STOCHASTIC PROCESSES IN EGONOMIC MODELLING OPERATIONS

By

T. ARTIKIS Ph. D
Graduate School of Industrial Studies
Piraeus, Greece

G. ARTIKIS M. B. A
Agricultural Bank of Greece

Introduction

Constructing theories that can explain the phenomena observed is the main purpose of science. Economics is no exception. A theory is called a model if it is expressed in mathematical form. For example the economist may wish to know how changes in prices affect exports in order to predict what will happen when the exchange rate depreciates. Such a study would involve establishing a mathematical relationship between exports and prices. A set of such relationships is an Economic model. An Economic model can be viewed as transforming a set of exogeneous variables into a set of endogenous variables. Economic models which determine the values of certain endogeneous variables, given the values of the exogoneous variables without specifying how the variables evolve through time are called static models. These models are used in Economics as an abstraction from the changing conditions in order to understand in part how the forces work in the very short ran.

From their structure static models cannot describe cyclical fluctuations or growth and decline of economic variables. Thinking in static terms, an economist might fall into the trap of believing that the economy is intrisically stable in the sense of producing nearly constant or very smooth time peths for Gross National Product, employment, the price level and so on, only if the exogeneous variables such as money supply are kept constant. Furthermore static models might also lead to the conclusion that if money supply is controlled to grow at constant percentage rate the resulting economic aggregates will also follow smooth growth paths. Anyone with experience in the dynamic character of economic systems realizes that this presupposition may not be valid. The resulting time path of Gross National Product from a one-step increase in government expenditures is not likely to be smooth. Neither will it necessarily converge to a constant value as time goes on. From the above reasons it becomes obvious that ignoring timing relationships in economic systems it is very likely for the economist to get false conclusions. These defficiencies of static models gave rise to the dynamic economic models.

A dynamic theory specifies the relations among the economic variables at different points of time and can thus yield the time paths of the endogeneous variables. The solution of a dynamic theory, being a set of functions of time is more difficult to study or even to characterise than the solution of abstract theory. Since the value of each endogeneous variable changes through time we wish to know whether a certain variable increases or decreases with time, whether it oscilates, and whether it may eventually reach an equilibrium value. For the problems treated by dynamic models, particularly those having to do with fluctuations in the economy or business cycles, a set of equilibrium values is not necessarily required. It is possible to be continously fluctuating.

Many important economic systems are extremely complex and their description by dynamic models arises the question of how much of the flavour of the original system is retained in the mathematical description. One of the ways to account for the various inaccuracies in translating the real economic system into a tractable dynamic model is to assume that the system is subjected to random disturbances of one type or another. The defects of the economic system, the errors of the economic machine and the consumer's decisions are some other cases where random elements enter. In truth the random elements are inherent in the conditions under which the economic system has to work. Stohastic dynamic models deal with the stochastic time paths generated by dynamics systems in which random elements enter. A simple and useful way to transform a dynamic model into a stochastic dynamic model is to add random factors to the equations of the dynamic model. These factors, which are used to study economic quantities varying in time in a random manner, are known in probability theory as stochastic processes. A stochastic process can be defined quite generally as any collection of random variables X(t), teT defined on a common probability space, where T is a subsect of the real line and is thought of as the time parameter set [2]. Since the evolution from static deterministic economic models to dynamic stochastic models is still taking place today it will be interesting to see the applications of stochastic processes in economic modelling operations. Operational research, analysis of time series, stochastic control theory, the explanation of Pareto law and Econometrics, are the main points of contact between stochastic processes and economic modelling operations.

Operational Research

Operational research is a marginal subject and — in practice at least — it tends to lure the economist away from his main purpose, which is the analysis of the working of economic system. But it offers him advantages which he usually misses, namely concrete tasks, and work in continuous contact with practical requirements and data. The main points of contact between operational research and stochastic processes are queueing and replacement problems [4].

In queueing situations, problems arise either because of too much demand on the facilities, in which case there is an excess of queueing time, or too little demand, in which case there is too much idle time for the facilities. The practical problem is to obtain an optimum balance between the costs associated with waiting and idle time. The mathematics involved are relatively straightforward, provided that arrivals and service times follow the negative exponential distribution. Once this simplifying assumption is removed, the mathematics tends to become extremely complex. However the advent of the electronic computer and the consequential ability to carry out Monte Carlo type simulations on a massive scale have completely revolutionized the approach to queueing problems.

Replacement problems fall into two categories. In the first the decision is generally between replacing a large piece of capital equipment or having it overhauled and hoping that it will last sufficiently long to be able to make the decision afresh at a suitable later date. The other type of replacement problem concerns depreciating items which are rather cheaper. In both these cases it can be seen that the element of probability enters into the calculation in the form of a renewal process. The point where operational research and economic theory meet to establish a new route has not yet been reached.

TIme Series

In the last thirty years the theory of time-series has been transformed into a new subject. The transformation is due to the introduction of probabilistic ideas into what was formerly treated deterministically. A time-series evolving probabilistic ideas is a stochastic process generated by a system of stochastic difference equations. The simplest case of time series is described by the stochastic difference equation.

$$X(t + 2) + a X(t + 1) + b X(t) = Z (t + 2) \qquad (1)$$

where a and b are parameters and Z represents a random variable. The time-series approach to the modelling of economic systems makes use of the following methods and technics.

1. Decomposition methods. These methods have been used by economists for the purpose of adjusting time series for seasonality and the predicting the business cycle. From the point of view of accuracy these methods seem to be producing adequate results even though this statement has not been proved by extensive research findings.

2. Autoregressive and moving-average methods which have been used extensively by economists in the field of forecasting. Although their theoretical rigorousness and completeness are surpassed they have lagged in actual application. The complexity of the method hinders its utilization.

3. Filtering techniques are the most promising of time series methods and their usage in Economics will increase in the future. Their main purpose is to estimate current states of the economic system.

4. Exponential smoothing models were utilized by business enterprises. Their main contribution has been in the area of short-term forecasting of inventories and the sale of individual items or products. Their utilization cost is extremely low, while their accuracy is at least as good as that of autoregressive and moving average methods. Their disadvantage is that there are several models available from which the Economist must select the one best suited to his data and personal preferences.

Stochastic Control Theory

The application of control theory in Economics is a recent one. In a deterministic framework control theory assumes that a dynamic system can be completely described at time t by its state. The state is a set of n usually interrelated variables evolving with time. We denote it by a vector

$$x(t) = (x_1 (t), \dots x_n (t))$$

Considering the case of continuous time the evolution of the system can be described by the n differential equations

$$\frac{d\,x\,(t)}{dt} = f(x\,(t),\ u\,(t),\ t) \qquad (2)$$

where the functions f(...) are twice differentiable. The function

$$u\,(t) = (u_1\,(t), \dots, u_m\,(t))$$

is called control function. The object of control theory is to choose the control function in order to optimize a stated objective function or measure of performance.

When the economist can assign objective or subjective probability laws to the uncertainties of the economic system described by (2) then stochastic control theory is applied. In this case the system (2) has to be changed into a stochastic differential system of the form

$$\frac{d\,x(t)}{dt} = f(x(t),\ u(t),\ w(t),\ t) \qquad (3)$$

where w(t) represents uncertainties and will be represented by a stochastic process. Stochastic modelling in control theory relies on the theory of stochastic differential equations in the sense of I T O (3). These equations are the exact counterpart of ordinary deterministic differential equations. One of the main difficulties of stochastic control theory comes from the fact that open — loop controls are not equivalent, as is the case of deterministic control theory. Inventory and production control, finance, marketing, and control of natural resources are some applications of stochastic control theory in Economics.

The Pareto Distribution

The Pareto distribution is encountered in many fields of economics. Income and wealth are well-known instances. The distribution of firms according to capital, sales, or employment, is another instance. The Pareto distribution was first explained by D. C. CHAMPERNOWNE [1] as the result of a stochastic process. He uses the ergotic character of some stochastic processes; by virtue of this the process, if it continues long enough, produces a stable distribution which is independent of the initial distribution. The result of the process depends only on the probabilities of transition from one state to the other. The explanation of the Pareto law depends on the probability theory by which order in the mass is produced out of individual chaos by the very fact of the random character of individual action.

Econometrics

Time series methods are not interested in why something is happening. The only other alternative to a such mechanistic approach is to attempt to explain the behavior of a certain factor through variations in a number of independent variables. This can be done through a system of multiple regression equations which, by their nature, are explanatory.

Such an approach was very popular in '50s and '60s and came mainly from economists; it resulted in huge econometric models, often consisting of hundreds or even thousands of variables. The main uses of these econometric models are, to provide economic forecasts, to explain the dynamic behavior of the economy in question, and to help make better quantitative economic policies.

The basic tenet of econometrics is that economics deals with random processes and hence to many econometricians it may seem superfluous to ask for stochastic approach. The most realistic econometric models are those of which the regression coefficients are stationary stochastic processes [5].

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