A NOTE ON THE MULTIPLIER

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The multipliers of a model comprising an expenditure and a monetary sector differ from the multipliers of a model in which a production - employment sector has been added. However, in the limiting cases, i.e. when the IS curve is interest inelastic, or when the LM curve is interest infinitely elastic, the same multipliers are derived from either the two - or the three-sector model.

For the two-sector model we assume that the following standard relationships hold :

$$Y = C + I + G \quad (1)$$

$$C = C(Y, i) \quad (2)$$

$$I = I(Y, i) \quad (3)$$

$$G = \bar{G} \quad (4)$$

$$L = L(Y, i) \quad (5)$$

$$M = \bar{M} \quad (6)$$

Where :

Y = income

C = consumption

I = investment

G = government expenditure

L = liquidity preference

M = money supply

i = rate of interest

and all the variables are expressed in real values, except $\bar{M}$, the bar above a variable denoting that the variable is determined exogenously.

Substituting (2), (3) and (4) into (1) we obtain the equation of the expenditure sector, i.e. the IS curve :

$$Y = C\ (Y, i) + I\ (Y, i) + \bar{G} \tag{7}$$

From (5) and (6) we obtain the equation of the monetary sector, i.e. the LM curve:

$$L\ (Y, i) = \frac{\bar{M}}{P} \tag{8}$$

where P denotes the price level. Differentiating totally (7) and (8) and arranging in matrix form we have :

$$\begin{bmatrix} (1 - C_Y - I_Y) & -(C_i + I_i) \\ L_Y & L_i \end{bmatrix} \begin{bmatrix} dY \\ di \end{bmatrix} = \begin{bmatrix} d\bar{G} \\ \frac{d\bar{M}}{P} - \frac{\bar{M}dP}{P^2} \end{bmatrix} \tag{9}$$

where the parameters with a subscript denote the partial derivative of the relevant function with respect to the variable in subscript. From (9) we obtain the solution :

$$dY = \frac{L_i d\bar{G} + (C_i + I_i) \left(\frac{d\bar{M}}{P} - \frac{\bar{M}_{dP}}{P} \right)}{(1 - C_Y - I_Y)\ L_i + (C_i + I_i)\ L_Y} \tag{10}$$

where $C_Y > 0, \quad I_Y > 0, \quad C_i < 0, \quad I_i < 0$

$L_i < 0, \quad L_Y < 0$

$| C_i + I_i | < 1,$

so that the denominator in (10) is a negative number, $\Delta < 0$

From (10) we obtain the multipliers :

$$\frac{\partial Y}{\partial \bar{G}} = \frac{L_i}{\Delta} > 0 \tag{11}$$

$$\frac{\partial Y}{\partial \bar{M}} = \frac{(C_i + I_i)}{\Delta} > 0 \tag{12}$$

For the limiting case of interest inelastic IS curve we write $C_i = I_i = 0$ and obtain the multipliers :

$$\frac{\partial Y}{\partial \bar{G}} = \frac{1}{(1 - C_Y - I_Y)} \tag{13}$$

$$\frac{\partial Y}{\partial \bar{M}} = 0 \tag{14}$$

And for the limiting case of interest infinitely elastic LM curve, we substitute $L_i \rightarrow \infty$ into (11) and (12) and obtain the multipliers :

$$\frac{\partial Y}{\partial \bar{G}} \rightarrow \frac{1}{1 - C_Y - I_Y} \tag{13a}$$

$$\frac{\partial Y}{\partial \bar{M}} \rightarrow 0 \tag{14a}$$

Consider now the introduction of the production-employment sector in the model. The following variables are added :

N = labour employment

W = money wage rate

Y'_N = marginal product of labour

Y''_N = derivative of Y'_N

The following relationships are satisfied :

$$Y'_N = \frac{W}{P} \tag{15}$$

$$Y = Y(N) \tag{16}$$

Where $Y'_N > 0$ and $Y''_N < 0$.

Equation (16) is the supply side of the model. Assuming that W is determined endogenously, full employment is guaranteed and therefore

$$dY = Y'_N \ dN = 0$$

The model consists of three equations : (7), (8) and (16). Differentiating totally and arranging in matrix form we have :

$$\begin{bmatrix} (1 - C_Y - I_Y) & -(C_i + I_i) & 0 \\ L_Y & L_i & \frac{\bar{M}}{P^2} \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} dY \\ d_i \\ dP \end{bmatrix} = \begin{bmatrix} d\bar{G} \\ \frac{d\bar{M}}{P} \\ 0 \end{bmatrix} \tag{17}$$

Solving (17) we obtain

$$dY = 0 \tag{18}$$

so that all the income multipliers become zero.

In the limiting case, the system in (17) becomes inconsistent ; for example, for $C_i = I_i = 0$ and for r (A) rank of the matrix of the co-efficients and r (Ab) the rank of the adjugate matrix, we have that r (A) $<$ r (Ab). Nevertheless the expenditure and monetary multiplier work just the same: by partitioning (17) we obtain (9) and the multipliers provided by the latter system. Therefore, when one of the limiting cases is present in a three-sector model of full-employment equilibrium, the production sector plays a passive role: Equilibrium can be reached by shifts in the other two sectors only.

Assuming now that the money wage rate is determined exogenously, $\overline{W}$, we derive the following relationships from equations (10) and (11) respectively:

$$dY - Y'_N \, dN = 0 \tag{19}$$

$$dY''_N \, dN + \frac{\overline{W}}{P} dP = \frac{d\overline{W}}{F} \tag{20}$$

The model, in matrix form, now becomes:

$$\begin{bmatrix} (1 - C_Y - I_Y & -C_i + I_i) & 0 & 0 \\ L_Y & L_i & 0 & \frac{\overline{M}}{P^2} \\ 1 & 0 & -Y'_N & 0 \\ 0 & 0 & Y''_N & \frac{\overline{W}}{P^2} \end{bmatrix} \begin{bmatrix} dY \\ di \\ dN \\ dP \end{bmatrix} = \begin{bmatrix} d\overline{G} \\ \frac{d\overline{M}}{P} \\ 0 \\ \frac{d\overline{W}}{P} \end{bmatrix} \tag{21}$$

from which we obtain:

$$dY = \frac{(C_i + I_i) \, Y'_N \frac{1}{P^2} \quad \frac{\overline{W} d\overline{M}}{P} - \frac{\overline{M} d\overline{W}}{P} + L_i Y'_N \frac{\overline{W}}{P^2} \; d\overline{G}}{(1 - C_Y - I_Y) \, Y'_N \, L_i \frac{\overline{W}}{P^2} + (C_i + I_i) \frac{1}{P^2} (L_Y Y'_N \overline{W} - Y''_N \overline{M})} \tag{22}$$

Therefore

$$\frac{\partial Y}{\partial \overline{G}} = \frac{L_i Y'_N \frac{\overline{N}}{P^2}}{\Delta_i} > 0 \tag{23}$$

And

$$\frac{\partial Y}{\partial \overline{M}} = \frac{Y'_N \, (C_i + I_i) \frac{1}{P^2} \frac{1}{P}}{\Delta_i} > 0 \tag{24}$$

Where $\Delta_i < 0$, the denominator in (22).

In the limiting case the system again becomes inconsistent. However by partitioning we obtain the multipliers in (13) and (14). The production-employment sector continues to play a passive role.

If, instead of using the 4-equation three sector model, we turn to the 2-equation Aggregate Demand - Aggregate Supply analysis, we would obtain the same results :

Equation (10) is the differential of the aggregate demand function. From equation (20) we obtain :

$$dN = \frac{Pd\bar{W} - \bar{W}dP}{P^2 Y''_N} \tag{25}$$

which substituted into equation (19) gives the differential of the aggregate supply function :

$$dY = \frac{Y'_N \; (Pd\bar{W} - \bar{W}dP)}{P^2 Y''_N} \tag{26}$$

Equation (10) and (26) form the system :

$$\begin{bmatrix} (1-C_Y-I_Y)\,L_i+(C_i+I_i)\,L_Y & \dfrac{\bar{M}(C_i+I_i)}{P^2} \\ Y''_N P^2 & Y'_N\,\bar{W} \end{bmatrix} \begin{bmatrix} dY \\ dP \end{bmatrix} =$$

$$= \begin{bmatrix} L_i d\bar{G} + (C_i + I_i)\,\dfrac{d\bar{M}}{P} \\ Y'_N\,Pd\bar{N} \end{bmatrix} \tag{27}$$

which gives :

$$dY = \frac{Y'_N\bar{W}\left[\,L_i d\bar{G} + (C_i + I_i)\,\dfrac{d\bar{M}}{P}\,\right] - Y'_N\,\dfrac{\bar{M}(C_i + I_i)}{P}\,d\bar{W}}{Y'_N\bar{W}\,(1 - C_Y - I_Y)\,L_i + [C_i + I_i)\,L_Y \quad - Y''_N\bar{M}\,(C_i + I_i)} \tag{28}$$

and

$$\frac{\partial Y}{d\bar{G}} = \frac{Y'_N \bar{W} L_i}{\Delta_2} > 0 \tag{29}$$

$$\frac{\partial Y}{\partial M} = \frac{Y'_N \bar{W} (C_i + I_i)}{\Delta_2} \quad \frac{1}{P} > 0 \tag{30}$$

$\Delta_2 < 0$, being the denominator in (28).

When the limiting cases are considered the system in (27) remains consistent and gives the multiplier in (13) and (14).

Therefore, in conclusion, the limiting cases being imperfections of the expenditure and monetary sectors, are contained within these two sectors, while the production-employment sector does not participate in the elimination of the anomaly.