UNIDO: A NEW PROSPECT

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The Faculty of Industrial Studies of Piraeus has decided to publish a volume in honour of Professor D.T. Collias. Those responsible for the publication asked me to contribute and in view of the successful activity of Professor D.T. Collias first in the ministry of Finance, second in the Faculty of Industrial Studies of Piraeus I accepted to do so with pleasure. During my participation in the General Assembly of the United Nations of 1973 where I was representing Greece in the second commission dealing with economic and financial problems I got the opportunity to learn a lot about the activities of the various membres of the United Nations' family. The reading of their reports and the discussion with those concerned were very useful. In view of the very unfavourable repercussions for the efforts of developing nations not producing oil to develop which became gradually felt in the last weeks of 1973 the author tried to find new ways for the recycling of the oil money provided the oil exporting countries will be willing to help. The author believes that United Nations Industrial Development Organisation may be very useful in this connection and his thoughts in this respect will be exposed here.

Unido is an important member of the United Nations family. New developments in the distribution of wealth and of income in the frame of the world economy may enlarge the possibilities and the activities of Unido.

I intend accordingly to deal briefly with the possibilities of Unido today in I, and with new prospects as a result of the trepling of the price of oil in II.

I. Unido's aim is to contribute to the quickest growth of industry in the developing countries. The latter have to face difficulties :

1) in recruiting the appropriate leaders,

2) in securing the indispensable know how,

3) in getting the appropriate capital in combination with the necessity to avoid great deficits of the balance of payments,

4) in persuading prospective buyers that their products are satisfactory,

5) in finding the finance needed to channel manufactured commodities to those needing same but unable to pay.

Unido is helping developing countries in items 1, 2 and eventually 4 but not in items 3 and 5. This great difficulty could not be neutralised by decisions of the United Nations despite the fact that their decision making depends from the third world. A decision of the United Nations cannot be enforced on the members who want to stay aloof and we cannot expect the Western countries who would be obliged to cover the greatest part of the expense involved to carry out decisions they did not approve. Until the sudden increase of the oil price and of the corresponding proceeds of the oil exporting countries the decisions of the United Nations implying expense depended on the good will of the Western countries. The Eastern European countries are unwilling to supply the appropriate support owing to their permanent balance of payments deficits.

On the other hand the Western countries are reluctant to do so not only for financial reasons but also because their desires in any direction are not adopted by the quasi totality of the 77 countries. The great distance between the decisions taken and the carrying out developed in this way. It seems however that the substantial rise of the oil price, provided it is not revused or is not followed by corresponding price rises of all other important commodities which seems to be (April 1974) rather improbable is giving Unido the unique chance to finance the industrial development of the countries of the third world and the sale of the manufactured commodities they produce. As a matter of fact if the oil producing countries decide to invest their increased sale proceeds in a rational way intensifying their own industrial development and that of the other countries of the third world and not to affect same to currency speculations or to the purchase of various assets in developed Western countries the prospects for Unido and the countries it advises, helps and supports will be bright.

Unido's budget is certainly not excessive both in absolute figures and in relation to its aims. The attempt to be allowed an independent budget has been turned down in the 1973 United Nations General Assembly. It seems however that Unido's expansion may be secured by a substantial support from the oil producing countries whose price increases contributed substantially to the deterioration of the balance of payments, of the barter terms of trade, of both the internal and the external equilibria prospects and, of the developing countries not producing oil. The latter's equilibrium could not even before this unfavourable development be secured easily. Now these countries have to face not only the tripling of their oil bill but also :

1) The gradual diminution of the foreign aid which they can expect only from countries with a substantial surplus of their balance of payments; this is not to be frequent after the sharp increases of the oil prices.

2) The gradual diminution of the demand for the majority of the exports of the developing countries under the unfavourable repercussions of the oil price rises as in many cases inventories are rather high and as the boom conditions have somewhat subdued.

3) The certitude that the prices of their export commodities will fall, as il happens already with wheat reversing so the baseless expectation that seller markets of practically all raw materials and foodstuffs which alt have their substitutes, will continue,

4) The great probability that their barter terms of trade will face a constan deterioration until boom conditions develop again in the Western countries.

Industrial development in the developing countries is unfavourably affected by the continuous difficulty of securing buyers for their manufactured commodities despite the progress noticed in a number of cases in the last two decades as a consequence of the impossibility of securing the appropriate labour force in a period of overemployment in the Western world. On the other hand it has not to be forgotten that many projects of extension and of expansion cannot be carried out as their prospects are poor whilst hundreds of millions of people are unable to finance the purchase of manufactured commodities. On the other hand if these poor people would have been enabled to purchase the manufactured commodities they need the cash flow to the latter's producers would have increased, their operation would have become profitable, their expansion would have been substantial as long as subsidies would not have been needed and their receipts from abroad would increase improving so their balance of payments.

II. If we admit that the oil exporting countries consider themselves obliged as may be noticed by recent statements of some representative leaders reproduced in the press to alleviate the problems imposed by their own policy on developing countries and on the latters' ability to industrialise by taking advantage of the recommendations and of the support of Unido the following possibilities are available:

a) place at Unido's disposal substantial amounts allowing same to secure to the developing countries all the support mentioned under 1 - 5 on a great scale on a gift basis,

b) place at the disposal of the world's poorest countries for a number of years on a gift basis or on a very low interest loan basis to be repaid let me say after twenty years amounts to be fixed in relation to their inhabitants under the condition that the proceeds will be used exclusively for the purchase of manufactured commodities in countries where the income per head and per year is lower than say $ 1.300 or even $ 1.500,

c) invest on a huge scale in various member countries of the group of 77

contributing so to the latter's external equilibrium and industrialisation,

d) induce the countries where the manufactured commodities will be ordered to exploit intensely the plants available and pose this as a condition of their inclusion in the supplying countries,

A) In judging these alternatives it seems that the most rational and the most promising is alternative A provided Unido is able to cope with the involments to be faced when its budget and so its activities are to be increased on a substantial scale. When it is known how much Unido has contributed to the industrialisation of developing countries with its actual rather modest budget it is easy to forsee how much Unido could do when secured with ample resources and of course with a greater number of advisers, experts and administrators.

There is no doubt that the problems to be solved will need in every case a separate approach. The designation of leaders and of experts is not an easy task because they must be acquainted with the particular problems of the industry and of the country concerned before assuming their duties or at least they must be sufficiently open minded to grasp same. They must not forget the special responsibility of an international official and they must avoid anything which may offend the country where they will be located. They will have to consider that the solution of each problem is influenced by the particularities of the country concerned and by the intricacies which are special in every case. Generalisations are certainly dangerous and will have to be avoided particularly when the know how is involved. As far as the latter is concerned Unido's responsibility will have to include the adequate training of local people in the country concerned so that the replacement in due time will be secured. No country is expected to admit that her industry will be on a permanent basis under the leadership of foreign experts even if they come as Unido's representatives or assistants. If Unido is to operate on such a big scale and to finance industrial developments it will be available on let us say a three or five years basis. A corresponding number of projects will be taken over with the best possible calculation of the amounts involved and with appropriate reserves in order to cope with substantial price rises as those which became customary in the last years. These will have to be expected of course on a minor scale if stagflation is to continue. On the other hand it seems it will be advisable for Unido to apply the practice of the World Bank, namely to limit its financing to the expenses involved in foreign currency whilst leaving the expenses in local currency to the country concerned. This will increase the chances of a wise choice by the latter. At the same time it does not seem advisable for Unido to start a bank of its own when the World Bank and its affiliated could take over the operational side of its financing but of course without interfering in Unido's decision making. There is no doubt that in the choice of the projects to be financed both economic and political considerations will be involved.

Unido's efforts to persuade prospective buyers that manufactured commodities produced in developing countries are of good quality and at least not of a lower quality than those supplied by the developed countries will be a difficult job. It will have to be supported by the producer's reduction of prices and grant of payment facilities so that they are competitive. On the other hand Unido's support for the preference of the manufactured commodities of developing countries by eventual buyers must be available only when its experts recommend same and when those in charge of Unido will be persuaded that their experts are really objective and cannot be influenced by those interested for the sale of their production on the world market.

Unido has to do its best to secure the full utilization of all plants available before launching in new investments particularly if financed by gifts or grants of the oil producing countries. The latter will in this way neutralise the unfavourable repercussions of the rise of the price of oil on the industrialisation efforts of the developing countries.

B) Considering the very low standard of life of those living in the poorest countries of the world many people consider that their lot may be alleviated only by pure gifts which of course constitute a burden for the donors. When however we consider that a substantial number of plants all over the developing world are not operating on the highest possible scale with the result that they cannot earn their depreciation, that they cannot secure the advantages of decreasing costs, that they do not encourage the further expansion of industrial investments, that they do not contribute to the satisfaction of the needs of the people nor to the absorption of those who are unemployed or underemployed nor to the improvement of the balance of payments of the country where they are located, this means a pure waste and a loss which may be avoided if the operation is carried out on the appropriate scale. It follows that if the oil - exporting countries would secure the appropriate finance for the delivery free of charge within limits to be fixed every year e.g. of $ United States 20 or 30 per inhabitant as shown in the United Nations monthly bulletins of statistics, manufactured commodities needed in the poorest countries of the world on the basis of orders of the governments concerned who will receive the countervalue for development purposes without being allowed to export, the expense involved would have to be cut by the countervalue of the advantages to be expected from the improvement of international liquidity as in this way the difficulties connected with the balance of payments in the great majority of developing countries will be neutralised. In this connection the most difficult will be to fix in a satisfactory and appropriate way the amounts which have to be spent on this purpose, the best distribution, the choice of the countries to be helped in this way, the freedom of choice to be left to them, last but not least, the countries whose industry will supply the commodities involved[1]. I am in favour of those

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1. Cf. D. J. Delivanis, Foreign aid and industrialisation, Balkan Studies 1969, pp. 113-126, and particularly pp. 120 - 126.

countries where the income per head and per year does not exceed $ United States 1.300 - 1.500.

C) When all plants available will be exploited to utmost capacity new investments will be needed in industry and in this connection the contribution of the oilproducing countries will be precious. As a matter of fact the substantial increase of the sale of manufactured commodities will improve the possibilities of autonomous investment of the firms and if needed of the governments concerned. The recourse to oil exporters' investment will come later. It will not be necessary then to act on a gift basis provided the studies needed are carried out in the appropriate way and without omitting any of the factors to be considered and provided the investment and later the operation are carried out in the appropriate rational way. The activity of Unido in this connection will be most helpful first in the choice, second in avoiding errors. Starting new plants will be available on a big scale if the delivery of manufactured commodities to the poorest countries is carried out as exposed before inasmuch as when those involved get accustomed to manufactured commodities interruption is rendered difficult by both economic and psychological factors whose importance is constantly increasing.

D) Let me add that Unido ought to stress the importance of the utmost exploitation of the capacity of the plants available to the governments of the countries where manufactured commodities will be ordered and paid by the proceeds of the grants of the oil exporting countries. As a matter of fact it is high time to stress the importance for the world economy to avoid waste. This happens when new investments are carried out whilst the plants available if operated at about 90 - 100 % and not at about 50 % of capacity would easily secure the commodities needed.

Conclusion. The decision of the oil exporting countries to do their best to reduce the substantial disadvantages of the oil price increase for the developing world constitutes a great help for same. There is no doubt that the rational employment of the amounts involved ought to be entrusted to Unido. The latter's activity could then be enlarged and include not only the recruitment of leaders and experts, the supply of know how in combination with the stressing of the quality of the manufactured commodities produced in the developing world, provided it is really so, but also the financing when other credit sources are missing and the delivery free of charge of manufactured commodities produced in developing countries to the governments of the poorest countries. The latter will not be allowed to export these commodities but will be empowered to affect the proceeds resulting from their sale within the home market for economic development purposes.

ΠΕΔΙΟΝ ΟΡΙΣΜΟΥ ΟΙΚΟΝΟΜΙΚΩΝ ΣΥΝΑΡΤΗΣΕΩΝ

Τοῦ κ. ΑΝΤΩΝΙΟΥ Χ. ΠΑΝΑΓΙΩΤΟΠΟΥΛΟΥ

Καθηγητοῦ τῶν Μαθηματικῶν τῆς Ἀνωτάτης Βιομηχανικῆς Σχολῆς Πειραιῶς

1. Εἰσαγωγὴ

Οἱ σημερινοὶ μελετηταὶ τῆς Οἰκονομικῆς παρατηροῦν ὅτι ἐνῶ τὰ κείμενα τῶν πρωτοπόρων τῆς Μαθηματικῆς Οἰκονομικῆς ἔχουν πλέον καταστῆ ἀπρόσιτα εἰς τοὺς ἀγνοοῦντας τὴν γλῶσσαν τῶν Μαθηματικῶν, ἐν τούτοις εἰς τὸν χῶρον τόσον τῆς κλασσικῆς βασικῆς οἰκονομικῆς παιδείας, ὅσον καὶ τῶν οἰκονομικῶν ἐφαρμογῶν, ἐξακολουθεῖ νὰ γίνεται παρουσίασις ὑποδειγμάτων ὡς ταῦτα ἐδόθησαν τὸ πρῶτον ὑπὸ τῶν δημιουργῶν των.

Οὕτως ἡ συνάρτησις ἐξακολουθεῖ νὰ παρουσιάζεται εἰς αὐτὰ ὡς ὁ τύπος παροχῆς τῆς ἐξηρτημένης μεταβλητῆς ἐκ τῆς ἀνεξαρτήτου τοιαύτης, δηλαδὴ ὡς τὸ ἕν ἐκ τῶν τριῶν στοιχείων τοῦ διδασκομένου σήμερον ὁρισμοῦ κατὰ τὸν ὁποῖον :

Κάθε μονοσήμαντος ἀπεικόνισις $f : A \to B$ ἑνὸς συνόλου A εἰς ἕνα σύνολον (ἐπὶ ἑνὸς συνόλου) B καλεῖται συνάρτησις μὲ πεδίον ὁρισμοῦ τὸ A καὶ πεδίον τιμῶν τὸ B.

Εἶναι φανερὸν ὅτι ἡ ἀλλαγὴ ἑνὸς ἐκ τῶν τριῶν ἀνωτέρω στοιχείων τῆς συναρτήσεως σημαίνει καὶ ἀλλαγὴν αὐτῆς. Ἐξ ἄλλου ἡ συναρτησιακὴ σχέσις f τῆς συναρτήσεως δύναται νὰ ἐκφράζηται καὶ μέσω ὡρισμένου τύπου. Εἰς τὴν περίπτωσιν αὐτὴν ὁ ἀνωτέρω συμβολισμὸς συμπτύσσεται εἰς τὸν f / A, ἐνῶ μὲ $f(A) = \{ y \in B : \exists x \in A \text{ μὲ } y = f(x) \}$ συμβολίζεται τὸ σύνολον τῶν τιμῶν τῆς συναρτήσεως.

Βεβαίως εἰς τὴν Οἰκονομικὴν αἱ τιμαὶ τῶν περισσοτέρων μεγεθῶν εἶναι μὴ ἀρνητικαί, δηλαδὴ ἀνήκουν εἰς τὸ σύνολον R_0^+, καὶ ὡς ἐκ τούτου δύναται νὰ ὑποστηριχθῆ ὅτι [2] :

α) Τὸ πεδίον ὁρισμοῦ τῶν περισσοτέρων οἰκονομικῶν συναρτήσεων n μεταβλητῶν εἶναι τὸ σύνολον $A = \underbrace{R_0^+ X R_0^+ X \cdots X R_0^+}_{n \text{ παράγοντες}}$.

β) Τὸ πεδίον τιμῶν τῶν περισσοτέρων οἰκονομικῶν συναρτήσεων εἶναι τὸ σύνολον $B = R_0^+$.

Ἡ ἄποψις ὅμως αὕτη ἐξυπηρετεῖ μόνον τὴν συνέπειαν τῆς διδασκαλίας τῶν Μαθηματικῶν εἰς τὰς Οἰκονομικὰς Σχολὰς [3] καὶ τὴν παρουσίασιν ἀσκήσεων τῆς Οἰκονομικῆς Ἀναλύσεως [1]. Καὶ τοῦτο διότι ἡ ἄγνοια τοῦ ἀκριβοῦς πεδίου ὁρισμοῦ τῶν οἰκονομικῶν συναρτήσεων δυνατὸν νὰ ὁδηγήσῃ τὴν οἰκονομικὴν σκέψιν εἰς λανθασμένα ἀποτελέσματα.

Οὕτως ἐπὶ παραδείγματι αἱ κατωτέρω συναρτήσεις ζητήσεως :

$$D = 160 - p_{,} / R_0^+$$
$$D = 160 - p / [0,40]$$
$$D = 160 - p / [40,100]$$

ἐνῶ ἔχουν τὸν αὐτὸν τύπον, ἐν τούτοις δὲν ἐκφράζουν τὴν αὐτὴν θέσιν τοῦ καταναλωτοῦ.

2. Παραδείγματα

Ἐκ τῶν ἀνωτέρω προκύπτει ὅτι μόνον μὲ τὸ πεδίον ὁρισμοῦ καθίσταται πλῆρες τὸ ἁπλούστερον οἰκονομικὸν ὑπόδειγμα, ἡ συνάρτησις.

Οὕτω κατωτέρω παρουσιάζονται πλήρεις αἱ μορφαὶ μερικῶν βασικῶν συναρτήσεων τῆς οἰκονομικῆς θεωρίας :

$D = D(p) / A_D$ συνάρτησις ζητήσεως ἑνὸς ἀγαθοῦ μὲ πεδίον ὁρισμοῦ τὸ διάστημα τῶν δυνατῶν τιμῶν ζητήσεως αὐτοῦ.

$S = S(p) / A_S$ συνάρτησις προσφορᾶς ἑνὸς ἀγαθοῦ μὲ πεδίον ὁρισμοῦ τὸ διάστημα τῶν δυνατῶν τιμῶν προσφορᾶς αὐτοῦ.

$C = C(q) / A_C$ συνάρτησις ὁλικοῦ κόστους ἑνὸς ἀγαθοῦ μὲ πεδίον ὁρισμοῦ τὸ διάστημα τῶν δυνατῶν ποσοτήτων παραγωγῆς αὐτοῦ.

$R = R(q) / A_R$ συνάρτησις ὁλικῶν ἐσόδων ἑνὸς ἀγαθοῦ μὲ πεδίον ὁρισμοῦ τὸ διάστημα τῶν δυνατῶν ποσοτήτων πωλήσεως αὐτοῦ.

$Q = Q(K,L) / A_Q$ συνάρτησις παραγωγῆς ἑνὸς ἀγαθοῦ μὲ πεδίον ὁρισμοῦ τὸ καρτεσιανὸν γινόμενον τῶν διαστημάτων τῶν δυνατῶν ποσοτήτων τῶν συντελεστῶν K, L.

$U = U(q_x, q_y) / A_U$ συνάρτησις ὠφελιμότητος ἑνὸς καταναλωτοῦ μὲ πεδίον ὁρισμοῦ τὸ καρτεσιανὸν γινόμενον τῶν διαστημάτων τῶν εἰς εἰς τὴν διάθεσιν τοῦ καταναλωτοῦ δυνατῶν ποσοτήτων τῶν ἀγαθῶν Χ, Ψ.

Διὰ τὸ πεδίον ὁρισμοῦ τῶν οἰκονομικῶν συναρτήσεων ὑπάρχουν δύο διαδικασίαι. Ἡ πρώτη συνιστᾶ καθορισμὸν αὐτοῦ καὶ ἀναφέρεται εἰς τὴν περίπτωσιν κατὰ τὴν ὁποίαν τοῦτο δὲν ἕπεται τοῦ τύπου τῆς συναρτήσεως. Ἡ δευτέρα συνιστᾶ προσδιορισμὸν αὐτοῦ καὶ ἀναφέρεται εἰς τὴν περίπτωσιν κατὰ τὴν ὁποίαν τοῦτο ἕπεται τοῦ τύπου τῆς συναρτήσεως.

Κατ' ἀρχὴν ὁ καθορισμὸς τοῦ πεδίου ὁρισμοῦ μιᾶς οἰκονομικῆς συναρτήσεως προϋποθέτει πλήρη γνῶσιν τόσον τῶν οἰκονομικῶν, τεχνολογικῶν καὶ θεσμολογικῶν περιορισμῶν ἐπὶ τῶν μεταβλητῶν αὐτῆς, ὅσον καὶ τῆς συμπεριφορᾶς τῶν οἰκονομικῶς δρώντων.

Μὲ βάσιν τ' ἀνωτέρω ὁ ἐρευνητὴς τοῦ οἰκονομικοῦ φαινομένου θὰ καθορίσῃ τὸ πεδίον ὁρισμοῦ, συμφώνως πρὸς τὴν οἰκονομικὴν θεωρίαν.

Αἱ ἀνωτέρω παρατηρήσεις ἰσχύουν καὶ διὰ τὴν δευτέραν περίπτωσιν κατὰ τὴν ὁποίαν ζητεῖται νὰ προσδιορισθῇ ἐκ τῶν ὑστέρων τὸ πεδίον ὁρισμοῦ οἰκονομικῆς συναρτήσεως τῆς ὁποίας εἶναι γνωστὸς ὁ τύπος ἀπὸ οἰκονομετρικὰς μεθόδους. Εἰς τὴν περίπτωσιν αὐτὴν ἐξ ἄλλου, μία ἁπλῆ σχετικῶς μαθηματικὴ διαδικασία ἐπιτρέπει τὴν εὕρεσιν τοῦ πλησιεστέρου ὑπερσυνόλου τοῦ συνόλου Α. Στηρίζεται δὲ αὕτη εἰς τὴν ἰδιότητα $f(A) \subseteq R_0^+$. Οὕτως:

1) Διὰ συνάρτησιν ζητήσεως $D = 16 - p / A$ θὰ εἶναι $A \subseteq [0,16]$, διότι διὰ $p > 16$ προκύπτει $D < 0$.

2) Διὰ τὴν συνάρτησιν ἐσόδων $R = 4q - q^2 / A$ θὰ εἶναι $A \subseteq [0,4]$, διότι διὰ $q > 4$ προκύπτει $R < 0$.

3) Διὰ τὴν συνάρτησιν παραγωγῆς $Q = 8\chi\psi - \chi^2 - \psi^2 / A$ θὰ εἶναι $A \subseteq \{(\chi, \psi) \in R_0^+ \times R_0^+ : \chi^2 + \psi^2 \leqslant 8\chi\psi \}$, διότι διὰ ζεύγη (χ, ψ) μὲ $\chi^2 + \psi^2 > 8\chi\psi$ προκύπτει $Q < 0$.

3. Ἰσορροπία

Ἡ θεώρησις οἰκονομικῶν συναρτήσεων μετὰ πεδίου ὁρισμοῦ βοηθεῖ καὶ τὴν περιγραφὴν διερευνήσεων τῆς οἰκονομικῆς θεωρίας.

Ἕνα δεῖγμα διερευνήσεως τῆς ἰσορροπίας τῆς ἀγορᾶς, τῇ βοηθείᾳ τῶν συνόλων ὁρισμοῦ καὶ τιμῶν τῶν ἀντιστοίχων συναρτήσεων, παρουσιάζεται κατωτέρω.

Ἔστωσαν λοιπὸν $D = D(p) / A_D$ καὶ $S = S(p) / A_S$ αἱ συναρτήσεις ζητήσεως καὶ προσφορᾶς ἑνὸς ἀγαθοῦ. Κατὰ τὴν πρώτην ὁ καταναλωτὴς ἀγοράζει ὅταν ἡ τιμὴ τοῦ ἀγαθοῦ εὑρίσκεται εἰς τὸ διάστημα A_D. Κατὰ τὴν δευτέραν ὁ παραγωγὸς πωλεῖ ὅταν ἡ τιμὴ τοῦ ἀγαθοῦ εὑρίσκεται εἰς τὸ διάστημα A_S.

Ἤδη προκειμένου νὰ μελετηθῇ ἡ ἰσορροπία τῆς ἀγορᾶς διακρίνονται αἱ κάτωθι περιπτώσεις :

1) Ἐὰν $A_D = A_S = A$, δηλαδὴ αἱ συναρτήσεις ζητήσεως καὶ προσφορᾶς ἔχουν κοινὸν πεδίον ὁρισμοῦ, τότε ἰσχύει ἡ ἑπομένη συνθήκη :

|| Ἵνα ὑπάρχει ἰσορροπία πρέπει :
|| $D(A) \cap S(A) \neq \emptyset$.

Διὰ τὴν τιμὴν καὶ τὴν ποσότητα τῆς ἰσορροπίας θὰ ἰσχύουν :

$$p_0 \in A \text{ καὶ } q_0 \in D(A) \cap S(A)$$

2) Ἐὰν $A_D \neq A_S$, δηλαδὴ αἱ συναρτήσεις ζητήσεως καὶ προσφορᾶς δὲν ἔχουν τὸ αὐτὸ πεδίον ὁρισμοῦ, τότε ἰσχύει ἡ ἑπομένη συνθήκη :

Ἵνα ὑπάρχει ἰσορροπία πρέπει :

$A_D \cap A_S \neq \emptyset$ καὶ $D(A_D) \cap S(A_S) \neq \emptyset$.

Διὰ τὴν τιμὴν καὶ τὴν ποσότητα τῆς ἰσορροπίας θὰ ἰσχύουν :

$$p_{0\in} A_D \cap A_S \quad \text{καὶ} \quad q_{0\in} D(A_D) \cap S(A_S)$$

3) Ἐὰν $A_D \cap A_S = \emptyset$, δηλαδὴ αἱ συναρτήσεις ζητήσεως καὶ προσφορᾶς ἔχουν πεδία ὁρισμοῦ ξένα μεταξύ των, τότε δὲν ὑφίσταται ἰσορροπία συμφώνως πρὸς τὴν συνθήκην τῆς προηγουμένης περιπτώσεως.

Ἐὰν ὅμως ζητηθῇ νὰ ἐπιτευχθῇ ἰσορροπία **χωρὶς νὰ γίνῃ ἀλλαγὴ τῶν τύπων τῶν συναρτήσεων** τότε θὰ πρέπει :

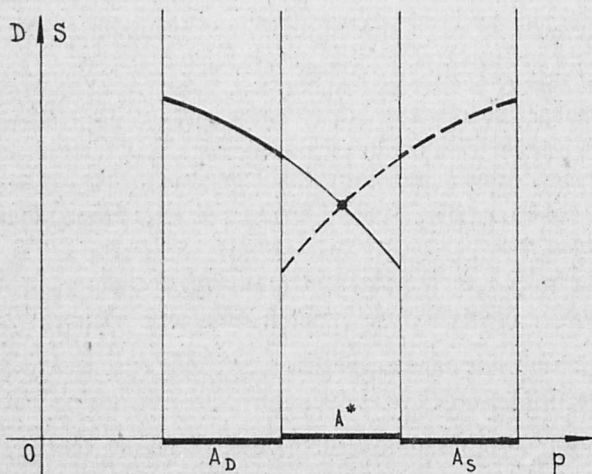α) Νὰ ἔχουν ἔννοιαν αἱ συναρτήσεις D/A^* καὶ S/A^*, ἔνθα A^* τὸ μεταξὺ τῶν A_D καὶ A_S διάστημα τιμῶν τοῦ p.

β) Νὰ ἰσχύῃ ἡ σχέσις :

$$D(A^*) \cap S(A^*) \neq \emptyset.$$

γ) Νὰ γίνῃ ἀλλαγὴ τῶν πεδίων ὁρισμῶν τῶν συναρτήσεων ἀπὸ A_D καὶ A_S εἰς $A_D^1, A_D^2, \ldots, A_D^n \subseteq A_D \cup A^*$ καὶ $A_S^1, A_S^2, \ldots, A_S^n \subseteq A_S \cup A^*$ ἀντιστοίχως ὥστε νὰ ἰσχύουν αἱ σχέσεις :

$$A_D^i \cap A_S^i = \emptyset, \ i = 1, 2, \ldots, n-1$$

$$A_D^n \cap A_S^n \neq \emptyset.$$

Ἐὰν τώρα τὰ σύνολα A_D^i, A_S^i $i = 1, 2, \ldots$, n τὰ ὁποῖα παριστοῦν τὰς

παραχωρήσεις ὡς πρὸς τὴν τιμὴν τῶν δύο παραγόντων τῆς ἀγορᾶς, ἐμφανί-
ζονται κατὰ τὴν i—οστὴν στιγμήν, τότε ἡ ἀνωτέρω ἰσορροπία ἐπιτυγχάνεται
κατὰ τὴν n—οστὴν στιγμήν.

Ἡ ἀνωτέρω διαδικασία συμφωνεῖ μὲ τὴν κλασσικὴν μέθοδον τοῦ Walras
ἐννοιολογικῶς καὶ γραφικῶς (δεδομένου ὅτι ἀνεξάρτητος μεταβλητὴ εἶναι ἡ
τιμὴ p τοῦ ἀγαθοῦ).

4) Ἐὰν $D(A_D) \cap S(A_S) = \varnothing$, δηλαδὴ αἱ συναρτήσεις ζητήσεως καὶ
προσφορᾶς ἔχουν σύνολα τιμῶν ξένα μεταξύ των, τότε δὲν ὑφίσταται ἰσορρο-
πία συμφώνως πρὸς τὴν συνθήκην τῆς δευτέρας περιπτώσεως.

Ἐὰν ὅμως ζητηθῇ νὰ ἐπιτευχθῇ ἰσορροπία **χωρὶς νὰ γίνῃ ἀλλαγὴ τῶν
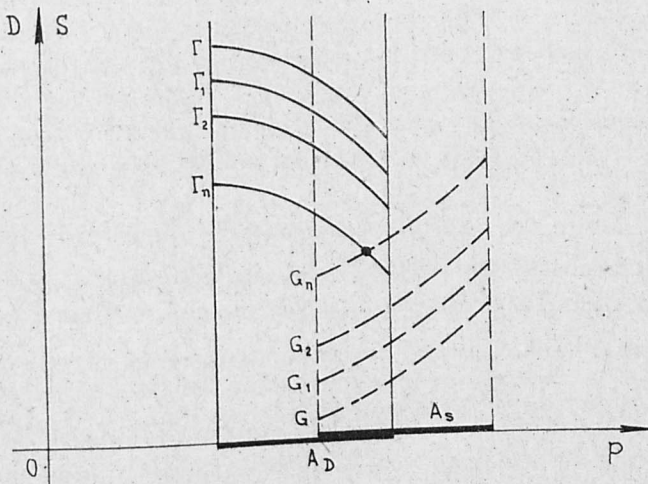πεδίων ὁρισμῶν τῶν συναρτήσεων** τότε θὰ πρέπει :

α) Νὰ ἰσχύῃ ἡ σχέσις :

$$A_D \cap A_S \neq \varnothing$$

β) Νὰ γίνῃ ἀλλαγὴ τῶν τύπων συναρτήσεως ἀπὸ D καὶ S εἰς
$D^1, D^2, \ldots, D^n$ καὶ $S^1, S^2, \ldots, S^n$ ἀντιστοίχως ὥστε νὰ ἰσχύουν
αἱ σχέσεις :

$$D^i(A_D) \cap S^i(A_S) = \varnothing, \ i = 1, 2, \ldots, n-1$$

$$D^n(A_D) \cap S^n(A_S) \neq \varnothing$$

Ἡ ἀνωτέρω ἀλλαγὴ δύναται νὰ πραγματοποιηθῇ καὶ διὰ τῆς δημιουρ-
γίας τύπων παρεχόντων οἰκογενείας καμπύλων $\Gamma_1, \Gamma_2, \ldots, \Gamma_n$ ἀντιστοίχως
$G_1, G_2, \ldots, G_n$ δυναμένων νὰ τμηθοῦν εἰς τὸ διάστημα $A_D \cap A_S$.

Ἐξ ἄλλου ἡ ἀλλαγὴ δύναται νὰ ἀναφέρεται εἰς τὸν ἕνα ἐκ τῶν δύο παρα-γόντων τῆς ἀγορᾶς, τοῦ ἑτέρου μὴ προβαίνοντος εἰς οὐδεμίαν παραχώρησιν· Ἐὰν τώρα οἱ τύποι τῶν D^i, S^i, $i = 1, 2, \ldots$ n οἱ ὁποῖοι δίδουν τὰς παρα-χωρήσεις ὡς πρὸς τὴν ποσότητα τῶν δύο παραγόντων τῆς ἀγορᾶς, ἐμφανί-ζονται κατὰ τὴν i—οστήν στιγμήν, τότε ἡ ἀνωτέρω ἰσορροπία ἐπιτυγχάνε-ται κατὰ τὴν n—οστὴν στιγμήν.

Ἡ ἀνωτέρω διαδικασία συμφωνεῖ μὲ τὴν κλασσικὴν μέθοδον τοῦ Marshall. Διαφέρει δὲ μόνον εἰς τὴν γραφικὴν παράστασιν, διότι διατηρεῖ ὡς ἀνεξάρτη-τον μεταβλητὴν τὴν τιμὴν p τοῦ ἀγαθοῦ. Τοῦτο ἔγινε διὰ νὰ μὴ εἰσαχθοῦν ἐπὶ πλέον σύμβολα. Δὲν ἀλλάσσει ὅμως τὰ συμπεράσματα τῶν ἀνωτέρω συλλογισμῶν.

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