

# INVESTMENT PLANNING MODELS FOR POWER GENERATING SYSTEMS

by J-EMMANUEL SAMOUILIDIS, National Technical University of Athens and KEPE

## I. INTRODUCTION

### 1.1. The Problem

The most important problem facing the electricity generating companies, known as rapid growth capital intensive corporations, is that of investment planning in generating equipment: ie that of determining the type and the size of the generating plants which will be constructed during the planning period so as to optimize the systems structure. The system is said to be of an optimal structure when it meets the demand in electrical energy under conditions of minimum cost.

The problem is difficult to solve mainly because of its two basic characteristics:

1. It is a problem of dynamical nature because it involves a set of interrelated decisions distributed in a time span of at least twenty years.

2. It is a problem of stochastic nature in which the level of uncertainty is very high, due to the relatively long life period of the installations and the rapid progress in the technology of power generation.

For the solution of the above problem there are two types of methods currently in use:

1. The marginal methods (méthodes marginales)
2. The global methods (méthodes globales).

Both methods have been mainly developed by Electricité de France, the French nationalized society for electricity.

### 1.2. Marginal Methods

According to this approach the decision to install a power station of given technical and economic characteristics is taken as an isolated decision by comparing it to a "reference plant", defined in terms of a standard equipment, eg, diesel burning thermoelectric equipment. The method attributes to each candidate plant a profitability coefficient which represents its economic value (1)<sup>1</sup>. The decision to construct the station is taken when the coefficient has an appropriate value.

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1. Figures in parentheses correspond to the references given at the end of the paper.

Though marginal methods can be used effectively for classifying individual projects within a given set, they do not guarantee the optimal structure of the production system. Their main disadvantages are:

- They are based on the assumption that the structure of the production system and therefore the utilization pattern of the plant under consideration are given.
- They require the choice of the reference plant to which each candidate project will be compared.

### **1.3. Global Methods (2)**

Through the global methods, based on the use of mathematical programming,<sup>2</sup> the problem of power plants selection is not solved separately for each individual project, but for the system as whole, so that the evolution of the structure of the system is determined.

A feasible investment program consists of a set of power plants which under a specified utilization pattern meets an electrical energy demand curve. Of all feasible programs the optimal program-eg. the least cost program - can be determined by the use of global methods. The program thus determined gives the evolution of the system, expressed in terms of the total power of each category of generating equipment (i.e. total installed power in hydroplants, total installed power in nuclear plants etc), which will be constructed during the planning horizon.

### **1.4. The Complementarity of the two Methods**

The marginal and global methods can be used as complementary methods for power plant selection. Assuming that the individual project, under consideration is part of the optimal structure defined by global methods, one determines the reference equipment as the one which can replace it in the optimal structure. Marginal methods are consequently used to find the profitability coefficient.

These two approaches, the marginal dealing with the marginal properties of individual projects, the global considering the production system as a whole, are used in Electricité de France, simultaneously, for production equipment planning (3), (4), (5).

### **1.5. The State of the Art**

The first attempt to use mathematical programming in the power plant selection problem was made in 1954 by Massé and Gibrat (2), who elaborated a simple linear programming model for the benefit of Electricité de France (E.D.F.). Further developments by members of the E.D.F. staff led to the elaboration of the "Three Plan Model" containing 224 constraints and 253 variables (6).

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2. In a *mathematical programming model* it is asked to find the values of a given set of variables which minimize or maximize an "Objective function" and satisfy a number of constraints expressed in the form of inequalities.

The French economists developed further a non-linear model, known as "Investment 85 Model" which included 46 constraints and 68 variables (5).

In Greece, the Operations Research Group of the Public Power Corporation elaborated, in 1968, a linear programming model the "LP 1968" containing 180 real variables and 220 constraints (7).<sup>3</sup>

The paragraph which follows gives a short description of the "LP 1968" model. In the final part of the paper a mixed integer model is proposed which when appropriately used offers a combination of the two approaches described above i.e. the global and marginal.

## 2. THE "LP 1968" MODEL

A short description of the linear programming model "LP 1968" is given below.

### 2.1. Statement of the Problem

It is required to determine the total installed capacity of all units to be introduced to the system, by category of equipment during a planning horizon extending from 1.1.73 up to 31.12.85, so that:

- the demand in electrical energy and power is met,
- the total cost of meeting this demand during the planning horizon is minimized.

### 2.2. The Planning Horizon

The planning horizon covers a period of 13 years starting on 1.1.1973 and ending on 31.12.1985. The reasons which led to the above time limits were the following:

- Decisions had already been taken on the plants to be constructed before 1973.<sup>4</sup>
- Year 1980 was considered, at that time, as a turning point in the evolution of nuclear technology: fast breeders were expected to attain technical maturity at that year.
- The period of seven years from 1973 to 1980 was unrealistically small for planning equipment of a lifespan of at least 30 years. It was thus decided to add a five year period, 1980-1985 which could constitute a type of "buffer" to the first seven-year period for which the model results could be considered valid.

The planning horizon was further divided in "uniform periods". Each period is characterized by an annual load duration curve which does not change during the period, and constitutes a point in the time axis in which the generating plants come

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3. *Linear Programming* (LP) is a special case of mathematical programming. In an LP model it is required to determine the values of a set of variables which minimize a cost function and satisfy a number of constraints, where both the cost function and the constraints are linear expressions of the variables. In the general case, the variables are continuous and may assume any positive value. In some models however the variables can only be discrete (e.g. 0, 1, 2); these models are named *integer models*. Models including both discrete and continuous variables are known as *mixed integer models*.

4. It is reminded that the model was elaborated in the first half of 1968.

into operation. In an ideal situation each uniform period should cover just one year. However it was decided for computational reasons to consider 3 three-year periods (1973-74-75, 1976-77-78, 1979-80-81) and 1 four-year period (1982-83-84-85).

### **2.3 The Decomposition of the Load Duration Curve**

It was assumed that the annual load duration curve can be represented by only six characteristic points. In other words, this meant that the year can be considered as consisting of six "seasons" during which the electrical load is constant. The load duration curve was thus approximated by a succession of six step functions.

"Seasons" were determined as following:

- Peak season
- Average load season
- Low load season

Distinct "seasons" were determined for the winter and the summer semesters.

### **2.4. Types of Technology**

The various techniques which can be used to produce electricity were grouped into three types of technology; different categories of equipment were distinguished in each type of technology as following:

a. Conventional thermal equipment. The following categories according to the type of fuel used were considered

- Lignite plants burning low cost coal (e.g. of the Piolemais type).
- Lignite plants burning high cost coal (e.g. of the Aliveri type).
- Thermoelectric plants burning diesel
- Gas turbine plants

b. Nuclear equipment, including the following three categories:

- Natural-uranium graphite system representing the industrially mature generation.
- Advanced air cool system representing the generation of reactors under development.
- Fast breeders which being still in the stage of research, were considered only for the 1980-85 period.

c. Hydroelectric plants, which were classified according to two criteria namely the mode of operation and the investment cost.

According to the first criterion:

- Peak load plants operating during peak hours.
- Average load plants operating during peak and average load hours.

According to the second criterion

- High capital cost plants
- Low capital cost plants

Through combination of the above classifications, four different categories of hydroplants were obtained.



## 2.5. The Planning Variables

The model includes two types of decision variables

- The strategic or investment variables
- The tactical or operational variables

The strategic or investment variables represent the total quantity of each category of equipment to be installed. To be more specific:

- For all thermoelectric equipment – conventional and nuclear – each strategic variable represents total capacity to be constructed in a uniform period.
- For all hydroelectric equipment each strategic variable represents total hydraulic energy to be utilized during a year of average hydraulicity.

The tactical or operational variables represent quantities of energy produced by each category of equipment. It is assumed by the model that equipment of the same category will have the same pattern of operation.

## 2.6. The Model Constraints

The model constraints expressed that:

- The sum of energies produced by all categories of new equipment and existing plants must be large enough to meet the demand level in each "season" during an average year.
- The sum of energies produced by all categories of new equipment and existing plants must be large enough to meet the demand level in each "season", during a year of high demand and low hydraulicity.
- For certain categories of equipment, as for lignite and hydro plants, the amount to be constructed is limited by the existing inventory of coal deposits and the availability of suitable sites respectively.
- For each category of thermoelectric equipment there exists a maximum utilization coefficient whose value is a function of programmed maintenance and the rate of equipment failures.

All constraints are expressed as linear functions of the model variables.

## 2.7. The Objective Function

The objective function expressed in mathematical form the selection criterion of the optimal program. It was assumed that the total cost of meeting the electricity demand during the 13-year planning horizon was the suitable criterion: the optimal program is the one that minimizes this cost. The cost of meeting the demand in electrical energy was analyzed in two component:

- the investment cost which was expressed as a linear function of the capacity for each category of equipment
- the operation cost which was expressed as a linear function of the energy produced by each category of equipment

All costs, incurring during the course of the planning horizon, were discounted to a present value.

## **2.8. Critique of the Model**

All models are approximations of reality. Their solution should therefore be treated as an approximation to the really optimal solution too. There are model opponents that go so far as to suggest that model solutions are of no practical value, their only contribution is, they argue, that model building helps in a precise and clear, understanding of what the problem is. Though it is believed that the solution to this model might be of some practical value, the model is not free of deficiencies.

The most essential of the model's weaknesses are as following:

- Each electricity generating unit has its particular technical and economic characteristics· this is especially true for hydroplants. The model eliminates these diversities assuming totally uniform plants grouped in various categories.
- Unit costs, both investment and operating, can be approximated by constants only for a unit of given capacity; for plants of different size they may vary significantly, and there is always an economy of scales factor. These considerations are not taken into account by the model, in which constant unit cost, independent of the plant capacity, are assumed.

The wish to eliminate the first of the above two deficiencies of the "LP 1968 Model" led to a Mixed Integer Model the "MI 1974 Model".

## **3. The Mixed Integer Model**

### **3.1. The Basic Concept**

The basic concept is very simple. A inventory of all individual projects that could possibly be constructed during the planning horizon is prepared. The list includes many more projects than will be finally installed; otherwise there would be no selection problem. Each project is an individual plant with its own particular economic and technical characteristics; there is no need to group projects in uniform categories. E.g. a thermoelectric plant, burning lignite in Ptolemais of the installed capacity of 400 MW investment cost of \$150 million and operating cost of \$1.3/MWh, may be one of the projects in the list. The energy to be produced by this plant is not a given characteristic; it is a decision tactical variable which will be specified by the model solution.

Each project is represented in the model by a strategic variable which in mathematical terms is a binary variable; this means that in the solution to be obtained it can assume either one of two values:

- either 0, in which case the corresponding plant should not be constructed.
- either 1, in which case the corresponding plant should be constructed.

An example of a mixed integer model "the MI 1974" is given in the Appendix to this paper for illustrative purposes.

### **3.2. The Main Features**

The general model structure is similar to the "LP 1968" model. Both models

have a flexible structure which can easily be adapted to the requirements of the specific problem.

The "MI 1974" model is designed rather as an illustrative example adopted to the Greek situation and not as a ready to be used model. Its planning horizon covers only one "uniform period" of, say, 3 years. The load duration curve is decomposed into three "seasons" the peak, average and low loads.

The types of technology in electricity generation are as in the "LP 1968 model", ie:

- Conventional thermoelectric equipment including three lignite plants, three diesel plants and one gas turbine.
  - Nuclear equipment including three plants
  - Hydroelectric equipment including three hydroplants
- Each of the fourteen plants is represented by the following variables:
- by a strategic variable which represents the decision to construct or not the plant; this variable is restricted to only two values 0 or 1<sup>5</sup>
  - by a set of three, in general, tactical variables representing the amount of energy to be produced by the plant in any of the three seasons.

The model constraints and the objective function have the same form in both the linear and mixed integer models.

### 3.3. The Two Models Combined

In the power plants investment planning procedure, the two models, given above, offer themselves for a useful combination.

Planning departments in electricity supply companies can usually identify individual plants, completely defined, which are candidates for construction in the near future; the decision to introduce these units to the system depends however on the evolution of the system structure in the long term, for which individual projects cannot be identified.

The planning model will therefore have two parts:

- One part which will include individual completely specified projects as decision variables. These projects are candidate for construction during the near future, say in the next 6 years. This part of the model will be a mixed integer model.
- A second part which will represent the evolution of the system after the first six-year-period. The plants to be constructed then will not be identified as individual projects, but they will be grouped in uniform categories. This part will have the form of a linear programming model-integer variables will of course still be present.

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5. This does not hold for the gas turbine station whose technical characteristics are not prespecified and the corresponding strategic variable represents the plant capacity.

## 4. CONCLUSIONS

4.1. Two approaches currently in use for investment planning in electricity generating are: the broadly used marginal and the less frequently used global approach. The former deals with the marginal properties of individual projects, while the latter considers the production system as a whole.

4.2. Efforts to design a production system of minimum cost, independently of how this cost is defined, must be based on simultaneous use of both these approaches.

4.3. This paper presents an optimization model, of the mixed-integer type, which can be used in order to combine the two approaches. The model includes individual projects, completely specified, on which a decision will be taken, to construct them or not; it also includes strategic variables representing the total quantity of each category of equipment. The solution to the model will thus indicate: a. the individual projects to be put in operation in the near future and b. the evolution of the optimal structure of the electricity production system in the long range.

4.4. The model is flexible and its form is independent of the optimization criterion. It lends itself to the use of any criterion which can be expressed as an explicit function of the plant capacity and the energy produced. Thus the discounted cost criterion, used in the illustrative example, can be replaced by a social cost criterion.

4.5. The model can be parametrized and a large number of parametric solutions can easily be obtained. The sensitivity of the plans to changing prices of primary energy sources and other factors can thus be thoroughly studied. This property renders the approach a particularly useful tool in periods as the present, in which both the investment selection criteria and the relative price structure of energy sources are in constant fluidity.

## REFERENCES

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THE MIXED INTEGER '74 MODEL

No	Left Hand Side	Sign	Right Hand S
i = 1	$\sum_{j=1}^3 Y(n, j, 1) + \sum_{j=1}^3 Y(\lambda, j, 1) + \sum_{j=1}^3 Y(f, j, 1) + \sum_{j=1}^4 Y(h, j, 1) + Y(t)$	$\geq$	W(1)
i = 2	$\sum_{j=1}^3 Y(n, j, 2) + \sum_{j=1}^3 Y(\lambda, j, 2) + \sum_{j=1}^3 Y(f, j, 2) + \sum_{j=1}^4 Y(h, j, 2)$	$\geq$	W(2)
i = 3	$\sum_{j=1}^3 Y(n, j, 3) + \sum_{j=1}^3 Y(\lambda, j, 3)$	$\geq$	W(3)
i = 4	$\sum_{j=1}^3 X(n, j) E(n, j, 1) + \sum_{j=1}^3 X(\lambda, j) E(\lambda, j, 1) + \sum_{j=1}^3 X(f, j) E(f, j, 1) + \sum_{j=1}^4 \bar{Y}(h, j, 1) + K(t, 1) X(t)$	$\geq$	$\bar{W}(1)$
i = 5	$\sum_{j=1}^3 X(n, j) E(n, j, 2) + \sum_{j=1}^3 X(\lambda, j) E(\lambda, j, 2) + \sum_{j=1}^3 X(f, j) E(f, j, 2) + \sum_{j=1}^4 \bar{Y}(h, j, 2) + K(t, 2) X(t)$	$\geq$	$\bar{W}(2)$
i=6, 7, 8	$X(n, 1) E(n, 1, s) - Y(n, 1, s)$	$\geq$	0, s=1, 2, 3
i=9, 10, 11	$X(n, 2) E(n, 2, s) - Y(n, 2, s)$	$\geq$	0, s=1, 2, 3
i=12, 13, 14	$X(n, 3) E(n, 3, s) - Y(n, 3, s)$	$\geq$	0, s=1, 2, 3
i=15, 16, 17	$X(\lambda, 1) E(\lambda, 1, s) - Y(\lambda, 1, s)$	$\geq$	0, s=1, 2, 3
i=18, 19, 20	$X(\lambda, 2) E(\lambda, 2, s) - Y(\lambda, 2, s)$	$\geq$	0, s=1, 2, 3
i=21, 22, 23	$X(\lambda, 3) E(\lambda, 3, s) - Y(\lambda, 3, s)$	$\geq$	0, s=1, 2, 3
i=24, 25	$X(f, 1) E(f, 1, s) - Y(f, 1, s)$	$\geq$	0, s=1, 2
i=26, 27	$X(f, 2) E(f, 2, s) - Y(f, 2, s)$	$\geq$	0, s=1, 2
i=28, 29	$X(f, 3) E(f, 3, s) - Y(f, 3, s)$	$\geq$	0, s=1, 2
i=30, 31	$X(h, j) E(h, j) - Y(h, j, 1) + Y(h, j, 2)$	$\geq$	0, j=1, 2
i=32, 33	$X(h, j) E(h, j) - Y(h, j, 1)$	$\geq$	0, j=3, 4
i=34	$K(t) X(t) - Y(t)$	$\geq$	0
i=35	$X(h, 1) \bar{E}(h, 1) - \bar{Y}(h, 1, 1) + \bar{Y}(h, 1, 2)$	$\geq$	0
i=36	$X(h, 2) \bar{E}(h, 2) - \bar{Y}(h, 2, 1) + \bar{Y}(h, 2, 2)$	$\geq$	0
i=37	$X(h, 3) \bar{E}(h, 3) - \bar{Y}(h, 3, 1) + \bar{Y}(h, 3, 2)$	$\geq$	0
i=38	$X(h, 4) \bar{E}(h, 4) - \bar{Y}(h, 4, 1) + \bar{Y}(h, 4, 2)$	$\geq$	0

# THE MIXED INTEGER '74 MODEL NOTATION

## 1. INDICES

- k: type of technology  
= n: nuclear,  $\lambda$ : lignite, f: diesel, h: hydro,  
t: gas turbine.  
j: individual project within one type of technology  
s: season, ie period of constant load  
= 1: peak, 2: average, 3: low

## 2. STRATEGIC VARIABLES

- Binary variables (0 or 1): X (k, j)  
Nuclear: X ( $\pi$ , 1), X ( $\pi$ , 2), X ( $\pi$ , 3)  
Lignite : X( $\lambda$ , 1), X ( $\lambda$ , 2), X ( $\lambda$ , 3)  
Diesel : X (f, 1), X (f, 2), X (f, 3)  
Hydro : X (h, 1), X (h, 2), X (h, 3), X (h, 4)  
Continuous variable: X(t): gas turbine

## 3. TACTICAL VARIABLES

- Energy produced: average year Y (k, j, s), Y(t)  
Energy produced: dry year, Y (k, j, s), Y (t)

## 4. PARAMETERS

- Electricity demand: average year, W(s)  
Electricity demand: dry year, W (s)  
Maximum energy: average year E (k, j, s)  
Maximum energy: dry year E (k, j, s)  
Utilization coefficient: K(t) only for gas turbines  
Annual capital cost:  $D_I$  (k, j)  $k \neq t$   
Unit Operating cost:  $D_E$  (k, j)  $k \neq t$   
Unit annual capital  
cost:  $d_j$  (t) for gas turbines  
Unit operating cost:  $d_e$  (t) for gas turbines

## 5. MODEL STATISTICS

- No of variables: 53 Integer variables; 13, Continous var.: 40  
No of constraints: 38