A SURVEY OF EMPIRICAL STUDIES ON THE TERM STRUCTURE OF INTEREST RATES

By NICHOLAS C. BALTAS (Ph. D.)

Agricultural Bank of Greece

The purpose of this paper is to analyse the relevance of recent empirical research on interest rate determination, in which focus is mainly placed on a Keynesian formulation and the more recent theories on the term structure of interest rates, a subject much discussed in economic literature.

Despite the volume of literature on the structure of interest rates, very little significant statistical work appears to have been undertaken; at least, the percentage of material that could be described as statistical is very small indeed. In spite of the existence of an extensive literature, stretching at least as far back as Marshall, no really unified, formal, theoretical framework for discussion of the structure of interest rates is available.

Ever since the publication of Keynes' «General Theory», economists have been fascinated by the concept of «liquidity preference» and Keynes' new interpretation of the forces that determine, to his mind, the long-term rate of interest. Recent contributions on «liquidity-preference» have dealt with both its theoretical and empirical aspects. Very recently, a tendency is becoming apparent to take account of liquidity in a much broader sense. The presence of financial intermediaries outside the commercial banking system has brought this about.

An attempt at elaborating on the influence of non-bank-financial intermediaries, via their holding of financial assets in general, on the long-term rate of interest has been made by J. L. Ford and T. Stark (1). They used the simple Keynesian interest rate determination function substituting the stock of money with weighted liquid assets :

$$(1) \qquad R_t = f\left[\left(\frac{L}{Y} \right)_t \right]$$

where R : is the long-term rate of interest, in that case 2 $^{1}/_{2}$ per cent consol yield

L : weighted liquid assets

Y : gross domestic product

t : time units

They fitted data from 1948-63, for the U.K. using an exponential as well as a power function,

(2)
$$\log R_t = \underset{(0.096)}{2.392} - \underset{(0.0013)}{0.0123} \left(\frac{L}{Y}\right)_t$$

$$\bar{R}^2 : 0.856, \quad d = 0.81$$

(3)
$$\log R_t = \underset{(0.372)}{5.596} - \underset{(0.087)}{0.960} \log \left(\frac{L}{Y}\right)_t$$

$$\bar{R}^2 = 0.889, \quad d = 0.97$$

Both equations provide good statistical results* with the exception that the value of the Von Neumann ratio for equation (2), at the 5 per cent level, is inconclusive. What we could draw as a conclusion is that «liquidity» appears to have a considerable influence on the rate of interest. In their attempt to improve the statistical results and to devise a theoretically more satisfactory equation, introducing some kind of expectational factor into the interest rate determination, the findings would have to be considered very poor. The estimators were not statistically significant or they had the wrong sign.**

(4)
$$\log R_t = \underset{(1.16)}{6.40} - \underset{(0.26)}{1.14} \log \left(\frac{L}{Y}\right)_t - \underset{(0.058)}{0.043} \log (r)_t$$

Another extremely ingenious method of what ultimately amounts to including an expectational factor is that adopted by Ball (2). He has adopted Friedman's

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* $\bar{R}^2$ is the correlation coefficient adjusted, $d = \frac{\delta^2}{\delta^2}$ is the Von Neumann ratio, which is used here as the test for first order autocorrelation of the disturbances.

** On the a priori ground, it is expected that the coefficient of r — the Treasury Bill rate taken as «short - term» rate of interest - to be positive. For instance, under «normal» conditions a rise in r would automatically bring about a rise in R, «long-term» rate of interest — in this case 2 $^{1}/_{2}$ per cent Consol Yield—because of investors shifting out of bonds. Any way, the importance of the expected short rate will depend upon how long investors believe any given expected short rate is going to last for. If there are expectations, generally held, concerning the future short rate, then given the actual values of these, the question that arises is what time span investors attach to them.

concept of permanent income in invoking the idea of normal rate of interest, R^n_t. He defined

$$(5) \qquad R^n_t = \sum_{i=0}^{\infty} \lambda^{i+1} R_{t-i}, \quad 0<\lambda<1$$

«The normal rate is defined as a weighted sum of past rates of interest with geometrically declining weights....since it is plausible that the weight attributable to a given rate should decline with distance away from the present». The statistical results obtained from using a function of the kind $R_t = \varphi \left[R^n_t, \left(\frac{L}{Y}\right)_t, \right]$ are very good

$$(6) \qquad \log R_t = 1.49 - 0.0139A + 0.111 R_{t-1}$$

$$\bar{R}^2 = 0.915, \quad d = 1.68$$

where $A = \left[\left(\frac{L}{Y}\right)_t - \lambda \left(\frac{L}{Y}\right)_{t-1} \right]$

Ball assumes that the value of λ is 5, because, as he has argued, there can be such a thing as a permanent rate of interest.

This empirical study suggests that the more reliable type of function is theoretically the one which includes some kind of expectational variable. It would be, therefore, quite interesting to examine briefly the most important theories concerning the interrelation of long and short term rates of interest, in the sense of the functional relationship among yields of securities, which differ only in their terms to maturity (or as it is usually described in economic literature, the term structure of interest rates*).

Two opposite theories have been the subject of serious consideration :
(1) The expectation theory or expectation hypothesis, which has been formulated by Fisher and restated by Hicks (3) and Lutz (4). According to this theory the relationship of the long rate to the short rate of interest depends exclusively upon expectations concerning future-term rates.** Lutz states that the sole explanation of the long-term rate is an average of expected short-term rates. It follows, there-

* In the neoclassical tradition of monetary theory, this relationship was explained as the outcome of expectations about future changes in rates of interests. Keynes, in the «General Theory», did not deal explicitly with this problem; he did so implicitly by truncating it into the choice between holding cash — a zero - interest loan for a minimal period of time —and consols; Keynes' followers extended his analysis to include a variety of securities and risk quality.

** Suppose that all individuals expect interest rates to be higher in the future than they are now, although all do not necessarily expect the same rise in rates. As a result, lenders would now demand higher long - term rates because there is the option of lending for a short time now, waiting for the appearance of the higher spot rates, and then buying bonds at the higher rates, and vice versa.

fore, that if short-term rates are expected to rise the long-term rate at the present time will exceed the actual short-term rate. Conversely, if short rates are expected to fall the long-term one will be lower than the present short-term rate (2). The liquidity preference theory asserts that forward rates contain a liquidity premium and in consequence, adherents of this theory deny that current forward rates provide unbiased forecasts of subsequent spot rates, as postulated by the expectation theory. Therefore, forecasts of future rates embodied in the present structure systematically overestimate the spot rates that materialise.

Empirical tests (5) of the expectation theory are based on the necessity to demonstrate some relation between the forward rates and the subsequent spot rates. This leads to an examination of the accuracy of the forecast provided by the forward rates. Among the first to study the predictive accuracy of forward interest rates was Macauley in 1938. Before the establishment of the Federal Reserve, there was a pronounced seasonal in the movement of call money rates. Macauley found that time money rates did anticipate this seasonal. However, he was unable to discover evidence of successful forecasting beyond the seasonal. Kassel in 1965 repeated Macauley's test using data for 27 and 55 day bills for the period 1959-61 and confirmed Macauley's findings that the market anticipated the seasonal in these rates.

Kessel (6) gives more recent evidence of forecast accuracy. For the period January 1959 to March 1962, Kessel compares the forward rates with the subsequent spot rates which are supposed to predict short-term (up to six months) U.S. Treasury Bills. According to his evidence, forward rates are upward-biased forecasters, which means that actual spot rates are less than estimated spot rates. Furthermore there is autocorrelation which leads us to suspect either the omission of some important factors, which are quite likely to participate in the determination of spot rates, or the forward rate is not at least the unique determining factor. Therefore, Kessel's empirical evidence implies only that future rates roughly predict the subsequent spot rates. In addition, since forecast errors are often sizeable and persistently in one direction, one is drawn to the conclusion that market expectations implied by the forward rates tend to adjust slowly.

Another important contribution as far as the formation of market expectations is concerned has been made by Meiselman (7). He postulated that market expectations are revised according to the size of the error between the forecast of the spot rate given by the forward interest rate and the spot rate that subsequently appeared. Thus the forecast error in period t, E_t, is defined as follows :

$$E_t = R_{1,t} - r_{1,t-1,t} \qquad (9)$$

where $R_{1,t}$ is the spot interest rate on a one-period loan in period t, and $r_{1,t-1,t}$ is the forward interest rate as quoted in period t-1 for one period that will start in period t. According to the market expectations theory, the latter forecasts the former. Meiselman postulates that there is a linear approximation between the change of expectations for one period loan that will start in period $t + n$;

$$r_{1,t,t+n} - r_{1,t-1,t+n} = a_n + b_n E_t \quad n = 1,2,3\ldots \tag{10}$$

This model can say nothing about the level of expected interest rates. It can only tell how expectations change. Equation (10) does not fully explain the revision of expectations, and it is necessary to introduce a residual $u_{1,t,t+n}$ to express that forward rates are responsive to other factors besides the forecast error E_t. Therefore, with the introduction of the residual, (10) becomes;

$$r_{1,t,t+n} - r_{1,t-1,t+n} = a_n + b_n E_t u_{1,t,t+n} \quad n = 1,2,\ldots \tag{11}$$

Meiselman fits (11) by least squares using annual data, (1901-54) for high grade corporate bond yields for maturities of one to eight years, as estimated by Durand. He found that b_n, which measures the responsiveness of forward rates to forecasting error, and R^2 decline as n increases. This means that current experience has a greater impact on near-term expectations and that the force of the impact diminishes as the expectation horizon lengthens. The decline can be attributed to the increasing haziness of the distant future and, therefore, the diminished relevance of expected forward rates to investors' decisions. Another important point is that a_n is close to zero. The result seemed to deny the claim of adherents to the liquidity hypothesis that forward rates provide biased estimates for spot rates, since in the former there is a liquidity premium, although Wood (1963) (8) and Kessel (1965) (6) independently pointed out that if $a_n = 0$, it constitutes a necessary but insufficient condition for the absence of a liquidity premium.*

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* This can easily be proved.

Let $R^*_{t,t+u}$ be the spot rate expected to rule in period $t + n$, at time t. We assume that the forward rate is related to the expected spot rate

$$R^*_{t,t+n} = r_{t,t+n} - L_{t,t+n}$$

where, $L_{t,t+n}$ denotes the liquidity premium at time t for a loan that will begin at time $t+n$. Then,

$$r_{t,t+n} = R^*_{t,t+n} + L_{t,t+n} \tag{1}$$

Similarly,

$$r_{t-1,t+n} = R^*_{t,t+n} + L_{t-1,t+n} \tag{2}$$

Subtracting (2) from (1)

$$r_{t,t+n} - r_{t-1,t+n} = R^*_{t,t+n} - R^*_{t-1,t+n} + L_{t,t+n} - L_{t-1,t+n} \tag{3}$$

According to Meiselman's hypothesis,

Buse (9) attempted to test the Meiselman model using the January averages of yields on British government securities for the period 1933 - 63. He found that b_n, reaction coefficients, and R^2, correlation — coefficient, decline systematically as n increases. This means that expectations about more distant rates are more firmly held and less responsive to forecast error than expectations about rates closer to the present. Buse's test is in agreement with Meiselman, except that a_n is clearly different from zero. The parameters of this model provide implications for an alternative version of the expectation theory known as «liquidity premium hypothesis». This hypothesis was first specified by Hicks.* Wallace (1964) also calculated for U.S. Treasury bond rates using quarterly data for the period 1946-62. He included a dummy variable in an attempt to account for the Federal Reserve Treasury accord in 1951. Although Wallace found that the error term explains the revision of expectations, he also discovered that the intercepts, a_n, are positive and much larger than their standard errors. The residuals were found to be autocorrelated in his regressions.

A further attempt to specify the expectations mechanism involved in the determination of the term structure of interest rates has been provided by Malkiel (10). He assumes that the interest rate of any maturities will be expected to fall if it is high relative to historic levels and to rise if it is low. Malkiel modifies the pure expectations model by including transactions costs, assumed absent by Lutz. His analysis revealed that the structure of transactions costs observed presently in debt markets imparts a positive bias to the slope of the yield curve. The asymmetrical effects of trading costs on the demand side of the market imply that investors with long holding periods are essentially indifferent between short and long issues at the same yield. Short-holding period investors, on the other hand, always prefer short to long issues. Transactions costs do not have the same effect on bond suppliers. Suppliers who need funds over long periods prefer long to

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$$r_{t,t+n} - r_{t-t+n} = a_n + b_n (R_{1,t} - r_{1,t-1,t}) \qquad (4)$$

Substituting (3) into (4) and taking into account that

$$r_{t-1,t} = R^*_{t-1,t} + L_{t-1,t} \text{ we get}$$

$$R^*_{t,t+n} = R^*_{t-1,t+n} = (L_{t,t+n} - L_{t-1,t+n} - b_n L_{t-1,t}) + b_n (R_t - R^*_{t-1,t})$$

In our case, $a_n = L_{t,t+n} - L_{t-1,t+n} - b_n L_{t-1,t}$. Therefore, a_n will be zero either if $L_{t,t+n} = L_{t-1,t+n} = L_{t-1,t} = 0$, which means that liquidity premiums are zero, or if by chance, $L_{t,t+n} - L_{t-1,t+n} = b_n L_{t-1,t}$.

* He asserts that the forward interest rates implied by equation are biased estimates of the expected one - period rates. Hicks attributes this bias to two separate factors; the first, due to risk aversion,is related to a risk premium to offset the possibility of capital loss if the funds are required before the final redemption date.The premium is usually assumed to be a positive increasing function of maturity. The second follows from the existence of transaction costs of investment,

short issues, since the transactions costs per unit of time are lower the longer the issue. Malkiels' analysis suggests that as long as the holding periods of demanders tends to be shorter than the issue periods of bond suppliers, the yield curve will rise sharply and then level off.

Ut to now, results do not provide strong evidence in favour of the expectations theory, insofar as the explanation of the term structure is concerned. The main two reasons are on the one hand the bias of the forward rates as forecasts of the spot rates and on the other hand the maturity composition of debt.*

As we have already mentioned the other approach to the term structure of interest rates is liquidity preference theory. According to Kessel, liquidity premium varies directly with the level of interest rates. His argument is based on the premise that nearer-term bonds are less susceptible to risk of principle than longer-term bonds, and hence provide their owners with more liquidity. At a high interest rate, there is a high opportunity cost to holding money. Hence, if both short and long-term bond yields were to increase by the same amount, there would be a tendency to substitute short-term for long-term to secure the greater liquidity of the former. Kessel therefore concludes that the spread between short and long-term rates should widen at high interest rates.

In support of his argument, he regressed the forecast error-the difference between the actual and the forward rates - using two time series data. The first regression,

$$R_{t+n} - r_{t,\,t+n} = -\,.672 + .473\,R_t$$
$$\text{t - ratios} \qquad (4.080) \quad (8.650) \qquad D\text{-}W = 0.79$$

is based on 138 weekly observations of the 91-day Treasury bill rate, January 1959 to February 1967. The second one,

$$R_{t+n} - r_{t,\,t+n} = -\,.140 + .218\,R_t$$
$$\text{t - ratios} \qquad (2.160) \quad (6.900) \qquad D\text{-}W = 0.41$$

is based on 137 monthly observations of the 28-day Treasury bill rate, October 1949 to February1961. His test is based on the assumption that the forecast errors are independent of liquidity premiums; thus he was unable to estimate the actual liquidity premiums. The existence of autocorrelation in residuals of the above two regressions lead us to suspect the omission of some important factors. The fact that some other factors influence the changes in the forward rates - or changes

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* The expectations theory argues that changes in the debt composition can affect expectations. This argument is inconsistent with Modigliani's and Sutch's (11) empirical evidence. They could not find any relation between the composition of the Federal debt and the term structure of rates. Using a different expectations model — one which relates the spread between long and short-term interest rates to a weighted sum of lagged short-term rates and to the term composition of the Federal department rates — they found that the components of the debt have opposite signs and nearly equal magnitudes. This implies a trivial net effect on the term structure of rates.

in expectations - combined with the Kessel hypothesis provides a strong implication in favour of liquidity premiums. Any way, the troublesome question still remains of identifying the part of the change in forward rates due to changes in the liquidity premiums and that due to hitherto unexplored causes of changes in expectations.

No firm conclusions can be drawn about the validity of either the expectations theory or the liquidity preference theory based on the relults of the work reviewed. None of these results is decisive, whilst some of the studies contain errors, shortcomings and paradoxes. The assumption of universal risk aversion does not provide a logical foundation for the liquidity preference theory and may be questionned. Whilst risk aversion may cause borrowers and lenders to hedge, this does not necessarily mean that it will result in a systematic bias in favour of short-term securities. However, the liquidity preference theory may be more consistent with the data, even though its rational background is imperfect.

On the other hand, although the empirical content of the expectations theory-that forward rates are unbiased predictors of future spot rates - is a direct consequence of the substantive assumptions, the empirical evidence does not support it. The new expectations models, based on error-leading mechanisms, were developed precisely because the empirical tests of the expectations theory seemed to indicate that the forward rate is a biased estimate of the spot rate. Furthermore, they provide evidence that forecast errors are of-ten sizeable and persist in one direction, and that market expectations may adjust only slowly. However, positive constant terms-that exist in yield curves for U.S. and U.K. government securities-are inconsistent with the identification of forward rates as forecasts, since expected rates are assumed to remain unchanged when expectations are realised.

The empirical research does not therefore enable us to decide whether the expectations or liquidity preference frameworks provide a better organising principle. Further work remains to be done in «explaining» the formation and change of expectations. Future research should expand on the expectations theory examining the other factors that influence expectations. Hints for these factors can be indicated through residuals by examining them in an econometric sense. Kessel's results indicate that important factors have been omitted. As we have already pointed out extrapolative and regressive elements operate jointly in the formation of expectations. In other words, risk aversion might influence the behaviour of investors in appraising the attractiveness of securities of different maturities. Moreover, the degree of risk aversion might vary widely among investing institutions, particularly among commercial banks. However, the future research will not have to ignore the fact that expectations are not uniform even amongst professional speculators and investors: it seems that expectations are less than perfectly elastic, so that there is always the probability of getting better predictions and a more satisfactory theory of the structure of interest rate determination.

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