THE SERIOUSNESS OF THE CAPACITY INDIVISIBILITY PROBLEM IN INVESTMENT DECISIONS *

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1. Introduction.

The capacity indivisibility problem has frequently been an important issue in discussions on efficient investment policies for public enterprises [1]. In this paper we formulate a model to measure the seriousness of this problem, based on an estimate of the additional investment costs, in present worth terms, which have to be incurred in order to ensure that capacity is always sufficient to meet expected peak demand. Specifically, the model can be used to measure the seriousness of the indivisibility problem in particular sections of an interconnected power system, where new indivisible equipment of a certain type should be installed at the time when the total capacity of the existing equipment is just been utilized, in order to ensure that excess demand in these sections requiring rationing will never appear [2].

The installed capacity of a certain type of equipment depends, as is known,

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1. See, for example, M. Boiteux (ref. 1), H. S. Houthakker (ref. 3), R. Turvey (ref. 7. 8) and O. Williamson (ref. 9).

2. H. B. Chenery (ref. 2) considered the problem of optimal timing of lumpy investments by formulating a model based on the interplay between economies of scale and an expected linear growth in demand for capacity. A. S. Manne (ref. 4) extented Chenery's model by considering: a) probabilities in place of a constant rate of growth in demand and, b) the economies and the penalties involved in accumulating backlongs of unsatisfied demand. While the object of these studies was to determine the optimal size of the plant to be installed, the present paper is concerned with measuring the seriousness of the indivisibility problem, when the capacity required to be installed is only available to the industry at a standardized plant size.

on the peak demand likely to be presented at the section of the system where this equipment is located[1]. If existing capacity is just sufficient to meet peak demand, a given growth in the latter can only be met by installing new equipment. While, however, the growth in peak demand may usually be considered as continous, installed capacity can only be increased in steps, as new equipment is introduced by discrete amounts. This poses the problem of choosing between an early introduction of the equipment in order to fit capacity to expected peak demand and the postponment of the installation until the time when the growth in peak demand can justify the efficient use of the new capacity.

The first solution results in higher investment costs due to permanent existence, at the particular section of the system, of excess capacity, as new indivisible equipment is introduced at the time when peak demand becomes just equal to installed capacity. The second solution, however, leads to a deterioration of the quality of the service rendered to the consumer fed from this section of the system, since it may require arbitrary voltage reductions or power cuts during peak load periods. It is evident that, the more serious the indivisibility problem is, the higher will be the additional investment costs to be incurred in order to ensure that capacity shortages requiring rationing of excess demand will never appear.

2. The Model for Measuring the Seriousness of the Indivisibility Problem.

Provided that it is the consumer who will finally be charged with a higher price, because of the additional investment costs due to the early introduction of a new indivisible equipment, it is worth determining the magnitude of these costs when we evaluate investment proposals in expanding the capacity of particular sections of an interconnected power system. For this purpose, the formulation of the model is made with reference to a given type of equipment, Z, located at the section W of the system. Therefore, the method is based on estimating the additional investment costs, which need to be incurred in order to ensure that the installed capacity of the indivisible equipment Z is always sufficient to meet the expected peak demand at W.

These costs, taken as a proportion of the total investment costs which would have to be incurred if Z were perfectly divisible, so that the increase in capacity coincided with the growth in peak demand, can then be considered as a measure of the seriousness of the indivisibility problem of the equipment in question.

For the formulation of the model the following relationships are used with respect to the case considered :

1. The required reserve capacity is regarded as part of the expected peak demand.

(a) The peak demand in growing exponentially at a rate m, i.e.,

$$D(t) = D_0 \ (1 + m)^t$$

(b) The initial capital cost is \$ k per unit of capacity and it is constant.

(c) The operational life of the equipment is L years.

(d) The discount rate r is constant throughout.

(e) New equipment is installed : i) in perfectly divisible amount, ii) in discrete amount of S capacity units.

Given these relationships, the rate of increase in peak demand is

$$D_0 \ (1 + m)^t \ \ln (1 + m)$$

On the other hand, the worth of the capital costs of installing a new unit of capacity and its subsequent replacements to infinity, at the time when the new unit is installed, is equal to :

$$K(t) = k \ [1 - (1 - r)^{-L}]^{-1} = K, \text{ constant}$$

Thus, the worth at dt of the capital costs to be incurred in installing in dt the required new capacity at section W and its subsequent replacements under case (e) i. will be :

$$KD_0 \ (1 + m)^t \ \ln (1 + m)dt$$

and the present worth of the total capital costs to be incurred from now to infinity will be :

$$C_1 = \int_0^{\infty} KD_0 \, (1 + m)^t \ (1 + r)^{-t} \ \ln \ (1 + m) \ dt \qquad (1)$$

where, C_1 is the present worth of all capital costs incurred from now to infinity for capacity increases and replacements.

Let now be assumed that new equipment can only be installed according to case (e) ii., i.e. in discrete amounts of S capacity units. In order to have always sufficient capacity at section W, new plant should be installed in concecutive points of time $T_0, T_1, T_2, \ldots.,$ where T_j $(j = 1, 2, \ldots.)$ is estimated, given T_{j-1} from the relation :

$$D(T_j) - D(T_{j-1}) = S$$

or

$$D(T_j) - D(T_{j-1}) = D(T_{j-1}) \left[(1+m)^{T_j - T_{j-1}} - 1 \right] = S \qquad (2)$$

when, $\quad D(T_0) = D_0 , \qquad T_0 = 0$

The worth at T_j of the capital costs incurred for the introduction at T_j of a new plant and its subsequent replacements to infinity will be :

$$KS$$

and the present worth of the sequence will, therefore, be egual to :

$$C_2 = \sum_{j=0}^{\infty} KS\ (1 + r)^{-T_j} \tag{3}$$

The measure ot the seriousness of the indivisibility problem of equipment Z located at the point W of the system can be expressed as :

$$C_2/C_1 = 1 + u$$

where, u represents the proportion of the additional investment costs, which should be incurred in order to ensure that the capacity of Z, which can be installed in descrete amounts, is always sufficient to meet expected peak demand.

3. Simplification of the Model.

It is difficult to handle eq. (3) for estimating C_2, since this would require T_j to be calculated for $j = 1$ to $j = \infty$. However, this equation can be simplified if we are willing to assume that the predetermined size of the indivisible plant, which is introduced in successive points of time, is rising at the same rate as the growth rate of peak demand [1].

Using eq. (2), it can easily be shown that this assumption results in $T_j - T_{j-1} = T$ say, for all $j = 1, 2 \ldots\ldots$ In fact, eq. (2) can be written as :

$$D(T_j) - D(T_{j-1}) = D(T_{j-1}) \left[(1+m)^{T_j - T_{j-1}} - 1 \right] = S_j = S_{j-1}\ (1+m)^{T_j - T_{j-1}}$$

$$\text{or, } D_0(1+m)^{T_{j-1}} \left[(1+m)^{T_j - T_{j-1}} - 1 \right] = S_0(1+m)^{T_{j-1}} (1+m)^{T_j - T_{j-1}}$$

where, S_j is the predetermined size of the plant to be installed in time T_j. Reducing this equation, we can obtain :

$$(D_0 - S_0)\ (1 + m)^{T_j - T_{j-1}} = D_0$$

$$\text{and,} \quad (1 + m)^{T_j - T_{j-1}} = \frac{D_0}{D_0 - S_0}, \text{ constant}$$

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1. T.N. Srinivasan (ref. 5) considered the problems of optimal capacity expansion like A.S. Manne, except that demand was assumed to grow at a constant geometric rate over an infinite horizon. He pointed out that it is optimal to install a plant at each point of a sequence of equally spaced time points. Because, however, of the geometric pattern of growth in demand, in his model like our assumption, the size of plant to be installed at each such time point is growing exponentially at a rate equal to the growth rate of demand.

Having $T_j - T_{j-1} = T$, constant for $j = 1$ to $j = \infty$, we can estimate T from

$$T = \frac{\ln\left(\frac{D_0}{D_0 - S_0}\right)}{\ln(1+m)}$$

Therefore, eq. (3) can now be written [1].

$$C_2 = \sum_{j=0}^{\infty} KS_0 (1+m)^{jT} (1+r)^{-jT} \qquad (3.1)$$

Since C_2 is a sum, we shall modify C_1 into a sum as well. Thus letting

$$(1+m)(1+r)^{-1} = R$$

eq. (1) can be written :

$$C_1 = KD_0 [\ln(1+m)] \left[\int_0^T R^t\,dt + R^t \int_T^{2T} \frac{R^t}{R^T}\,dt + R^{2t} \int_{2T}^{3T} \frac{R^t}{R^{2T}}\,dt + \ldots\ldots \right]$$

However, since.

$$\int_0^T R^t\,dt = \int_T^{2T} \frac{R^t}{R^T}\,dt = \int_{2T}^{3T} \frac{R^t}{R^{2T}}\,dt = \ldots\ldots = \frac{R^T - 1}{\ln R},$$

the above equation can take the form [2]:

$$C_1 = KD_0 \Big[\ln(1+m)\Big] \frac{R^T - 1}{\ln R} \sum_{j=0}^{\infty} R^{jT} \qquad (1.1)$$

Putting now KS_0 of eq. (3.1) at the left of the summation symbol, substituting R and dividing this equation by eq. (1.1), we may finally obtain :

$$C_2/C_1 = \frac{S_0}{D_0 \ln(1+m)} \frac{\ln R}{R^T - 1} = 1 + u \text{ for } R \neq 1$$

and (4)

$$C_2/C_1 = \frac{S_0}{D_0 \ln(1+m)} \frac{1}{T} = 1 + u \text{ for } R = 1$$

From the last equation we can calculate the proportion of the additional investment costs, in present worth terms, which should be incurred in order to ensure that capacity shortages at the section W of the system will never appear. Therefore, eq. (4) can be used as the measure of the seriousness of the existing capacity indivisibility problem.

1. As may be seen C_1 and C_2 are infinite for $m \geqslant r$. This however does not prevent estimating their ratio.

2. $\frac{R^T - 1}{\ln R}$ is valid for $R > 0$ and $\neq 1$. If $R = 1$, the integrals are simply equal to T. On the other hand, R is always positive.

It should be noted that this measure is based totally on investment costs. If however new capacity is more efficient than existing capacity, the introduction of the indivisible plant will result in better operating conditions for the system, since the excess capacity of the new plant will be used to replace less economical capacity. Thus, if we are willing to take into account the resulting savings in total operating costs of the installed capacity, then the proportion of the additional costs due to capacity indivisibility will be lower than that shown in eq. (4). Obviously, this proportion would be even lower if the predetermined size of new plant were expected to rise at a rate lower than the growth rate of demand.

Considering now the parameters entered in eq. (4), it can be deduced that the seriousness of the indivisibility problem, as measured by the value of u, is less when,

— the installed capacity of the equipment is large compared with the predetermined size of the new plant, i.e. when the ratio C_0/D_0 is small

— the growth rate of peak demand is high, and

— the discount rate is low.

Eq. (4) does not include K. This is due to the fact that this equation gives the additional investment costs to be incurred because of the indivisibility problem, as a proportion of the total investment costs which would be incurred if the capacity were perfectly divisible.

4. A Numerical Application of the Model.

Although the proposed model can be applied with reference to various sections of an interconnected power system and more generally, to any system where capacity indivisibilities represent a problem, for expository purposes we consider here a simble example referring to a section of an electricity distribution network [1]. In particular, let us consider the section W (i.e. the sum of distribution transformers feeding a village), where the individual equipment Z (distribution transformers) is located.

We make the following assumptions :

— $m = 0.11$ is the annual growth rate of peak demand

— $r = 0.10$ is the annual discount rate

1. For example, in the electricity generation section, the indivisibility problem may refer to the predetermined size of a new generating plant in relation to the installed capacity of the whole system.

— $D_0 = 2000$ KVA (killo-volt ampers) is the present peak demand and

— $S_0 = 400$ KVA is the present standardized size of a distribution transformer, which is available to the industry.

On the basis of these assumptions, we have:

$$R = (1 + m)(1 + r)^{-1} = 1.11/1.10 = 1.00909$$

and,

$$T = \frac{\ln\left(\frac{D_0}{D_0 - S_0}\right)}{\ln(1 + m)} = \frac{\ln\left(\frac{2000}{2000 - 400}\right)}{\ln 1.11} = 1.747$$

Substituting these values to eq.(4) we have:

$$C_2/C_1 = \frac{S_0}{D_0 \ln(1 + m)} \quad \frac{\ln R}{R^T - 1} = \frac{400}{2000 \ln 1.11} \quad \frac{\ln 1.00909}{1.00909^{1.747} - 1}$$

which finally gives

$$C_2/C_1 = 1.08834 = 1 + u$$

and, $u = 0{,}08834$.

Hence, the additional investment costs, in present worth terms, which should be incurred in order to ensure that installed capacity of the distribution transformers at the section W of the system is always sufficient to meet expected peak demand at that section, accounts for 8.83 per cent of the investment costs which would be incurred if the capacity of the transformers were perfectly divisible.

5. Allowances for Expected Technical Progress.

So far the formulation of the model was based on the assumption that the investment costs per unit of new capacity is constant. However, allowances for future technical progress can easily be incorporated into the model. The result, as shown below, is equivalent to raising the discount rate and this will increase the additional investment costs to be incurred in order to fit the indivisible capacity exactly to demand [1].

In order to spell this out let us assume that v denotes the rate of reduction of investment costs, that is the annual rate of expected technical progress. Then the investment costs of introducing a unit of new capacity at time t will be:

$$k(t) = k_0 (1 - v)^t$$

1. R. Turvey points out that the result of an allowance for expected technical progress is to raise long-run marginal cost, because the allowance is equivalent to raising the discount rate. See ref. 4, p. 56.

and the present worth, at that time, of the total investment costs of the first introduction of the unit of new capacity and its subsequent replacements into the infinity will be :

$$K^*(t) = \frac{k_0 (1 - v)^t}{1-(1 - v)^L \ (1 + r)^{-L}} = K^* (1 - v)^t$$

where K^* is constant.

Eq. (1) can now be written :

$$C_1^* = \int_0^\infty K^* D_0 (1 + m)^t \ (1 + p)^{-t} \ \ln(1 + m) dt$$

where, $(1 + p) = (1 - v)^{-1} (1 + r)$

Thus, letting $B = (1 + m) \ (1 + p)^{-1}$, eq (1.1) becomes :

$$C_1^* = K^* D_0 \left[\ln(1 + m) \right] \frac{B^T - 1}{\ln B} \sum_{j=0}^{\infty} B^{jT} \qquad (5)$$

On the other hand, eq. (3.1) can now be written :

$$C_2^* = \sum_{j=0}^{\infty} K^* S_0 (1 + m)^{jT} (1 + p)^{-jT} \qquad (6)$$

Finally, dividing eq.(6) by eq.(5) we obtain the following formulation of the model :

$$C_2^* / C_1^* = \frac{S_0}{D_0 \ln(1 + m)} \quad \frac{\ln B}{B^T - 1} = 1 + u^* \qquad \text{for } B \neq 1$$

$$\text{and,} \quad C_2^* / C_1^* = \frac{S_0}{D_0 \ln(1 + m)} \quad \frac{1}{T} = 1 + u^* \qquad \text{for } B = 1 \qquad (7)$$

From this formulation it is easily seen that the incorporation of future technical progress into the model is equivalent to an increase in the size of the discount rate, which results to raising the additional investment costs to be incurred in order to fit the indivisible capacity exactly to expected peak demand.

Thus, if we are willing to take into account, in the numerical example considered in the previous section, the expected technical progress by assuming that, the investment costs per unit of capacity of distribution transformers is expected to be reduced annually at a rate of 3 per cent, then we have $v = 0.03$ and

$$B = (1 - v) (1 + r)^{-1} (1 + m) = \frac{0.97 \times 1.11}{1.10} = 0.97882$$

Substituting the appropriate values in eq. (7) we obtain :

$$C_2^* / C_1^* = \frac{S_0}{D_0 \ln(1 + m)} \quad \frac{\ln B}{B^T - 1} = \frac{400}{2000 \ \ln 1.11} \cdot \frac{\ln 0,97882}{0.97881^{1.747} - 1}$$

which finally gives :

$$C_2^* / C_1^* = 1.11765 = 1 + u^*$$

and $$u^* = 0.11765$$

It is seen, therefore, that when expected technical progress is taken into account, the percentage of the additional investment costs rises from 8.83 to 11.77 per cent.

6. Conclusion.

The proportion of the additional investment costs, in present worth terms, which must be incurred in order to follow a policy of expanding the capacity of a section of a system, by installing equipment of predetermined sizes whenever demand catches up with existing capacity was suggested as a measure of the seriousness of the capacity indivisibility problem. The measure can be improved by taking into account the savings in operating costs, which result in the system from the replacement of the existing less economical capacity by the excess capacity of the new indivisible equipment over the time that the growth in demand is not sufficient to utilize its total capacity.

A possible use of this measure could be its application as a criterion to determine the appropriate time for installing a new indivisible plant. In particular, it may be decided that a policy of expanding the capacity whenever demand catches up with existing capacity will be followed only when the estimated proportion of the additional costs is lower than a maximum acceptable rate. Thus, if this proportion is found to be higher [1], then it may be appropriate to postpone the installation of the new plant until the time at which the proportion of the additional costs becomes equal to the level of the acceptable rate.

1. As we have noted, this may be due to one or more of the following factors : a) the expected growth in demand is low, b) the predetermined size of the plant which is availabl to the industry in large in relation to the existing capacity, c) the rate of discount is high and, d) the rate of future reduction in costs, owing to expected technical progress, is high.

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