OPTIMUM EXCESS RESERVES, SPEED OF ADJUSTMENT AND CREDIT EXPANSION

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I. INTRODUCTION

In their article published in 1961,[1] Orr and Mellon investigated the impact of the uncertainty upon the expansion of credit. Their procedure was to treat a bank which extends new credit in a volume which maximizes the expected profits of the bank. Their model can be summarized as follows:

It is assumed that there is a positive return, i, on credit expanded, the interest earned on new loans made during the period in response to the excess reserves on hand at the beginning of the period. It is also assumed that there is a cost whenever the bank falls below its reserve requirement. This cost is a composition of a lump-sum, M, and a penalty, r, on each dollar of reserve shortage. The loss of reserves, L, is assumed to be a random variable, so that expected profits, P, are given by the expression

$$P = iD - M \int_{v}^{\infty} f(L)dL - r \int_{v}^{\infty} Lf(L)\,dL \qquad (1)$$

where D stands for demand deposits and v is derived from the condition of legally required reserves: $R - L \geqslant \rho(D\text{-}L)$, by solving, in the case of the equality, for L:$v = (R\text{-}\rho D) / (1\text{-}\rho)$, and where R is the level of excess reserves at the beginning of the period. The assumption is made that banks try to maximize their expected profits, P, given by equation (1). The optimum level of D is derived after a series of painful evaluations and successive approximations, by setting the first partial of (1) with respect to D equal to zero, and solving for D, given the values of the parameters.

One of the major criticisms that this article has drawn[2] is the assumption in this model that the entire response of reserves and the adjustment of credit take place within one period. This exposition rules out completely the relevance and the possibility of the period-by-period type of expansion described in the textbooks, which is consistent with observed behavior.

1. Orr and Mellon [4].
2. Miller [3], p. 1120.

Furthermore, if the return on new credit, i, exceeds the penalty rate, r, the bank maximizes profits by expanding D indefinitely. Orr and Mellon do not particularly emphasize this point.[3] On the other hand, as one can draw from the experience of European banks, absence of any reserve requirement or absence of penalties in this respect does not necessarily imply an indefinite expansion of credit.

Orr and Mellon treat the probability of running short of excess reserves as an exogenous variable, independent of the behavior of the bank, which serves only to determine the level of expected profits of the bank. There is no mechanism by which this probability can be reduced, and the fact that the banks might be concerned also with the so-called "non-pecuniary" costs of running out of excess reserves is completely ignored. This probability, of course, could be reduced either by increasing the level of excess reserves or by increasing the speed of adjustment to the required level of excess reserves. Both higher excess reserves and a higher speed of adjustment entail costs, so that both these costs should be considered in the banks' decisions.

Credit expansion or contraction constitutes a policy in the hands of the government which uses it in relation with the overall economic policy. Random disturbances in the level of excess reserves means disturbances in the expected rate of credit expansion, which may intervene with the targets of monetary policy and consitute an undesirable impediment to the attainment of these targets. From the bank's point of view, given the net return on deposits, it would be highly efficient to make excess reserves equal to zero at any period of time, and thereby obviate the need for holding excess reserves to supply more credit in the economy. On the other hand, however, no bank would wish to invest resources in a stock of excess reserves so large as to enable it to face any possible outflow of reserves of any conceivable size. At some point the benefit conferred by an extra dollar of excess reserves is less than the gain obtained from expanding credit using that dollar of reserves. The benefit derived from holding excess reserves is simply the avoidance of the costs of adjustment. A reserve policy is immediately related to credit expansion policy; since any disturbance in the credit market must either be financed immediately by making use of excess reserves or be eliminated by a faster adjustment within the banks, a reserve requirement legally fixing required reserves necessarily entails some rate of adjustment on the part of the banks. The observed phenomenon in American banking that the adjustment is almost instantaneous cannot be taken as given, since the level of reserves and the speed of adjustment are actually interdependent and determine the rate of credit expansion simultaneously.

This interdependence is the main feature of the model proposed below. It is based on the assumption that the bank wishes to maintain a given probability of running out of excess reserves and that this goal can be accomplished with different combinations of reserve levels and rates of adjustment. These two policy instruments used by

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3. Miller [3], p. 1129. Also Cooper [1], p. 743, footnote 5.

banks entail costs for the bank, given a positive marginal net return from credit expansion. These costs are of a different nature; a larger level of excess reserves reduces the level of credit expansion and thus reduces the expected profits of the bank, whereas a higher speed of adjustment increases the variability of credit available and thus increases the variability of profits. By assuming that the utility of the bank is a function of both the expected value and the variability of profits, it is possible to derive the optimum speed of adjustment and the optimum level of excess reserves and credit expansion of the bank.

It is reasonable to assume that the bank derives utility from the level of profits it can make and disutility from any possible deviations from the expected level of profits, the deviations being randomly caused by the market. Therefore, the utility, U, of the bank is a function of the expected level of profits, $E(\pi)$, and of the variability of these profits measured by the square root of the variance σ_π^2; i.e.

$$U = U\ [E(\pi),\ \sigma_\pi]$$

Assuming that the net return from every unit of credit, D_t, generated during the period is positive, profits are a monotonically increasing function of D_t, so that the utility function can be expressed:

$$U = U\ [E(D),\ \sigma_D]$$

where $E(D)$ is the expected level of credit generated during the period and σ_D is the measure for the variability of credit. The expansion of credit depends upon the level of excess reserves, so that both $E(D)$ and σ_D depend upon the expected level and the variability of excess reserves. The utility maximizing procedure of the bank will determine the optimum level of credit expansion, the optimum speed of adjustment, and the optimum level of excess reserves. We need, therefore, to make some assumptions concerning the bank's behavior, which will allow us to determine the constraint or feasibility locus of the bank's behavior.

II. THE ADJUSTMENT MECHANISM

The basic assumption of the following model is that the bank wishes to maintain a desired level of excess reserves so that it can meet any unexpected disturbance in the flow of reserves. A particular level of excess reserves is desired not only because there is a legally required reserve ratio or a penalty in the case of not meeting this ratio (for there might be no reserve requirement), but mainly because if the bank runs out of excess reserves this could be a sign of unsound banking, with undesirable consequences in the market. Whenever the actual stock of excess reserves deviates from the desired level, the bank takes steps to bring it back to the target level.

Specifically, the bank desires to maintain a given stock of excess reserves, R_t^*, and each period it plans to reduce any discrepancy between R_t^* and its excess reserves at the beginning of the period, R_{t-1}, by a certain proportion, g, of this deviation. As long as excess reserves depart from their desired level, the bank wishes to induce a change, ΔR^*_t, in its excess reserves, which is given by:

$$\Delta R^*_t = g \cdot (R^*_t - R_{t-1}) \tag{1}$$

where $0 \leqslant g \leqslant 1$.

For example, if reserves at the end of last period were below the desired level, the bank will attempt in the current period to tighten its credit policy, thus improving its level of excess reserves by a certain proportion, g, of this gap.

Excess reserves will be adjusted both upward or downward. For simplicity, equation (1) assumes a completely symmetrical adjustment process, although a more realistic as well as more complicated model could be one that assumes an adjustment parameter which depends on the sign as well as on the magnitude of the departure.

Assuming now that the level of credit generated during the period t is D_t and that the legal reserve ratio is r, then the level of current reserves, after the occurance of the desired change, is legally efficient if:

$$R_{t-1} - \Delta R^*_t \geqslant r \cdot (D_t - R^*_t) \tag{2}$$

which can be written:

$$R_{t-1} - (1-r)\ \Delta R^*_t \geqslant rD_t$$

or, substituting $g(R^*_t - R_{t-1})$ from (1), we have:

$$\frac{1}{r}\,[R_{t-1} + (1-r)\,g\,(R_{t-1} - R^*_t)\,] \geqslant D_t \tag{3}$$

Equation (3) specifies that level of credit which can emerge at most, so that the desired level of excess reserves is achieved. For example, if $R_{t-1} - R^*_t > 0$, reflecting a surfeit of reserves, the bank will expand credit by an amount directly proportional to g and inversely proportional to r[4]. Note that the same relation holds if r is not legally required but it is institutionally determined by the criteria of sound banking. Note also that the usually considered relation $R_{t-1} \geqslant rD_t$ or $R/r \geqslant D$ emerges only if g=0. On the other extreme, if we assume, as Orr and Mellon do, that g=1, then:

$$\frac{1}{r}\,[R_{t-1} + (1-r) \cdot (R_{t-1} - R^*_t)\,] \geqslant D_t$$

$$\frac{1}{r}\,[\,(2-r)\,R_{t-1} - (1-r)\,R^*_t\,] \geqslant D_t$$

so that only R^*_t has to be determined.

4. The faster the adjustment process and the smaller the required reserve ratio the greater the expansion of credit, ceteris paribus.

III. LEVEL AND VARIABILITY OF RESERVES.

If the bank is confronted with *given* excess reserves at the beginning of the period, then with the policy-determined expansion of credit the actual change in the level of excess reserves will equal the desired change. Such certainty, however, is far from the real world where unexpected outflows of reserves occur. There is no reason for a bank to disregard such unexpected disturbances in its desired change of excess reserve levels, due to factors bearing elements of randomality.

We assume, therefore, that there is a random component, u_t, in the process of changing the level of excess reserves, so that the actual change ΔR_t becomes:

$$\Delta R_t = \Delta R^*_t + u_t \tag{4}$$

where u_t is the random component having zero mean and a constant variance σ^2_u, and is independently distributed.

Since, however, excess reserves at the beginning of the period have been assumed to be given and the expansion of credit is determined solely by the behavior of the bank, the disturbance term reflects an item which has no effect on the level of excess reserves at the beginning of the period, no effect on the composition of the bank's assets, and no effect on the credit expansion policy of the bank for the current period.[5] Such a disturbance might be caused, for example, by an unexpected rushing of customers to the bank and withdrawing all their deposits, causing an unexpected outflow of reserves.

Substituting (1) into (4) we have:

$$R_t = g(R^*_t - R_{t-1}) + u_t \tag{5}$$

$$R_t - R_{t-1} = g(R^*_t - R_{t-1}) + u_t$$

or, adding R_{t-1} to both sides:

$$R_t = gR^*_t + (1-g)R_{t-1} + u_t$$

On the assumption that u_t is independently distributed with variance σ^2_u, then the level of excess reserves is also independently distributed with a constant mean and with variance given by:

$$\sigma_R{}^2 = \frac{\sigma^2_u}{g(2-g)} \tag{6}$$

The variance σ^2_R is a measure of the variability of excess reserves. From equation (6) by differentiating we get:

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5. I wish to emphasize this point; the variance σ_u is external to the banking sector and independent of the operations of the banks.

$$\frac{d(\sigma_R^2)}{dg} = - \frac{2(1-g)}{g^2\ (2-g)^2}\sigma_u^2 < 0 \tag{7}$$

since g can take values between 0 and 1. This means that the variability of excess reserves is inversely related to the speed of adjustment g.

Furthermore, from equation (6), if $g = 0$, i.e. if the bank makes no attempt to maintain a particular level of excess reserves, the variance of excess reserves becomes infinite, since the level of excess reserves is then an infinite sum of random disturbances. On the other hand, if $g=1$, i.e. if the discrepancy between R_t^* and R_{t-1} is eliminated completely during the current period, the variance of R is at a minimum, as it can be seen from equation (6). In this latter case the variability of excess reserves is affected only by the variability of exogenously determined random elements.

IV. THE PROBABILITY OF RUNNING OUT OF EXCESS RESERVES.

Because it is a random variable, the level of excess reserves has a frequency distribution, represented in figure 1, with mean $R=E(R)$ and with variance given by equation (6).

What is of particular importance in this case is the probability of running out of excess reserves, i.e. when $R \leqslant 0$. This probability is given by the shaded area under f(R) to the left of $R=0$ in Figure 1. Negative excess reserves represent the case of either not meeting the legal reserve requirement and having to pay a penalty or the ca-

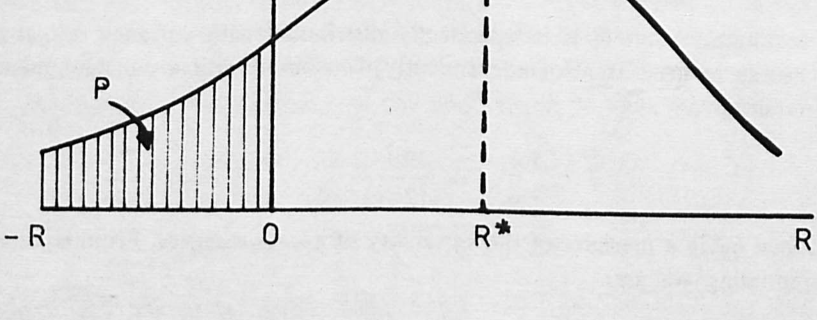

FIGURE 1.

se when borrowed reserves are used. It is assumed for our purposes that the **shape** of the f(R) curve is not affected by either the actual nor the desired level of excess reserves. An increase in the latter, for example, will merely shift the entire curve to the right by an amount equal to the increase of R^*, without changing the shape of the curve. If such is the case, then the probability of running out of excess reserves, P, is an inverse function of R^*. A shift of the entire curve to the right will decrease the shaded area to the left of $R=0$ in figure l. If there is no change in the variability – measured by the variance – of excess reserves, an increase in the desired level of excess reserves clearly reduces the likelihood that excess reserves will be exhausted.

On the other hand, for a given value of R^*, a decrease in the variability of excess reserves will reduce the value of P. Since by equation (6) σ_R^2 is itself a function of two parameters, g and σ_u^2, and since $d\sigma_R^2 / dg < 0$ and $d\sigma_R^2 / d\sigma_u^2 > 0$, then P is given by the functional relationship:

$$P = p(R^*, g, \sigma_u^2) \tag{8}$$

where

$$dP/dR^* < 0,\ dP/dg<0, \text{ and } dP/d\sigma_u^2 > 0 \tag{9}$$

The fact that the first partials of P with respect to R^* and g are both negative leads to the following conclusion: In order that the bank can maintain a given P, there is a trade-off between the average (desired) level of excess reserves and the speed of adjustment at which the bank responds or intends to respond to a gap between R_{t-1} and R_t^*. In other words, a given P can be maintained by reducing (increasing) the rate of adjustment and by keeping, on the average, more (less) excess reserves. The reduced (increased) g increases (decreases) the variance σ_R^2 of excess reserves and thereby increases (reduces) P, whereas the rise (fall) of R^* shifts the distribution to the right (left), thereby lowering (increasing) P.

If σ_u^2 is constant and P is assumed fixed or given, there is an explicit functional relationship between R^* and g. The values of R^* and g satisfying this relationship are plotted in Figure 2 and represented by the curve P_1P_1. Since there is a similar relationship for each value of P, we have a whole family of equal-probability curves. Obviously the value of P diminishes as we move upward and to the right.

From equation (9) it is clear that a rise in the variability of random disturbances can be met by either an increase in the desired level of excess reserves or by an increase in the rate of adjustment, so that the probability of running out of excess reserves remains the same.

V. COST OF ADJUSTMENT AND COST OF EXCESS RESERVES.

Obviously, if the cost of keeping excess reserves is greater than the cost of adjustment, the bank will attempt to increase the adjustment parameter, g, and decrease

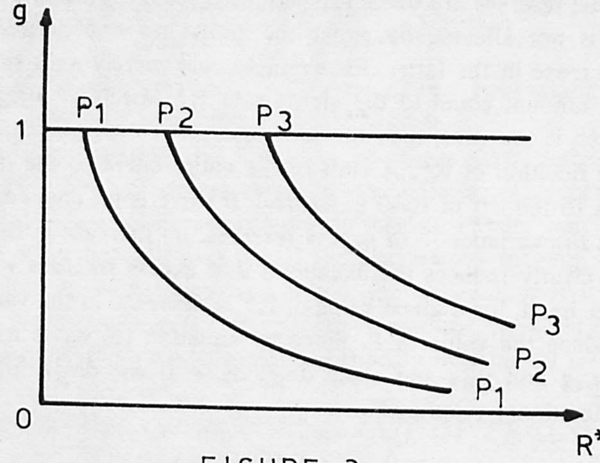

FIGURE 2.

the level of desired excess reserves up to the point where the probability of running out of excess reserves is considered acceptable.

Consider first the cost associated with holding a larger stock of excess reserves. This stock represents an allocation of assets away from alternative uses. Because we have assumed that the net return from a unit of credit expanded is positive, an increase in the level of excess reserves means a decrease in expected profits caused by the decrease of credit. Assuming that one dollar of reserves can generate k units of credit, then for every dollar of excess reserves credit is contracted by k units. If the desired level of excess reserves is R*, then the expected level of credit expansion will be kR* units less then it would have been had the bank decided to keep no excess reserves. The relation between the expected level of credit and the desired level of excess reserves is given by:

$$E(D) = D^{max} - kR^* \tag{10}$$

where D^{max} is the maximum level at which credit can be expanded in the case where no excess reserves are held. Thus, an increase in R* will reduce the chances of ever running out of excess reserves, as we have seen above, but at the same time it entails a decline in the expected level of credit and, therefore, in the expected level of profits, given the assumption that the net return from an additional unit of credit expanded is positive.

The cost associated with an increase in g is of a different nature. If the probability of running out of excess reserves is reduced not by a rise in excess reserves but rather by a faster adjustment, then there will be a larger variability around E(R), a

6. We assume throughout that the bank is a risk averter.

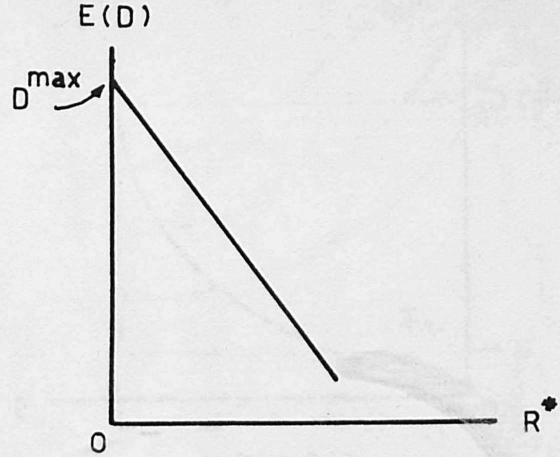

FIGURE 3.

larger variability around E(D) and, hence, a larger variability of excpected profits. This can be seen from equation (3) which can be written as

$$\frac{R_{t-1} + (1-r)\,gR_{t-1} - (1-r)\,gR^*_t}{r} = D_t \qquad (11)$$

and where the inequality sign has been omitted. This can be written as:

$$\frac{(1-g-rg)}{r}R_{t-1} - \frac{(1-r)g}{r}R_t = D_t \qquad (12)$$

where now R_t is an independent, normally distributed variable, with mean E(R) = R^*_t and variance σ^2_R given by equation (6). Therefore D_t is also a normally distributed variable with mean E(D) and variance σ^2_D given by:

$$\sigma^2_D = \frac{(1-r)^2}{r^2} g^2 \; \sigma^2{}_R = \frac{(1-r)^2}{r^2} \frac{g}{2-g} \quad \sigma^2_u \qquad (13)$$

The first derivative of σ^2_D with respect to g is:

$$\frac{d\sigma^2_D}{dg} = 2 \; \frac{[1-r]^2}{{}^2(2-g)^2} \cdot \sigma^2_u \qquad (14)$$

Since the derivative is always positive, σ^2_D is an increasing function of g such as depicted in figure 4.

It is now clear that the two behavioral instruments, g and R*, available to the bank in maximizing profits, while keeping a given probability of running out of reser-

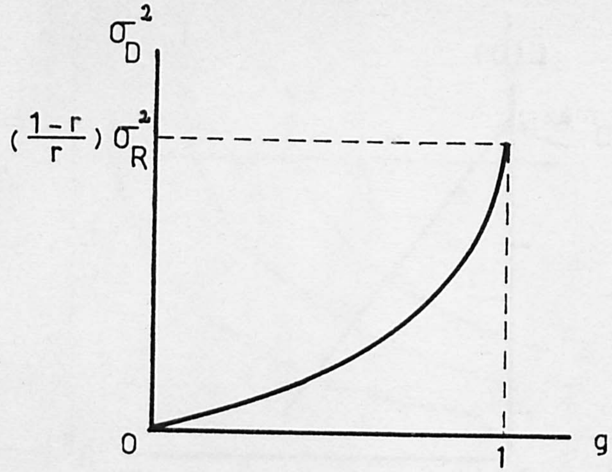

FIGURE 4.

ves, entail costs of quite a different nature. An increase in the rate of adjustment increases the **variability** of profits, while an increase in the level of excess reserves decreases the **level** of profits.

VI. PREFERENCE SURFACE AND FEASIBILITY LOCUS

The different costs have to be compared and then enter the optimizing decision of the bank. The subjective trade-off between the expected value and the variability of credit expansion is given by the bank's preferences which are determined by the utility function:

$$U = U[E(D), \sigma_D] \tag{15}$$

where $\frac{dU}{dE(D)} > 0$ and $\frac{dU}{d\sigma_D} < 0$

The indifference map corresponding to equation (15) is depicted in Figure 5.

These indifference curves reflect the fact that the marginal rate of substitution between E(D) and σ_D is positive and increasing, i.e. that the utility function of the bank is positively related to the level of credit expansion and negatively related to the variability of credit.

The feasibility locus represents the constraint posed to the bank in the optimizing conditions or, in other words, the technical trade-off between E(D) and σ_D that is possible for the bank to achieve, given the properties of R* and g, and the particular probability of running out of excess reserves that the bank wants to maintain. This technical trade-off can be written in general form as:

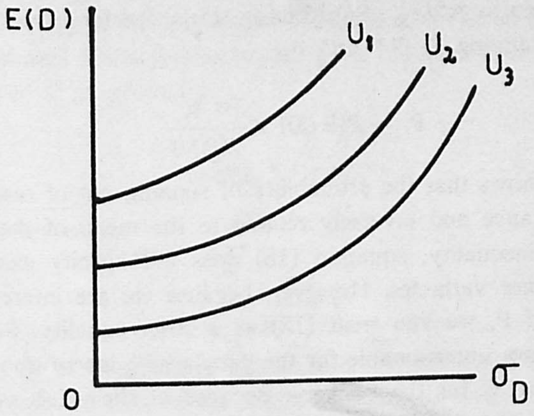

FIGURE 5.

$$E(D) = f(\sigma_D) \tag{16}$$

and is reasonably explained by the following line of reasoning. If the bank decides that its profit must be the highest possible, and therefore the level of credit expansion should be D^{max}, it will hold no reserves, and to maintain a given P it would adjust its excess reserves. To minimize P in the same time, of course, would be to adjust instantaneously, i.e. g=1. In this case, as results from the previous arguments, the variability of credit (and profits) would be extremely large. Any decrease in the variability can be achieved only by keeping a higher level of excess reserves, which necessarily implies a decline in E(D). For a given P, therefore, σ_D can be reduced only at the expense of E(D) and E(D) can be increased only if there is a concomitant rise in σ_D.

A more precise desctription of equation (16) requires that some assumption be made about the form of equation (8). As we have previously said, R is a random variable with a given distribution the mean of which is $E(R) = R^*$ and the variance of which is σ_R. If we assume that the distribution of R is symmetrical around R^*, then by making use of Chebychev's inequality[7] we have the following relation, for $R<R^*$:

$$[P(R^* - R) \geqslant t\,\sigma_R] \leqslant 1/2t \tag{17}$$

Since $t = R^* - R/\sigma_R$ and since we are interested in those values of R which are less

7. Chebychev's inequality can be stated in general terms as:

$$[p(x-\bar{x}) \geqslant t\sigma_x] \leqslant 1/t^2$$

and it asserts that given the distribution of any random variable, x, the probability of observing a value of x which is at least $(t\sigma_x)$ away from the mean, $\bar{x}$, is at most $1/t^2$.

than zero, we have $t = R^*/\sigma_R$. Substituting this value for t into (17), we obtain the following relation among P, R^* and σ_R :

$$P = P(R\leqslant 0) \leqslant \frac{\sigma^2_R}{2(R^*)} \tag{18}$$

This relationship shows that the probability of running out of reserves, P, is directly related to the variance and inversely related to the mean of the distribution of R. Because it is an inequality, equation (18) does not specify exactly how much P varies with the other variables. However, because we are interested mainly in the maximum value of P, we van treat (18) as a strict equality. Such a conservative evaluation of P is not unreasonable for the people who worry about the adequacy of excess reserves, that is for the bankers. Because of their risk aversion they would regard the maximum probability of running out of excess reserves, for given R^* and σ_R , as the relevant magnitude in determining excess reserve adequacy. Not to mention, of course, the obvious simplification that this assumption makes possible. Therefore, equation (18) will be treated as an equality in the remaining of this paper.

We substitute now from equation (6) for σ_R^2 in equation (18) so that:

$$P = \frac{\sigma_u^2}{2g(2-g)\ (R^*)^2} \tag{20}$$

Equation (20) clearly shows the inverse relation of P to R^* and the commensurate relation between P and g, since $0\leqslant g\leqslant 1$. If $g=1$, then the probability of running out of excess reserves is solely a function of the level of the desired reserves. If $g=0$, this probability approaches infinity since the bank does not try to close the gap caused by the discrepancy between the actual and the desired level of reserves. Furthermore, the partial derivatives of P with respect to R, g and σ_u are such that the conditions in equation (8) are satisfied. In order to make explicit the relationship between g and R^*, we can write equation (20) as:

$$g\ (2-g) = \frac{\sigma_u^2}{2P\ (R^*)^2} \tag{21}$$

Solving the quadratic equation (21) for g and taking the negative root,[8] we end up with:

$$g = 1 - \sqrt{[\ 1 - \frac{s_u^2}{2P\ (R^*)}\ ]} \tag{22}$$

Equation (22) is the algebraic expression for the **equal-probability curves** pictured in Figure 2. When R^* becomes very large, then the speed of adjustment needed to

8. The reader can easily verify that the positive root of the quadratic equation (21) has no meaning, since g takes values between zero and one.

maintain any probability of running out of excess reserves is very small. Again, when the speed of adjustment is instantaneous, i.e. $g=1$, the level of excess reserves needed to maintain a given P is given by:

$$R^* = \sqrt{\frac{\sigma_u^2}{2P}} \qquad (23)$$

Alternatively, we can write:

$$P = \frac{\sigma_u^2}{2(R^*)^2} \qquad (24)$$

i.e. the probability of running out of excess reserves, when the adjustment is instantaneous, is an inverse function of the desired (expected) level of reserves. However, as we have seen, a greater speed of adjustment means a larger variability of excess reserves, a larger variability of the expansion of credit, a larger variability of profits. Given the preference structure of the bank, all we need is to specify the feasibility locus of the bank, given the restrictive relationship between the level of excess reserves, the speed of adjustment and the probability of running out of excess reserves. The optimum, then, for the bank can be determined by the tangency of the utility function with the constraint of the bank represented by the feasibility locus. Let us start with the relationship between E(D) and R^* described by the above equation (10):

$$E(D) = D^{max} - kR^* \qquad (10)$$

From equation (20) we can obtain an expression for R^* in terms of the remaining parameters:

$$R^* = \sqrt{\frac{\sigma_u^2}{2Pg(2-g)}} \qquad (25)$$

which we can substitute into equation (10):

$$E(D) = D^{max} - \frac{k\sigma_u}{\sqrt{2Pg(2-g)}} \qquad (26)$$

In equation (13) we can rearrange terms[9] so that g is expressed as a function of σ_D :

$$g = \frac{2\,r^2\,\sigma_D^2}{r^2\,\sigma_D^2 + (1-r)^2\,\sigma_u^2} \qquad (27)$$

Substituting for g from (27) into equation (26), and after simplifying and rearranging terms,[10] we get a relationship which shows the maximum feasible E(D) corresponding to a given σ_D :

9, 10. The derivation of equations (27) and (28) has been omitted. They can be provided by this author upon request.

$$E(D) = D^{max} - \frac{k}{2\sqrt{2P}} \left[\frac{r}{1-r}\sigma_D + \frac{1-r}{r}\frac{\sigma_u^2}{\sigma_D} \right] \tag{28}$$

Equation (28) represents the **feasibility** locus or **the constraint** of the bank. Because we have assumed that σ_u^2 is constant, all of the variation in σ_D is due to changes in g. The feasibility locus is pictured in Figure 6.

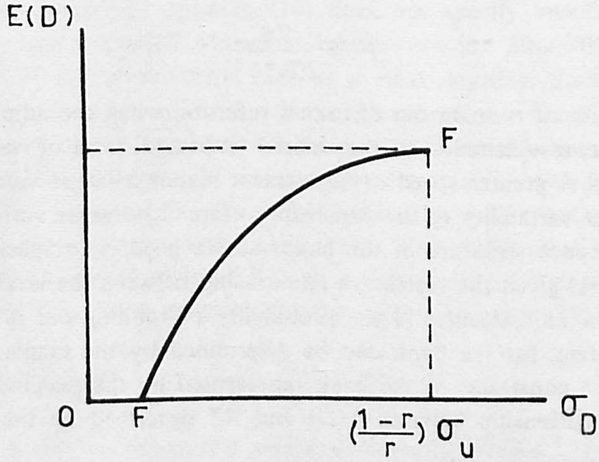

FIGURE 6.

Taking the first derivative of E(D) with respect to σ_D we have:

$$\frac{dE(D)}{d\sigma_D} = \frac{k}{2\sqrt{2P}} \left[\frac{1-r}{r}\frac{\sigma_u^2}{\sigma_D^2} - \frac{r}{1-r} \right] \tag{29}$$

It is clear that because σ_D is related to g only, and because g ranges between zero and one, the slope of the feasibility locus will range between infinity and zero. If $\sigma_D = 0$ (i.e. when g=0) the slope is infinite. The slope of the curve plotted in Figure 6 will be zero when the bracketed term in the right hand side of equation (29) is zero. i.e. when:

$$\sigma_D^2 = \frac{(1-r)^2}{r}\sigma_u^2 \tag{30}$$

This will happen only when g=1, as it can be seen from equation (13). Furthermore, if $\sigma_D = 0$, the value of E(D) given by equation (28) will be minus infinity. Therefore, the curve in Figure 6. crosses the σ_D - axis at a point where σ_D is positive.

Note that because both E(D) and its first derivative depend on the value of P, both the position and the slope of the feasibility locus are related to the probability P that can be maintained with a given rate of adjustment g_0. This is shown in Figure 7. From equation (28) and (29) it can be seen that in order to maintain a probability P_2, which is smaller than P_1, while keeping the rate of adjustment (and thus σ_D) the same, the level of the desired (expected) excess reserves has to be increased from R_1 to R_2 while the feasibility locus shifts downwards and the expected level of credit expansion (and hence of profits) decreases. To prevent such a decrease in profits the bank can maintain a level of excess reserves R_1 as previously and increase the speed of adjustment to one. Of course, this also increases the variability σ_D of credit expansion from σ_0 to $1-r/r \cdot \sigma_u$, and thus increases the variability of profits.

Similarly, a change in the required reserve ratio, whether legally set or determined by banking practices, will change both the position and the slope of the feasibility locus, while at the same time it will change the position and the slope of the relationship between g and σ_D.

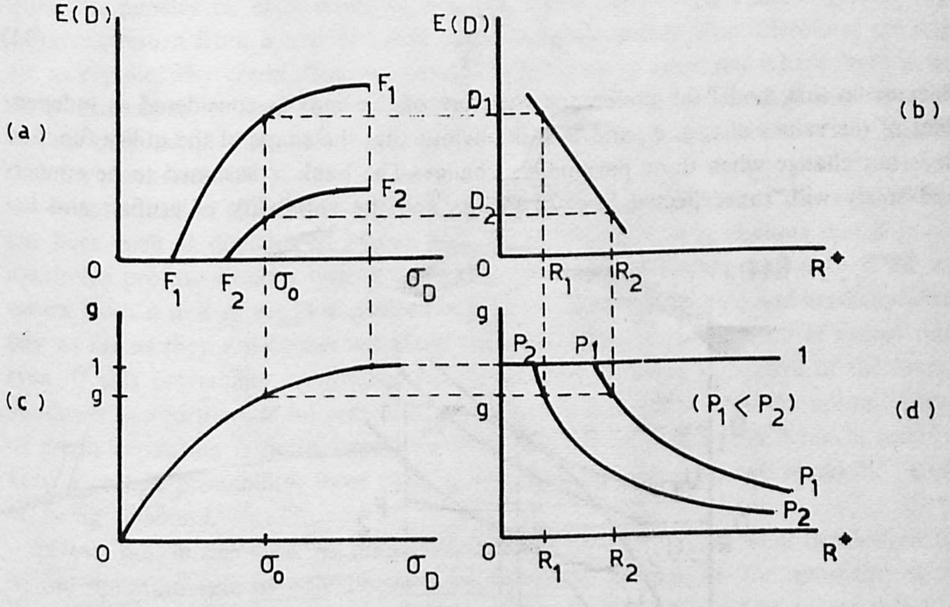

FIGURE 7.

11. See Orr and Mellon [4] p. 619, footnotes to their table 1.

VII. OPTIMUM VALUES OF EXCESS RESERVES, CREDIT EXPANSION AND RATE OF ADJUSTMENT.

To examine how the optimum values of the decision variables change when there is a change in one of the parameters, the preference structure of the bank has to be brought into the scene. That is, the utility function $U = U[E(D), \sigma_D]$ is to be superimposed on the surface of Figure 7(a).

This is done if Figure 8, which is the combination of Figures 5 and 7(a). Assume that the feasibility locus, which corresponds to the specific values of r, P and σ_u, is F_1F_1. The point of tangency of F_1F_1 with the highest possible indifference curve U_1 determines the optimum level of credit expansion D_1, and therefore, if we make use of Figure 7, it determines also the optimum level of excess reserves, $R^* = E(R)$. It also determines the desired variability of credit, and hence it determines, the optimum speed of adjustment g, from Figure 7(c), and the variability of excess reserves, σ_R, from equation (13). It should be noted that the variability of excess reserves, σ_R, is different from the variability of credit expansion for different values of the rate of adjustment, g, and for different values of the required reserve ratio r. The two variablities will be the same only in case where:

$$g = \frac{r}{1-r} \tag{31}$$

Because in this model the preference structure of the bank is considered as independent of the values of r, g, σ_u and P, it is obvious that the shape of the utility function does not change when these parameters change. The bank is assumed to be concerned solely with the expected level of profits and the variability of profits; and be-

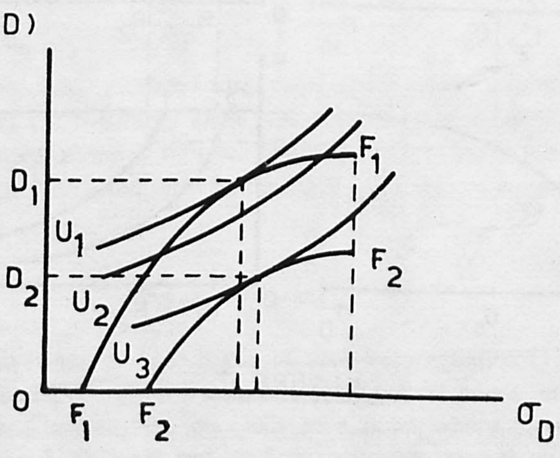

FIGURE 8.

cause we have assumed a positive net rate of return from every unit of credit expanded, this is the same like saying that the bank is concerned solely with the expected level and the variability of credit.

VIII. THE PROFIT MAXIMIZING BANK.

Les us now turn to the specific case where banks try to maximize profits without any concern as to the variability of profits. This means that the utility function will be solely dependent upon the expected level of credit, i.e.

$$U = U\,[E(D)] \tag{31}$$

We still retain the assumption that the net return from a unit of credit expanded is positive. This case is analogous to the assumption that underline Orr and Mellon's article, namely that banks maximize expected profits. Note, however, that with their model, the possibility that the net rate of return from credit expanded be positive leads to the "paradox" that the profit maximizing bank "should expand credit indefinitely(!)" Particularly, in their formulation the bank will seize expanding profits only when the penalty on each dollar of reserves which the bank is short is greater than the gross return from a unit of credit expanded. Orr and Mellon, therefore, are unable to explain why credit does not expand indefinitely in countries where there is not such restrictions as legal reserve requirements or penalties on reserve inadequacies. In fact they admit that their analysis "ignores such nonquantified factors as reputation for soundness".[12]

From equation (31) it can be seen that the resulting indifference curves are straight lines such as depicted in Figure 9(a). From Figure 9, it is obvious that if banks maximize profits, without regard as to the **variability** of profits, **and even if the net return from a unit of credit expanded is positive, they will not expand credit indefinitely** as far as they are concerned about the probability of running out of excess reserves. If this probability is watched by the market as being indicative of the bank's soudness – and there is no reason to assume that it is not – then the optimum level of credit expansion is finite, excess reserves will still be held by the bank in order to keep a certain probability, P, of running out of excess reserves, and avoid the "cost" of being unsound.

Note, that in this case, no matter what the value of P is, and what the desired R* is, the optimum rate of adjustment is always equal to one, i.e. for optimality there must always be an instantaneous adjustment to the desired levels of excess reserves, otherwise profits will not be maximized. This is an obvious deduction from the assumption of this model that the rate of adjustment is independent of the level of desired

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12. Orr and Mellon [5] p. 1121.

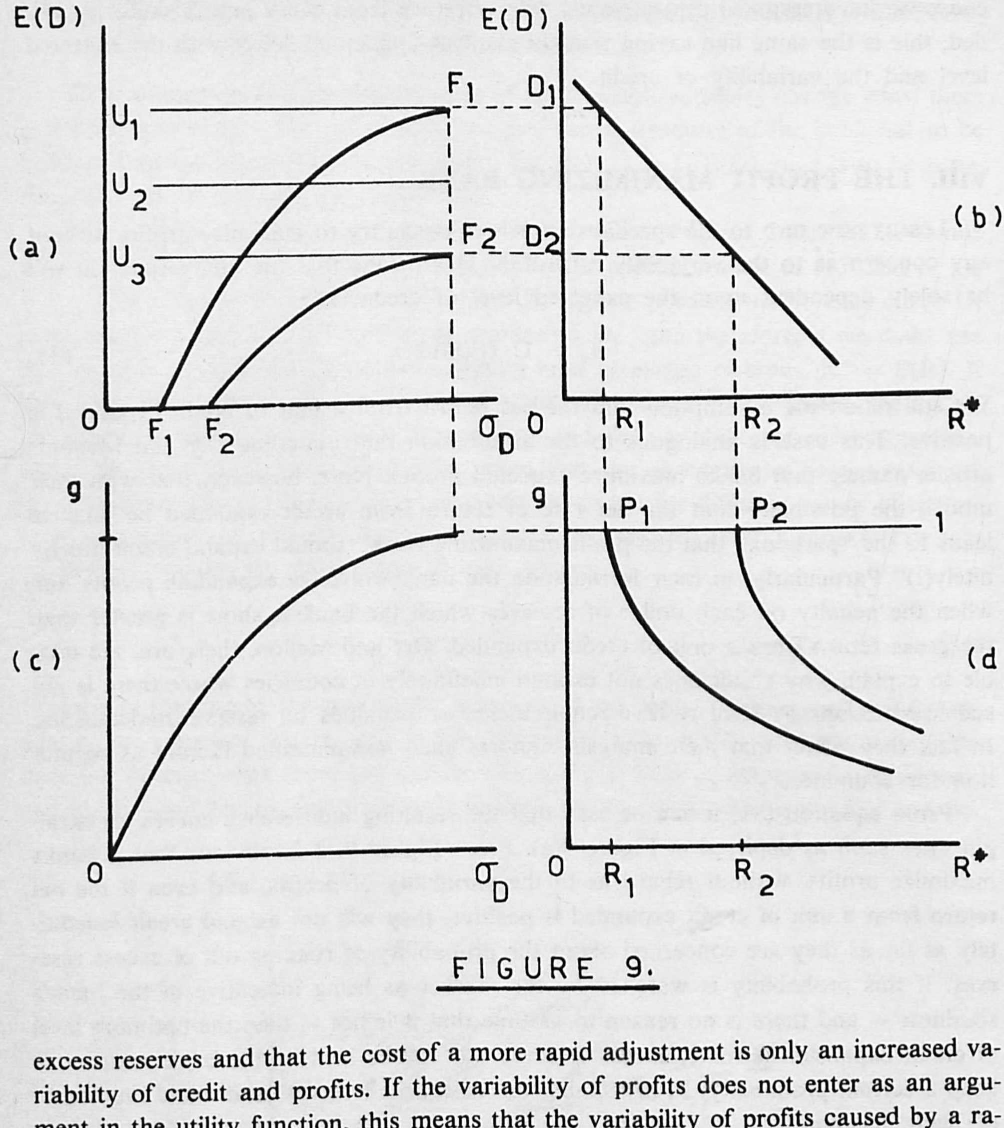

FIGURE 9.

excess reserves and that the cost of a more rapid adjustment is only an increased variability of credit and profits. If the variability of profits does not enter as an argument in the utility function, this means that the variability of profits caused by a rapid adjustment, although it reaches its maximum value, it does not concern the bank. The volume of optimum excess reserves depends solely on the degree of desired "soundness". It must also be noted, that although the bank is concerned about its soundness, the probability P does not need to appear as an argument in the utility function. If P is determined by market practices, i.e. by the public's confidence about the general economic situation of the country or the operations of the bank, this is a good reason for accepting that P is determined by an independent decision of the bank,or that it is exogenously determined and acts solely as a constraint to the decision on what is the optimum level of credit expansion, or of excess reserves, and the optimum rate of adjustment. Even if P does appear in the utility function, the general conclusions are not by any means invalidated. The indifference curves will be replaced by indifference surfaces and, if we still wish to maintain the simplicity of a two-dimensional diagram, both the feasibility locus and the indifference curves would shift, for different values of P.

IX. SMALL BANKS AND LARGE BANKS.

We turn now to the consistency of the above model with the "... observed bahavior of the larger ...banks in our system whose aim is to hold virtually no excess reserves".[13] There are two aspects in this case. First, it is conceivable that, ceteris paribus, the large bank is more concerned with the expected level of profits and less concerned with the variability of profits than the small bank. In terms of our model this means that the indifference curves representing the preferences of the large bank will be more horizontal than the indifference curves representing the preference of the small bank. In other words, given the values of all parameters, the marginal utility from the expected level of credit expansion is greater and the marginal disutility from the variability of credit is smaller for the large bank than for the small bank. Therefore, given the set of variables that determines the feasibility locus, the point of tangency with an indifference curve will be at a larger expected level of credit expansion in the case of a large bank than in the case of a small, as shown in Figure 10.

In this figure U_L, U_S represent the indifference curves respectively of the large and of the small banks. They are both tangent to the same feasibility locus, since we have assumed that all other parameters are the same. For the same probability P, therefore, and because of different preference structures, the ratio of desired excess reserves to the level of credit is larger for the smaller banks than for the larger banks; i.e. $R_S^* / D_S > R_L^* / D_L$

In other words, it should be expected that **the ratio of excess reserves to demand deposits is inversely related to the size of the bank.**

Furthermore, as it can be seen from Figure 10, because of different preference structures, and within the present assumptions, the optimum rate of adjustment for the large bank, g_L, is larger than the one corresponding to the small bank, g_S. Thus, in addition to a smaller excess reserve ratio one should expect **a faster rate of adjustment for large banks** to their desired (optimum) level of excess reserves, an implication which is also consistent with observed behavior.

It should be noted that in a profit maximizing framework of the type that Orr

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13. Miller [3] p. 1120.

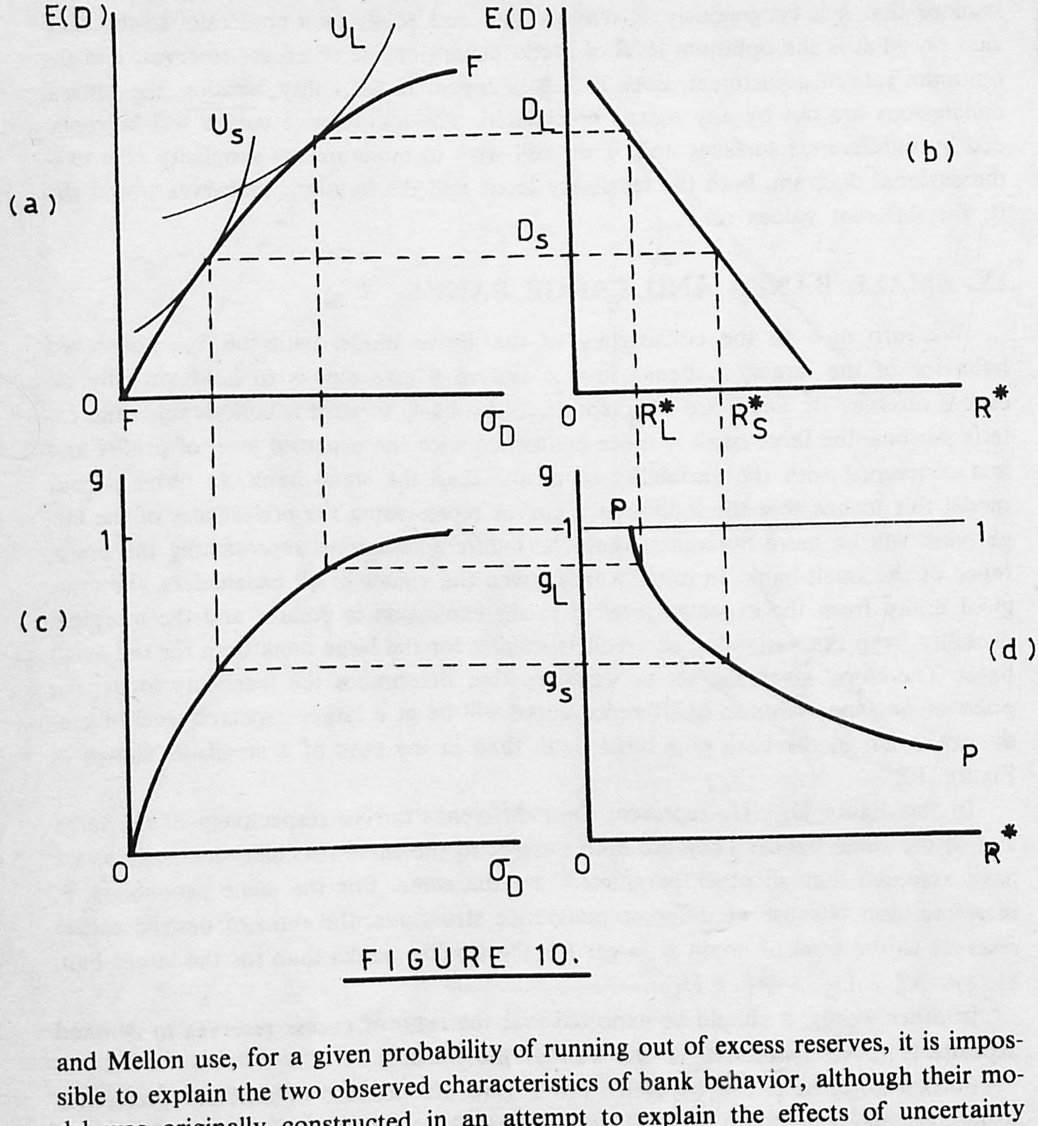

FIGURE 10.

and Mellon use, for a given probability of running out of excess reserves, it is impossible to explain the two observed characteristics of bank behavior, although their model was originally constructed in an attempt to explain the effects of uncertainty upon bank behavior.

Another possible explanation for the observed difference between large and small banks can be found in their degree of "soudness". A given probability P, that the market considers acceptable for a large bank, could be considered as very large for a small bank. In this case, one should expect that the small bank will usually try to maintain a smaller probability of running out of excess reserves, for fear of being suspected of unsound banking. In this case the feasibility locus will be lower than for the large bank and, even if we assume identical preference structures, the resulting optimum will entail a higher R^{*}/D ratio for the small bank, as it can be seen in Figure 7. The effect on the optimum rate in this case cannot be foressen a priori, but it depends on the marginal rate of substitution between E(D) and σ_D , as dictated by the utility function. However, if we consider the combined effect of different preference structures and different degrees of soundness, the implications of the present model are unambiguous.

X. CONCLUSION.

In this paper we have attempted to construct an appropriate framework within which bank behavior involving uncertainty should be analyzed. Particular attention has been focused on the probability of running out of excess reserves as a determinant of credit expansion. This probability **can** be controlled by the bank and can be maintained at acceptable levels by an appropriate combination of excess reserve levels and adjustments to the desired levels of excess reserves. The possibility of a slower or a faster adjustment process should by no means be ignored in the case of stochastic reserve losses and credit expansion, since it is related to the variability of credit expansion and to the variability of profits of the bank. It has been shown that in a decision involving uncertainty the rate of adjustment is an important behavioral factor, unless the bank is indifferent as to the variability of its profits.

It has also been shown that there is actually no justification in asserting that if the net return from credit expansion is positive the bank should expand credit indefinitely. Such an assertion, although criticized by reviewers of Orr and Mellon's article has been consequently ignored by the literature. There is no reason, however, for ignoring such nonquantified factors as reputation for soundness in a model explaining bank behavior, even if the bank is not at all concerned with the variability of its profits, as shown in paragraph VIII.

Furthermore, the model proposed above explains why the ratio of **total** reserves to total credit expansion is larger for small banks and smaller for large banks, while the rate of adjustment is faster for large banks and slower for small banks. Both explanations are consistent with observed bank behavior and have been unduly ignored by writers in the tradition of Orr and Mellon.

Although definite conslusions about the sensitivity of the decision variables to changes in the parameters cannot be reached unless the utility function in equation (15) is made more specific, it is possible to form some general conclusion in this respect by maximizing utility as given by (15) subject to the constraint given by (28). Since the maximizing procedure involves a system of equations of a degree higher than one in σ_D the particular solution is considered beyond the scope of this monograph.

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