ON THE POSSIBILITY OF ATTAINING A SOCIAL OPTIMUM THROUGH DECENTRALIZATION

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Introduction.

Theoretical welfare economics has not paid much attention to the derivation of decentralized mechanisms for attaining a socially optimum position. In fact most of the analysis runs in terms of an assumed *omniscient* planner who is able to determine directly the socially optimal position, on the basis of detailed knowledge of everyone's preference and production systems. The planner's welfare function is of the Bergson type incorporating, in most cases, the «individualistic ethic» that individual preferences count.

On the other hand, in the literature on decentralized planning the concept of a *distribution economy* has been developed [1]. If, for simplicity, we limit our attention to a pure exchange economy, in which the total existing quantities of the various consumable goods are given, then we can describe the distribution economy model as follows :

Given the fixed total quantities, x_j , $j = 1, \ldots, n$, of the n goods and their prices, p_j , which are endogenously determined, the economy's total income, $Y = \sum_{j=1}^{n} p_j x_j$, can take any a priori fixed value i.e.,

$$Y = \sum_{j=1}^{n} p_j \ x_j = \text{constant} = 1. \tag{1}$$

(this is the price normalization rule since only relative prices are relevant). Decentralized planning in the economy proceeds by income allotments to the various individuals, δ_i, $i = 1, \ldots, m$, which are also a priori given and satisfy

$$\sum_{j=1}^{n} P_j \ x_j = 1 = \sum_{i=1}^{m} \delta_i. \tag{2}$$

Each individual determines his choices by maximizing his ordinal utility function subject to his income restraint, i.e., by

$$\max_{x_{ij}} u^i \ (x_{i1}, \ldots, x_{in})$$

$$(3) \text{ subject to } \sum_{j=1}^{n} p_j \, x_{in} \leqq \delta_i ,$$

where, in each round of the iterative process, all prices are given by the planning office. An equilibrium in the above distribution economy is attained when prices are such that total demand for each good equals its given supply,

$$(4) \qquad \sum_{i=1}^{m} x_{ij} \leqq x_j \qquad j = 1, \ldots, n$$

The distribution economy framework has been generalized so as to include not only production but also, and what is more important, public goods. In the latter case separate income distributions for private and vote distributions for public goods may be considered in an effort to overcome the problem of true relevation of individuals' preferences.

The didtribution economy and its implied social welfare function :

The distribution economy model starts from a priori given income allotments without inquiring about their derivation from an overall social welfare function.

The problem examined in this paper is whether we can delineate the class of social welfare functions which are implied by and lead to any given income distribution. More precisely, the question is about the specific properties of the implied social welfare function so that the solution of the central planning problem will coincide with that of the decentralized distribution economy with any given income allotments. The δ_i's must, of course, be among the parameters of that function, which may or may not incorporate the individualistic ethic.

To be more concrete we prove the following theorem :

Theorem : If the indifference curves of all individuals are homothetic, the implied social welfare function is given by

$$W = W(u^1, \ldots, u_m) = u^{1\,\delta_1} \,.\, u^{2\,\delta_2} \ldots u_m^{\delta_m} ,$$

where $\delta_1 \geqq 0, \quad \sum_{i=1}^{m} \delta_i = 1$ are the income allotments.

Remark : note that each individual's preferences may be different as long as his indifference curves are homothetic. Mote also that W is a Cobb-Douglas function because of the price normalization rule.

Proof : Assume a well behaved general individualistic ordinal social welfare function $W = W\ (u^1, \ldots, u^u)$ and consider the central planning problem

$$\underset{x_{ij}}{\text{Max}} \quad W(u^1, \ldots, u^m)$$

(5) subject to

$$x_{1j} + x_{2j} + \ldots + x_{mj} \leqq x_j \,,\ j = 1, \ldots, u$$

where u^i are well behaved general ordinal utility functions and x_j are the fixed total quantities of the various goods. Letting μ_j be the lagrangean multipliers of the restraints, the first-order conditions are

(6) $$W_i\, u^i_j \leqq \mu_j \quad \text{with} \quad W_i\, u^i_j\, x_{ij} = \mu_j\, x_{ij} \,,\ i = 1, \ldots, m \text{ and } j = 1, \ldots, n,$$

where the subscripts in W and u_i denote partial derivatives w/r to the corresponding variables. If we define $p_j = \mu_j / W$ then (6) becomes

(6') $$\frac{W_i}{W}\, u^i_j \leqq p_i \text{ wjth } \frac{W_i}{W}\, u^i_j\, x_{ij} = p_j\, x_{!j} \,.$$

Adding up all the equalities in (6') we get

(6'') $$\frac{W_1}{W}(u_1{}^1\, x_{1i} + \ldots + u_n{}^1\, x_{in}) + \ldots + \frac{W_m}{W}(n^m_1\, x_{m1} + \ldots u^m_n x_{mn}) = = \sum_{j=1}^{n} p_j\, x_j$$

Now if, we consider homothetic indifference curves, then we may select utility indices which are homogenous of degree one, for which

$$u^i \equiv u^i_1\, x_{i1} + u^i_2\, x_{i2} + \ldots + u^i_n\, x_{in} \,.$$

Then (6'') becomes

(6''') $$\frac{W_1}{W}\, u^1 + \ldots + \frac{W_m}{W}\, u^m = \sum_{j=1}^{n} p_j\, x_j = 1$$

by our normalization rule (1).

Thus W must be a homogeneous function of degree one in the u^i 's.

On the other hand we get the following first-order conditions from the individual maximization problems :

(L) $$u^i_j \leqq \lambda_i\, p_j \qquad \text{with } u^j_i\, x_{ij} = \lambda_i\, p_j\, x_{ij}$$

for each individual. Adding up the equalities and using the special property of all u^i 's we get

$$(7') \qquad u^i = \lambda_i \, \delta_i \qquad \text{or } \lambda_i = \frac{u^i}{\delta_i}$$

The λ_i 's are the usual marginal utilities of income for each individual. Thus inequalities in (7) become

$$(7'') \qquad \delta_i \, u^i_j / u^i \leqq p_j \, .$$

The inequalities in (6') and (7'') are the first-oder conditions of the central and the decentralized problems, respectively. If we compare the two, we see that they coindide if and only if $\frac{W_i}{W} = \frac{\delta_i}{u^i}$ or, in other words iff the elasticities of W are

$$(8) \qquad \frac{W_i \, n^i}{W} = \delta_i \; , \qquad i = 1, \ldots, m \, ,$$

constant and equal to δ_i . Q.E.D.

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(1) See E. Malinvaud «Lectures in Micpoeconomic Theory». chapter V.

(2) See e.g. P, H.M. Ruys «Public Goods and Decemfralization», Tilbure University Press, 1974.