a}$$

According to Pollak (1970) the $p_i\gamma_i$'s can be interpreted as a "physiologically necessary" component of $\tilde{e}_{it}$ and $\alpha_i e_{it-1}$ as the "physiologically component".(2)

With this generalization the GLES model takes the form

$$e_{it} = p_{it}\gamma_i^* + \beta_i(y_t - \sum_{j=1}^{n} p_{jt}\gamma_j^*) + \alpha_i e_{it-1} - \beta_i \sum_{j=1}^{n}\alpha_j e_{jt-1} \quad (i=1,...,n)\ (t=1,...,T) \tag{3}$$

which is the dynamic generalized linear expenditure system (DGLES). As we can see, this generalization of the GLES does not destroy the adding up (the budget constraint) criterion.

According to the DGLES model the expenditures on commodity i does not depend only on income and all prices, but it depends also on last period expenditures on commodity i and on last period expenditures on all other commodities.

The use of the DGLES model permits us to compare the short- and long-run effects of income and own- and cross-price on expenditures.

The long-run equilibrium expenditures can be found by solving the above short-run expenditure functions (3) under the assumption that $e_{it}=e_{it-1}=e_i$, for all i. The "long-run" expenditure or "equilibrium" functions are of the form

$$e_i = p_i\tilde{\gamma}_i^* + \tilde{\beta}_i(y - \sum_{j=1}^{n} p_j\tilde{\gamma}_j^*) \quad (i=1,...,n) \tag{4}$$

where

$$\tilde{\gamma}_i^* = \gamma_i^* /(1-\alpha_i) \quad (i=1,...,n) \tag{5}$$

and

$$\tilde{\beta}_i = \delta_i p_i^\tau (1-\alpha_i)^{-1} / \{\sum_{j=1}^{n} \delta_j p_j^\tau (1-\alpha_j)^{-1}\} \quad (i=1,...,n) \tag{6}$$

These long-run expenditure functions can be found as follows. The first-order maximization conditions corresponding to the CES utility function, which is the basis of the DGLES model, are

$$\rho\delta_i^{(1-\rho)}(q_i-\gamma_i)^{(\rho-1)}=\lambda p_i \quad (i=1,...,n). \tag{7}$$

Solving the above equation (7) with respect to q_i, we get

$$q_i=\gamma_i+(\rho^{-1}\lambda p_i)^{1/(\rho-1)}\delta_i \quad (i=1,...,n), \tag{8}$$

which, multiplied by p_i, becomes

$$p_iq_i=p_i\gamma_i+(\rho^{-1}\lambda p_i)^{1/(\rho-1)}\delta_i p_i \quad (i=1,...,n). \tag{9}$$

Now in the long-run equilibrium the $p_i\gamma_i$'s given by $p_i\gamma_i=p_i\gamma_i^*+\alpha_i e_i$, where e_i is the long-run equilibrium value of e_{it}. Substituting the long-run equilibrium values of the minimum required expenditures into eq. (9) we obtain

$$e_i=p_i\gamma_i^*+\alpha_i e_i+(\rho^{-1}\lambda' p_i)^{1/(\rho-1)}\delta_i p_i \quad (i=1,...,n) \tag{10}$$

where λ' is the "long-run" Lagrange multiplier.

Solving with respect to e_i, and summing over i, we have

$$y=\sum_{i=1}^{n}p_i\gamma_i^*(1-\alpha_i)^{-1}+\lambda'\rho^{-1}\sum_{i=1}^{n}p_i^{e/(\rho-1)}\delta_i(1-\alpha_i)^{-1} \tag{11}$$

from which we obtain the "long-run" value of the Lagrange multiplier

$$\lambda'=\rho y\left\{\sum_{i=1}^{n}\delta_i p_i^{e/(e-1)}(1-\alpha_i)^{-1}\right\}^{-1}$$
$$-\rho\left\{\sum_{i=1}^{n}\delta_i p_i^{e/(e-1)}(1-\alpha_i)^{-1}\right\}^{-1}\sum_{i=1}^{n}\rho_i\gamma_i^*(1-\alpha_i)^{-1} \tag{12}$$

Now inserting (12) into (9) and using $\tau=\varrho/(\varrho\text{-}1)$ we obtain the long-run expenditure equations (4).

The sort-run income slope of commodity i is given by

$$\beta_i=\partial e_i/\partial y$$
$$=\delta_i\rho_i^{\tau}\left[\sum_{j=1}^{n}\delta_j\rho_j^{\tau}\right]^{-1} \quad (i=1,...,n) \tag{13}$$

while the long-run income slope for the same commodity is given by eq. (6). The $\tilde{\beta}_i$ is equal to β_i for $\alpha i=0$.

The sort-run income elasticity of commodity i is given by

$$\eta_i = (\partial q_i / \partial y)/(q_i / y)$$
$$= \beta_i / w_i \qquad (i=1,...,n) \tag{14}$$

where $w_i = e_i/y$ is the short-run average budget share, while the long-run income elasticity of the same commodity is

$$\tilde{\eta}_i = \tilde{\beta}_i / \tilde{w}_i \quad (i=1,...,n) \tag{15}$$

which becomes equal to ηi for $\alpha_i=0$, and where $\tilde{w}_i = e_i / y$ is the long-run average budget share.

The short-run uncompensated (Cournot) price elasticities are of the form

$$\eta_{ij} = \begin{cases} -1+(1-\beta_i)\{(1-\tau)\gamma_i^* q_i^{-1}+\tau\}-\tau(1-\beta_1)\alpha_i e_{it-1}e_{it}^{-1}, & for\ i=j \\ -\beta_i\gamma_j^*\rho_j e_i^{-1}-\tau\beta_j(1-\gamma_i^* q_{it}^{-1}-\alpha_i e_{it-1}e_{it}^{-1}), & for\ i\neq j \end{cases} \quad (i,j=1,...,n) \tag{16}$$

where $-\infty<\eta_{ij}<0$, for $0<\beta_i<1$, $q_i>\gamma_i^*$, $-\infty<\tau\leq 1$ and for $0<\beta_i<1$, $q_i<\gamma_i^*$, $1<\tau<+\infty$. The long-run uncompensated price elasticities η_{ij} are of the form

$$\tilde{\eta}_{ij} = \begin{cases} -1+(1-\tilde{\beta}_i)[(1-\tau)\tilde{\gamma}_i q_i^{-1}+\tau], & for\ i=j \\ -\tilde{\beta}_i\tilde{\gamma}_j^*\rho_j e_i^{-1}-\tau\tilde{\beta}_j\tilde{\beta}_j\left[y-\sum_{j=1}^{n}\rho_j\tilde{\gamma}_j^*\right]e_i^{-1} & for\ i\neq j \end{cases} \quad (i,j=1,...,n) \tag{17}$$

The compensated short-run and long-run price elasticities for the DGLES model can be obtained from the Slutsky equation

$$\eta_{ij}^* = \eta_{ij} + \eta_i w_j \quad (i,j=1,...,n) \tag{18}$$

2. The theoretical MDGLES model

In the DGLES model we assume that the "minimum required expenditure" depends only on the last period expenditures (the well-known "habit formation hypothesis in the consumer demand theory) [equation 2]. This defect of the DGLES model, when we believe that

$$\tilde{e}_{it} = p_{it}\gamma_i = p_{it}\gamma_i^* + \alpha_i\mu e_{it-1} + \alpha_i\mu^2 e_{it-2} + \ldots + \alpha_i\mu^N e_{it-N} \quad (i=1,...,n;t=1,...,\tau;\nu=1,...,N), \tag{19}$$

where $\tilde{e}_{it}$ is the "minimum required expenditure" for commodity i, e_{it-1}, e_{it-2},..., e_{it-N} are the past periods expenditures for commodity i, and γ_i^*'s, α_i's are parameters and the parameter μ is determined by Koyck's specification

$$\varkappa_{\nu i} = \alpha_i \mu^\nu \quad (i=1,...,n;\nu=0,...,N) \tag{20}$$

with $0\leq\mu\leq 1$.

Using equation (20) into (19) we have

$$\tilde{e}_{it} = p_{it}\gamma_i = p_{it}\gamma_i^* + (\kappa_{1i}e_{it-1} + \kappa_{2i}e_{it-2} + ... + \kappa_{Ni}e_{it-N}) \quad (21)$$

or

$$\tilde{e}_{it} = p_{it}\gamma_i = p_{it}\gamma_i^* + \alpha_i(1-\mu L)^{-1}e_{it-1} \quad (22)$$

where L is a lag operator and $(1-\mu L)^{-1} = \frac{1}{1-\mu L} = \mu^0 + \mu^1 + ... + \mu^N$.

Equation (22) will take the form

$$(1-\mu L)\tilde{e}_{it} = (1-\mu L)p_{it}\gamma_i^* + \alpha_i e_{it-1} \quad (23)$$

or

$$\begin{aligned}\tilde{e}_{it} &= \mu\tilde{e}_{it} + p_{it}\gamma_i^* - \mu p_{it-1}\gamma_i^* + \alpha_i e_{it-1} \\ &= \mu\tilde{e}_{it-1} + \gamma_i^*(p_{it} - \mu p_{it-1}) + \alpha_i e_{it-1}\end{aligned} \quad (23a)$$

For μ=0 equation (23a) takes the form

$$\tilde{e}_{it} = p_{it}\gamma_i^* + \alpha_i e_{it-1} \quad (i=1,...,n;t=1,...,T) \quad (23b)$$

which is the same with equation (2). In other words for μ=0 the MDGLES model becomes the DGLES model.

If μ=1 equation (23a) becomes

$$\tilde{e}_{it} = \tilde{e}_{it-1} + (p_{it} - p_{it-1})\gamma_i^* + \alpha_i e_{it-1} \quad (23c)$$

or

$$\Delta\tilde{e}_{it} = \tilde{e}_{it} - \tilde{e}_{it-1} = \gamma_i^*\Delta p_{it} + \alpha_i e_{it-1} \quad (23d)$$

From equation (23d) we see that if $p_{it}=p_{it-1}$, that is we have no inflation, $\Delta\tilde{e}_{it} = \alpha_i e_{it-1}$. This means that if we have no change in the relative prices, the changes at the "minimum required expenditures" depend only upon the "psychologically component". If we have also changes in relative prices then the changes in the "minimum required expenditures" depend upon the "physiologically necessary" component and the "psychologically component".(3)

Now if we divide equation (23a) by p_{it} we have

$$\gamma_i = \mu q_{it-1}\left(\frac{p_{it-1}}{p_{it}}\right) + \gamma_i^* - \gamma_i^*\mu\left(\frac{p_{it-1}}{p_{it}}\right) + \alpha_i q_{it-1}\left(\frac{p_{it-1}}{p_{it}}\right) \quad (i=1,...,n;t=1,...,T) \quad (24)$$

or

$$\gamma_i = \gamma_i^* + \alpha_i q_{it-1}\left(\frac{p_{it-1}}{p_{it}}\right)\left(1-\mu\frac{p_{it-1}}{p_{it}}\right)^{-1} \quad (i=1,...,n;t=1,...,T)^{(4)} \quad (24a)$$

Substitutions equation (23a) into equation (1) we have

$$e_{it} = p_{it}\gamma_i^* + \beta_i\left[(y_t - \mu y_{t-1}) - \left(\sum_{j=1}^{n} p_{it}\gamma_j^* - \mu\sum_{j=1}^{n} p_{jt-1}\gamma_j^*\right)\right]$$
$$- \mu p_{it-1}\gamma_i^* + (\mu + \alpha_i)e_{it-1} - \beta_i\sum_{j=1}^{n}\alpha_j e_{jt-1} \qquad (i,j=1,...,n;t=1,...,T) \qquad (25)$$

which is the **Multi-Dynamic Generalized Linear Expenditure System** (MDGLES). For $\mu=0$ the MDGLES becomes the DGLES model. As we can see the generalization of the DGLES model does not destroy the adding up (the budget constraint) criterion.(5)

The difference between DGLES model (3) and the MDGLES (25) is that using the lest model we assume that the "habit formation hypothesis" holds for the whole part periods and not only for the (one) last period. This means that the "psychological necessary" component of $\tilde{e}_{it}$ does not depend only upon the last period expenditures of the commodity i (e_{it-1}), but the consumer is affected of all the last periods expenditures, i.e. the $\tilde{e}_{it}$ is a function of e_{it-1}, e_{it-2},...,e_{it-N}. The habits are getting weaker as we go longer in the past (because $0\leq\mu\leq1$).

The other difference of the MDGLES model from the DGLES is that in the MDGLES we have to estimate one more parameter $\mu(0\leq\mu\leq1)$.(6)

The use of the MDGLES model permits us to compare the short- and long-run effects if income and own - and cross-price on expenditures (as we did using the DGLES model).

The long-run equilibrium expenditures can be found by solving the above short-run expenditure function (25) under the assumption that $e_{it}=e_{it-1}=e_{it-2}$ $...=e_{it-N}=e_i$, $p_{it}=p_{it-1}=p_{it-2}=...=p_{it-N}=p_i$, $y_t=y_{t-1}=y_{t-2}=...=y_{t-N}=y$, for all i commodities.

The "logn-run" expenditure or "equilibrium" functions are of the form

$$e_i = p_i\tilde{\gamma}_i^* + \tilde{\beta}_i\left[y - \sum_{j=1}^{n} p_i\tilde{\gamma}_j^*\right] \quad (i=1,...,n) \qquad (26)$$

where

$$\tilde{\gamma}_i^* = \left(\frac{1-\mu}{1-\mu-\alpha_i}\right)\gamma_i^* \quad (i=1,...,n) \qquad (27)$$

and

$$\tilde{\beta}_i = \left[\delta_i p_i^\tau\left(\frac{1-\mu}{1-\mu-\alpha_i}\right)\right] / \left[\sum_{j=1}^{n}\delta_j p_j^\tau\left(\frac{1-\mu}{1-\mu-\alpha_j}\right)\right] \quad (i,j=1,...,n) \qquad (28)$$

These long-run expenditure functions can be found as follows. The first-order maximization conditions corresponding to the CES utility functions, which is the basis of the MDGLES model (as it is of the DGLES) are:

$$\frac{\partial u}{\partial q_i} - \lambda p_i = 0 \text{ for all } i \tag{29}$$

which in the case of CES utility function are

$$\rho \delta^{(1-e)} (q_i - \gamma_i)^{(e-1)} = \lambda p_i \text{ for all } i \tag{30}$$

which is the same with equation (7). Solving the above equation (30) with respect to q_i, we get

$$q_i = \gamma_i + (\rho^{-1} \lambda p_i)^{1/(\rho-1)} \delta_i \quad (i=1,...,n) \tag{31}$$

which multiplied by pi becomes

$$p_i q_i = p_i \gamma_i + (\rho^{-1} \lambda p_i)^{1/(\rho-1)} \delta_i p_i \quad (i=1,...,n) \tag{32}$$

which is the same with equation (9).

Now in the long-run equilibrium the $p_i\gamma_i$'s are given by

$$p_i \gamma_i = p_i \gamma_i^* + \alpha (1 - \mu L)^{-1} e_i \quad (i=1,...,n) \tag{33}$$

or

$$(1 - \mu L) p_i \gamma_i = (1 - \mu L) p_i \gamma_i^* + \alpha_i e_i \tag{34}$$

or

$$(1 - \mu) p_i \gamma_i = (1 - \mu) p_i \gamma_i^* + \alpha_i e_i \quad (i=1,...,n) \tag{34a}$$

because the lag operator L does not work when we are in "equilibrium". Finally we get

$$p_i \gamma_i = p_i \gamma_i^* + \frac{\alpha_i}{1-\mu} e_i \tag{35}$$

or

$$\gamma_i = \gamma_i^* + \left(\frac{\alpha_i}{1-\mu} \right) q_i \quad (i=1,...,n). \tag{36}$$

Substituting equation (35) into (32) we obtain

$$e_i = p_i \gamma_i^* + \left(\frac{\alpha_i}{1-\mu} \right) e_i + (\rho^{-1} \lambda' p_i)^{1/(e-1)} \delta_i p_i \tag{37}$$

where λ' is the "long-run" Lagrange multiplier.
Solving with respect to e_i we have

$$[1-\alpha_i(1-\mu)^{-1}]e_i = p_i\gamma_i^* + (\rho^{-1}\lambda p_i)^{1/(e-1)}\delta_i p_i \tag{38}$$

or

$$e_i = p_i\tilde{\gamma}_i^* + \left(\frac{1-\mu}{1-\mu-\alpha_i}\right)\delta_i p_i^\tau (\lambda')^{1/(e-1)}\rho^{1/(e-1)} \tag{39}$$

where $\tilde{\gamma}_i^*$ is given by equation (27) and $\tau = \frac{\rho}{(\rho-1)}$. Now summing over i we have

$$y = \sum_{i=1}^{n} e_i = \sum_{i=1}^{n} p_i\tilde{\gamma}_i^* (\lambda')^{1/(e-1)}\rho^{1/(1-\rho)}\sum_{i=1}^{n}\delta_i p_i^\tau\left(\frac{1-\mu}{1-\mu-\alpha_i}\right) \tag{40}$$

from which we obtain the "long-run" Lagrange multiplier

$$\lambda' = \rho\left\{\sum_{i=1}^{n}\delta_i p_i^\tau\left(\frac{1-\mu}{1-\mu-\alpha_i}\right)\right\}^{(1-\rho)}\left(y-\sum_{i=1}^{n}p_i\tilde{\gamma}_i^*\right)^{(\rho-1)} \tag{41}$$

where $\tau = \frac{\rho}{(\rho-1)}$

Now using (41) into (32) we obtain the long-run expenditure equations (26). The short-run income slope of the commodity i is given by

$$\beta_i = \frac{\partial e_i}{\partial y} = \delta_i p_i^\tau\left[\sum_{j=1}^{n}\delta_j p_j^\tau\right]^{-1} \quad (i=1,\ldots,n). \tag{42}$$

The "long-run" income slopes for the same commodity is given by equation (28). The $\tilde{\beta}_i$ is equal to β_i for $\alpha_i=0$.
The short-run income elasticity of commodity is

$$\eta_i = \left(\frac{\partial q_i}{\partial y}\right)\left(\frac{y}{q_i}\right) = \beta_i / w_i \tag{43}$$

where wi$=e_i/y$ is the short-run average budget share. The "long-run" income elasticity of the same commodity is

$$\tilde{\eta}_i = \tilde{\beta}_i / \tilde{w}_i \quad (i=1,\ldots,n) \tag{44}$$

which becomes equal to η_i for $\alpha_i=0$, and where $\tilde{w}_i = \frac{e_i}{y}$ is the long-run average budget share.

The short-run uncompensated (Cournot) own-price elasticities are of the form

$$\eta_{ii} = \frac{\partial q_i}{\partial p_i}\cdot\frac{p_i}{q_i}$$
$$= -1 + (1-\beta_i)\left\{(1-\tau-\tau\mu p_{it-1}p_{it}^{-1})\gamma_i^* q_i^{-1} + \tau\right\}$$
$$-\tau(1-\beta_i)(\mu+\alpha_i)e_{it-1}e_{it}^{-1} \qquad (i=1,\ldots,\mathrm{n}) \qquad (45)$$

while the short-run uncompensated (Cournot) cross-price elasticities between qi and pj ($i,j=1,\ldots,n$, $i\neq j$) are of the form

$$\eta_{ij} = \frac{\partial q_i}{\partial p_j}\cdot\frac{p_j}{q_j}$$
$$= -\beta_i\gamma_j^* p_j e_{it}^{-1} - \tau\beta_i\left[1-\gamma_i^* q_{it}^{-1} + \mu\gamma_i^* p_{it-1}e_{it}^{-1} - (\alpha_i+\mu)e_{it-1}e_{it}^{-1}\right] \; (i,j=1,\ldots,\mathrm{n}) \; (i\neq j) \qquad (46)$$

where $-\infty<n_{ij}<0$, for $0<\beta_i<1$, $q_i>\gamma_i^*$, $-\infty<\tau\leq 1$ and for $0<\beta_i<1$, $q_i<\gamma_i^*$, $1<\tau<+\infty$, $0\leq\mu<1$.

For $\mu=0$ equations (45) and (46) becomes equal to equation (16). That is for $\mu=0$ the MDGLES becomes the DGLES model, there for the short-run elasticities (own- and cross-) are identical in these models.

The long-run uncompensated own-price elasticities n_{ii} are for the form

$$\eta_{ii} = \frac{\partial q_i}{\partial p_i}\cdot\frac{p_i}{q_i}$$
$$= -1 + (1-\tilde{\beta}_i)\left[(1-\tau)\tilde{\gamma}_i^* q_i^{-1} + \tau\right] \text{ for } i=j \qquad (47)$$

while the long-run uncompensated cross-price elasticities nij are

$$\eta_{ij} = \frac{\partial q_i}{\partial p_i}\cdot\frac{p_i}{q_i}$$
$$= -\tilde{\beta}_i\tilde{\gamma}_j^* p_j e_i^{-1} - \tau\tilde{\beta}_i\tilde{\beta}_j\left(y - \sum_{j=1}^{n} p_j\tilde{\gamma}_j^*\right)e_i^{-1} \quad \textit{for } i\neq j, \qquad (48)$$

As we see equations (47) and (48) are of the same form with equation (17). The difference between them is that the MDGLES model $\tilde{\gamma}_i^*$ and $\tilde{\beta}_i$, $\tilde{\beta}_j$ are given from equations (27) and (28), while for the DGLES model the parameters $\tilde{\gamma}_i^*$, $\tilde{\beta}_i$ and $\tilde{\beta}_j$ are given by equations (5) and (6) correspondently. For $\mu=0$

these price elasticities are the same; that is for μ=0 the MDGLES becomes the DGLES model.

Finally for the MDGLES model the compensated short-run and long-run price elasticities can be obtained from the Slutsky equation

$$\eta_{ij}^{*}=\eta_{ij}+\eta_{i}w_{j} \quad (i,j=1,\ldots,\mathrm{n}). \tag{49}$$

3. Conclusions

In 1970 the author presented (and published) the Generalized Linear Expenditure System (GLES) which is more general than Stone's Linear Expenditure System (LES). The GLES model was estimated and compared with Stone's LES and Houthakker's Indirect Addilog Expenditure System (IAES) and seems to be more attractive than LES and IAES models from the theoretical and empirical point of view. From the theoretical point of view the GLES (or GAMA) model is more attractive than the LES model because it permits the marginal budget shares to vary with changes in prices; it is also more attractive than the IAES model because the GLES model is a sharply-defined theoretical model, in the sense that its direct and indirect utility functions are know explicitly. From the empirical point of view the GLES model, generally speaking, is more attractive than the LES model, because it performs better than the LES model. To be sure, it does so at the cost of an additional parameter (τ) and computational complexity. The GLES model is also more attractive than the IAES model on the basis of their empirical performance. A defect of these models is that the demand functions are static. This affects the validity of functions for such commodities as clothing and durables, where inventories play an important role. A solution to this problem is to incorporate into the models dynamic elements, which will explain the cases when consumer's adjustment takes time.

In 1979 the author presented (and published) the Dynamic Generalized Linear Expenditure System (DGLES), that is more general than the GLES model. The GLES model has an obvious defect. It is a static model (like LES and IAES models). These models assumed that the average consumer never changes the quantities he feels committed to buy in the minimum required quantities. But, as the standard of living rises, one would expect committed quantities to rise. This is an important aspect of changes in taste and we can allow for these possibilities by making these parameters (the γi 's) functions of predetermined variables. This change in the GLES model does not destroy its basic properties. The DGLES model takes into account of the above mentioned considerations. In this model i introduced a habit formation hypothesis adjusted for the rate of inflation (eq. 2a).

According to DGLES model (eq. 3) the expenditures on commodity i does not depend only on income and all prices (like in the GLES model), but it depends also on **last** period expenditures on commodity i and on **last** period expenditures on all other commodities. The advantage of using the DGLES instead of the GLES model (or any other static model) is that it permits us to compare (and estimate) the short- and the long-run effects of income and own- and cross-price on expenditures. Also using the DGLES model instead of any other static model, it helps remove their defect that the standard of living is constant over time. From the empirical point of view the DGLES model is more attractive than the GLES model, because it performs better than the GLES model. It does so at the cost of an additional parameter (α_i) and computational complexity.

In this work the author presents the Multi-Dynamic Generalized Linear Expenditure System (MDGLES), which is more general than the DGLES model. The defect of the DGLES model is that the introduced habit formation hypothesis adjusted for the rate of inflation is valid only for one (the previous t-1) period of time. But this hypothesis needs to be corrected for such commodities as clothing and durables where i think, only the last period is not enough to explain consumer's habits. This generalization of the DGLES takes place when instead of using only last period (eq. 2a) to use as many as it takes past periods (t-1, t-2,..., t-N) to explain the habit formation hypothesis; that is instead of (eq. 2) to use (eq. 19).

The use of the MDGLES model does not destroy the adding up (the budget constrain) criterion. And this is very important for any complete demand system. The cost of using the MDGLES model instead of the DGLES model, from an estimation point of view, is that we introduce one additional parameter (μ) and a computational complexity. It remains to be seen which of the two DGLES and MDGLES models performs better, when we estimate and compare then.

In 1961 Houthakker stated that "...no **completely** satisfactory utility function (or system of demand functions) has yet been found. Perhaps there is none, but the search has hardly started and should be pursued".

In this work, following Houthakker, I present the MDGLES model, which I believe will help in searching for a completely satisfactory demand system.

Notes

1. The parameter p is related to the partial elasticity of substitution, as defined by Allen {1938, p. 504}. The partial elasticity of substitution with respect to the "supernumerary quantities" (q_i-γ_i) in this model is equal to (1-τ) for $-\infty<\tau\leq 1$ or (τ-1) for $1<\tau<+\infty$. In the LES model $\tau=0$, so this elasticity is equal to 1.

2. As a matter of fact, Pollak introduces past consumption into the GLES model in a different way. He makes the γ_i's functions of last-period quantities of that commodity. See also Phlips (1978).

3. The increase of the difference $\Delta\tilde{e}_{it}$ will depend more upon the habit formation hypothesis if the increase in price p_i is smaller.

4. For $\mu=0$ equation (24α) becomes $\gamma_i=\gamma_i^*+\alpha_i q_{it-1}\left(\frac{p_{it-1}}{p_{it}}\right)$ which is equation (2α) of the DGLES model.

5. By summing up equation e_{it} we have

$$\sum_{i=1}^{n} e_{it}=\sum_{i=1}^{n} p_{it}\gamma_i^*+y_t-\mu y_{t-1}-\sum_{j=1}^{n} p_{it}\gamma_j^*+\mu\sum_{j=1}^{n} p_{jt-1}\gamma_j^*-\mu\sum_{i=1}^{n} p_{it-1}\gamma_i^*$$

$$+\mu\sum_{i=1}^{n} e_{it-1}+\sum_{i=1}^{n}\alpha_i e_{it-1}-\sum_{j=1}^{n}\alpha_i e_{jt-1}=y_t$$

6. For simplicity here we use the same parameter μ for all i commodities. If we like to make thinks a little more difficult we could use μ_i (i=1,...,n) different parameter for each i commodity.

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