A DIAGRAMMATIC APPROACH TO THE FIRM'S DIVERSIFICATION PROBLEM

By

J. HASSID, Ph. D.

Institute of Economic and Industrial Research and the Athens Graduate School of Economics and Business Science

I. THE DECISION PROBLEM RESTATED

At the beginning of an article published some 20 years ago, M. R. Fisher pointed out that:

"... the contribution of existing theory nowhere appears more limited than in the explanation of the modus vivendi of the ubiguitious multi-product firm, the characteristic, not the atypical, unit of enterprise" [1, p. 293].

Several attempts have been made in recent years to discover the factors inducing firms to diversify. Very little effort has however been directed in formulating an analytical framework able of combining most, if not all, of the arguments suggested by various writers, in various places, at various points in time. Such an attempt is made here. In particular, starting from the same highly specific assumptions used by M. R. Fisher and by using methods very similar to his, we develop a simple model of diversification which possesses the above mentioned property.

Differentiation however between the new way of approaching the firm's diversification decision problem and that adopted by M. Fisher, begins in stating he problem itself. The two goods, X and Y, available as directions of production, are not anymore regarded simply as alternatives to which funds can be directed. On the contrary, from the firm's viewpoint—and that implies taking into consideration the views of all those involved in promoting its 'welfare' (production, finance and marketing staff), the two choices are regarded as fundamentally different.

In particular, while good X constitutes the area in which the firm has been a 'specialized' producer, i. e. the area with whose attributes, both the technological and the marketing ones, the firm is familiar, the production of good Y implies 'diversification' in the most general meaning of the term. Even if the opportunities for higher profits are better in the new area of activity there will, almost certainly, be some dispute whether the firm ought to diversify and if so as to the extent of diversification. Quite apart from the arguments associated with alleged advantages resulting from specialization and anticipated increases in the X market share, the conflict will also arise from the fact that while profit opportunities may seem to be better in the Y market they may at the same time be less certain since the problem recognizes that the variation of the expected price of Y may be greater than that of X. Clearly, whether the firm will diversify or not and also the decision as to the exact extent of diversification is a matter to be decided by relative bargaining power of all those within the firm aiming at profit maximization vis-a-vis those with objectives linked to how certain the profits to be gained are.

The procedure of the analysis is, broadly speaking, similar to that which the member of a firm's management, in charge of assessing the diversification proposal, is expected to follow in order to justify his final recommendations. The analysis, in spite of its abstract character, provides interesting insights to the diversification decision problem and to the implications of its solution on the extent to which a combination of objectives is to be met. The new formulation of the problem requires a two-stage solution.

In the first stage, the firm is assumed to reinvest the whole amount of funds, decided to be spent for expansion, in increasing production of X. As a result of this decision (specialized expansion), production of X increases by an amount determined by its production function, while at the same time, the same production function together with the predetermined expected price for X and the totally absorbed amount of funds, will determine the resulting increases in expected revenues and profits. Associated with these increases there will also be an increase in the variance of revenues and profits.

To determine the magnitude of all those increases we assume, after Fisher, the production function for X to be :

$$X = L_x^{\alpha}$$

with L_x indicating number of units of a single input required. The value of the exponent $(O < \alpha \gtreqless 1)$ determines whether the production of good X is subject to increasing, constant or decreasing returns to factor respectively. If w_x represents the unit price of L_x and F the totally absorbed amount of funds, the maximum feasible increase in X production will be:

$$X\max = \left(\frac{F}{w_x}\right)^{\alpha}$$

The magnitudes of the rest of the maximum feasible increases are easily derived as

Expected Revenues : $E(R)\max = \left(\frac{F}{w_x}\right)^{\alpha} . E(P_x)$

Expected Profits : $E(\Pi)\max = \left(\frac{F}{wx}\right)^{\alpha} . E(P_x) - F$

Variance of Revenues : $\sigma_R^2 \max = \left(\frac{F}{w_x}\right)^{2\alpha} . \sigma_x^2$

Variance of Profits[1] : $\sigma_{\Pi}^2 \max = \left(\frac{F}{w_x}\right)^{2\alpha} . \sigma_x^2$

Assuming that complete reinvestment is feasible and desirable, the second stage of the solution involves a gradual reduction in the amount of funds allocated in X-production and correspondingly, an increase in the funds diverted to the production of Y. During this gradual shifting of resources, we continuously observe the resulting changes in the expected value of profits and in their variance.

The mechanism by which reductions in the production of X result in increases in the production of Y is the following: By reducing X production by, say, one unit, there will be a release of funds equal to the number of input L_x units not required for the production of the last unit of X times their unit price w_x. This sum divided by the unit price w_y of input L_y, will determine the number of L_y units possible to be purchased. These, when put into production, will result in a positive change in the quantity of Y produced whose magnitude will be determined by the Y-production function.

Analytically, $\frac{dY}{dX}$, the change in Y resulting from a change in X, is determined by the relation:

$$\frac{dY}{dX} = \frac{dY}{dL_y} . \frac{dL_y}{dL_x} . \frac{dL_x}{dX}$$

---

1. Under the assumption of fixed total cost (F) it can easily be shown that $\sigma_{\pi}^2 \max = \sigma_R^2 \max$.

The third and the first term of the above relation are derived from the X and Y production functions respectively :

$$\frac{dL_x}{dX} = \frac{1}{\alpha} X^{\frac{1}{\alpha}} > 0$$

and, since the Y-production function is specified as :

$$Y = L_y^{\beta},$$

it also follows that :

$$\frac{dY}{dL_y} = \beta L_y^{\beta-1} > 0$$

The middle term is derived from our original assumption that F is totally absorbed for production purposes, i. e.

$$F = L_x w_x + L_y w_y$$

It follows, therefore, that :

$$\frac{dL_y}{dL_x} = \frac{w_x}{w_y} < 0$$

which, after transformation, becomes :

$$w_y dL_y = - w_x dL_x$$

This last relation indicates that if the entire F is to be spent, the amount released from the production of one good (X) must, at all times, be equal to the amount diverted to the production of the other good (Y).

The expected value of profits derived from the firm's total activity is determined by :

$$E(\Pi) = E(R_x) + E(R_y) - F = XE(P_x) + YE(P_y) - F$$

and therefore the change in $E(\Pi)$ resulting from a change in the production of X (accompanied by a change in the production of Y) will be :

$$\frac{dE(\Pi)}{dX} = E(P_x) + \frac{dY}{dX} E(P_y)$$

It follows that a change in the production of X will lead to higher, the same or lower $E(\Pi)$ depending on whether :

$$\frac{dY}{dX} \gtreqless - \frac{E(P_x)}{E(P_y)}$$

Since $\frac{dY}{dX}$ itself varies as we divert more and more funds from X to Y production, the changes in expected value of profits will not follow a monotonic path[1]. They may instead go initially through a negative (positive) stage, reach a turning point beyond which they start increasing (decreasing), reach again a zero value and continue increasing (decreasing) until the overall change in $E(\Pi)$ associated with complete diversification is achieved. This change is determined by comparing $E(\Pi)$ under specialization to that under complete diversification and it will be negative, zero or positive depending on whether :

$$\left\{\left(\frac{F}{w_x}\right)^{\alpha} E(P_x) - \left(\frac{F}{w_y}\right)^{\beta} E(P_y)\right\} \gtreqless 0$$

which could also be written as :

$$\frac{E(P_x)}{E(P_y)} \gtreqless \frac{\left(\frac{F}{w_y}\right)^{\beta}}{\left(\frac{F}{w_x}\right)^{\alpha}}$$

In general, one could say that the precise shape of the path that changes in $E(\Pi)$ will follow depends on the following four factors :

1. The ratio of expected prices for the two goods, expressed as : $\frac{E(P_x)}{E(P_y)}$

2. The nature of the 'production frontier' indicating the feasible combinations of X and Y. This frontier combines the properties of each of the production functions and it also reflects the absolute, as well as the relative, magnitude of the two production functions exponents α and β.

3. The unit prices of the two production inputs (w_x and w_y).

4. The total amount of founds available (F).

Figure 1 below provides a graphical illustration of the major points of the solution to the diversification decision problem discussed so far.

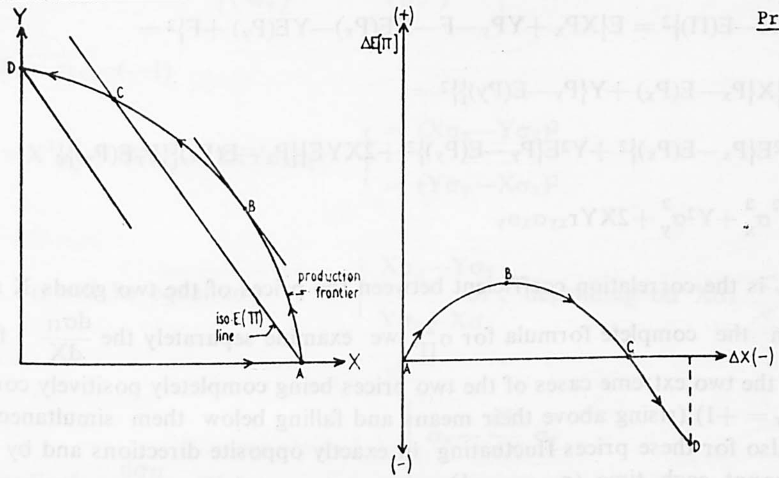
The other crucial variable whose changes we must observe is the variance of

---

1. $\frac{d^2Y}{dX^2} \neq 0$ and therefore $\frac{d^2E(\Pi)}{dX^2}$ will also be different from zero.

FIGURE 1

Derivation of the Diversifying Firm's Path of Changes in Expected Value of Profits



The 1st stage of the solution brings the firm to point A at which the expected value of profits has increased by : $\{E(R_x)\max - F\}$.

The second stage involves moving from A to point D, through B and C. Obviously, during the gradual diversion of funds from X to Y production, E(Π) increases for (x, y) combinations in the AB region, reach a maximum at point B and then starts decreasing for combinations in the BC region. At point C, expected value of profits is back to its original level (point A). For production combinations in the CD region, changes in E(Π) are negative and the minimum level is reached at point D, where :

$E(\Pi) = \{E(R_y)\max - F\}$. The overall change resulting from complete diversification is :

$\Delta E(\Pi) = \{E(R_y)\max - F - E(R_x)\max + F\} < 0$

profits (or its square root, defined as standard deviation). The complete formula for the variance of profits (σ^2_Π) is :

$$\sigma^2_\Pi = E\{\Pi - E(\Pi)\}^2 = E\{XP_x + YP_y - F - XE(P_x) - YE(P_y) + F\}^2 =$$

$$= E\{X\{P_x - E(P_x) + Y\{P_y - E(Py)\}\}^2 =$$

$$= X^2E\{P_x - E(P_x)\}^2 + Y^2E\{P_y - E(P_y)\}^2 + 2XYE\{\{P_x - E(P_x)\}\{P_y E(P_y)\}\}$$

$$= X^2\sigma^2_x + Y^2\sigma^2_y + 2XY r_{xy}\sigma_x\sigma_y$$

where r_{xy} is the correlation coefficient between the prices of the two goods X and Y.[1] Given the complete formula for σ^2_Π, we examine separately the $\frac{d\sigma_\Pi}{dX}$ function for the two extreme cases of the two prices being completely positively correlated ($r_{xy} = +1$) (rising above their means and falling below them simultaneously) and also for these prices fluctuating in exactly opposite directions and by the same amount each time ($r_{xy} = -1$).

1. For $r_{xy} = (+1)$

$$\sigma^2_\Pi = X^2\sigma^2_x + Y^2\sigma^2_y + 2XY\sigma_x\sigma_y = (X\sigma_x + Y\sigma_y)^2$$

$$\sigma_\Pi = \sqrt{\sigma^2_\Pi} = X\sigma_x + Y\sigma_y$$

It therefore follows that $\frac{d\sigma_\Pi}{dX} = \sigma_x + \frac{dY}{dX}\sigma_y$ and $\frac{d\sigma_\Pi}{dX} \gtreqless 0$ depending on $\frac{dY}{dX} \gtreqless -\frac{\sigma_x}{\sigma_y}$. Since $\frac{dY}{dX}$ is not constant, $\frac{d\sigma_\Pi}{dX}$ will follow a path which, depending on all the factors mentioned earlier (except the ratio of expected prices) and the relative magnitude of the standard deviations of the prices for the two goods, will not be a monotonic one. The overall change in the standard deviation of pro-

---

1. $r_{xy} = \frac{E\{\{P_x - E(P_x)\}\{P_y - E(P_y)\}\}}{\sqrt{E\{P_x - E(P_y)\}^2}\ \sqrt{E\{P_y - E(P_y)\}^2}}$. The value of the coefficient rxy, whuich could lie anywhere and —1, indicates the relation between fluctuations of the actual prices for the fwo goods, around their means.

fits resulting from complete diversion of funds depends again on :

$$\left\{\left(\frac{F}{w_y}\right)^{\beta} \sigma_y - \left(\frac{F}{w_x}\right)^{\alpha} \sigma_x\right\}$$

2. For $r_{xy} = (-1)$

$$\sigma_{\Pi}^2 = X^2\sigma_x^2 + Y^2\sigma_y^2 - 2XY\sigma_x\sigma_y \quad \begin{cases} = (X\sigma_x - Y\sigma_y)^2 \\ = (Y\sigma_y - X\sigma_x)^2 \end{cases}$$

and σ_Π will be equal to : $\begin{cases} X\sigma_x - Y\sigma_y \\ \text{or} \\ Y\sigma_y - X\sigma_x \end{cases}$, depending on $X\sigma_x \lessgtr Y\sigma_y$

Accordingly, $\dfrac{d\sigma_\Pi}{dx}$ will be equal to : $\left|\begin{array}{l} \sigma_x - \dfrac{dY}{dX}\sigma_y \\ \quad \text{or} \\ \dfrac{dY}{dX}\sigma_y - \sigma_x \end{array}\right.$

The iso-standard deviation lines in this case have a positive slope $\dfrac{\sigma_x}{\sigma_y}$, and therefore statements relating this slope to that of the production frontier $\left(\dfrac{dY}{dX}\right)$ (which is always negative) are irrelevant. With r_{xy} taking the extreme value of (-1), there will also be a $\sigma_\Pi = 0$ line (through the origin : $X = Y = 0$). The rest of the iso-σ_Π lines are divided into two groups. The first group, consisting of the lines described by the linear function :

$$\sigma_\Pi = X\sigma_x - Y\sigma_y,$$

will lie to the right of the $\sigma_\Pi = 0$ line, while the second group, including the lines described by :

$$\sigma_\Pi = Y\sigma_y - X\sigma_x,$$

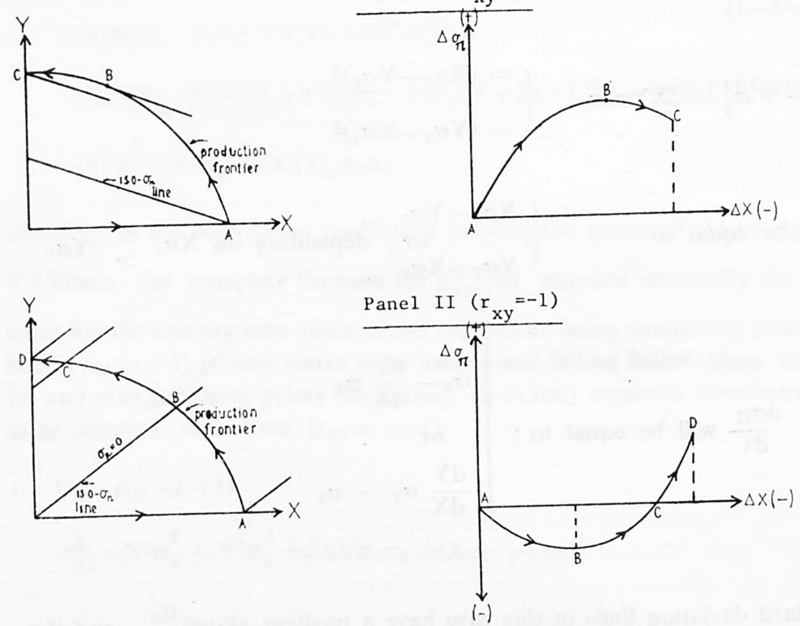
will lie to its left.

As we gradually divert funds from X-production the standard deviation of profits will decline until it reaches a value of zero, for the (X,Y) combination deter-

FIGURE 2

Derivation of the Diversifying Firm's Path of Changes in Standard Deviation of Profits

Panel I (r_{xy}=+1)



P a n e l I ($r_{xy} = +1$) : During the gradual diversion of funds from X to Y production standard deviation of profits increases for (x, y) combinations in the AB region. It reaches its maximum increase at point B and then it starts declining until the process of diversification is completed (point C). At the point the overall change in standard deviation is :

$$\Delta\sigma_\Pi = (Y\ max.\sigma_y - X\ max.\sigma_x) > 0$$

P a n e l II ($r_{xy} = -1$) : Changes in the standard deviation of profits are negative as production moves from A to point B. At B, where $\sigma_\Pi = 0$, the resultant change in standard deviation is exactly equal to :—(X max. σ_x). For (x, y) combinations in the BC region, standard deviation increases until it reaches its original level of (X max.σ_x), associated with point A. At that point $\Delta\sigma_\Pi = 0$. The upwards movement in standard deviation continues until complete diversification is achieved (point D) and the overall change effected becomes :

$$\Delta\sigma_\Pi = (Ymax.\sigma_x - Xmax.\sigma_x) > 0$$

mined by the intersection between the $\sigma_{\Pi} = 0$ line line and the production frontier. At this point the effected change in σ_{Π}, as a result of diversitication will be exactly equal to $-\left(\frac{F}{w_x}\right)^{\alpha} \sigma_x$. Beyond that point, standard deviation will start rising again, until the overall change, associated with complete diversification:

$$\Delta\sigma_{\Pi} = \left\{\left(\frac{F}{w_y}\right)^{\beta} \cdot \sigma_y - \left(\frac{F}{w_x}\right)^{\alpha} \cdot \sigma_x\right\}$$

is achieved.

Figure 2 below illustrates the relation between reductions in X-production and the associated changes in the standard deviation of profits for the two extreme values of the correlation coefficient (r_{xy}) between the prices of the two goods X and Y.

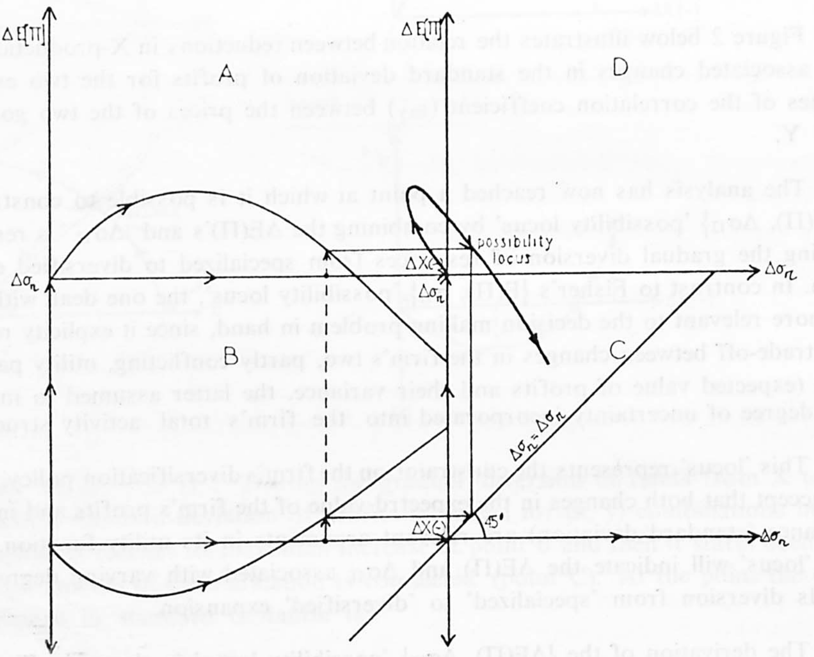
The analysis has now reached a point at which it is possible to construct a $\{\Delta E(\Pi), \Delta\sigma_{\Pi}\}$ 'possibility locus' by combining the $\Delta E(\Pi)$'s and $\Delta\sigma_{\Pi}$'s resulting during the gradual diversion of resources from specialized to diversified expansion. In contrast to Fisher's $\{E(\Pi), \sigma_{\Pi}\}$ 'possibility locus', the one dealt with here is more relevant to the decision making problem in hand, since it explicity reflects the trade-off between changes in the firm's two, partly conflicting, utility parameters (expected value of profits and their variance, the latter assumed to indicate the degree of uncertainty incorporated into the firm's total activity structure).

This 'locus' represents the constraint on the firm's diversification policy. Once we accept that both changes in the expectrd value of the firm's profits and in their variance (standard deviation) are relevant arguments in its utility function, then the 'locus' will indicate the $\Delta E(\Pi)$ and $\Delta\sigma_{\Pi}$ associated with varying degrees of funds diversion from 'specialized' to 'diversified' expansion.

The derivation of the $\{\Delta E(\Pi), \Delta\sigma_{\Pi}\}$ 'possibility locus' is shown in Figure 3. In parts A and B we reproduce two of the possible path shapes of $\Delta E(\Pi)$ and $\Delta\sigma_{\Pi}$, associated with gradual reductions in X production. The 45° line in part C allows us to transform $\Delta\sigma_{\Pi}$ from a vertically measured variable to an horizontally measured one and finally part D shows the derived 'possibility locus'. Its shape suggests that starting from a situation of completely specialized activity, diversion of resources results initially in rising expected value of profits and falling standard deviation. The increases in $E(\Pi)$ however reach a maximum and further increases in Y-production result in negative $\Delta E(\Pi)$, while at the same time standard deviation increases. The method by which the 'locus' has been derived implies that one can either decide on a particular level of resources diversion and then determine the $\Delta E(\Pi)$ and $\Delta\sigma_\Pi$ resulting from such a decision, or, alternatively, indicate any $\{\Delta E(\Pi), \Delta\sigma_\Pi\}$ combination and, by referring to the curves on the left side of the Figure, determine the associated reduction in X-production.

FIGURE 3

Derivation of the Diversifying Firm's $\{\Delta E(\Pi), \Delta\sigma_\Pi\}$ 'Possibility Locus'



The particular combination of $\Delta E(\Pi)$ and $\Delta\sigma_\Pi$ to be chosen, and therefore the optimal extent of diversification, depends on the firm's utility function, expressing the rate at which the firm is willing to trade changes in the expected value of profits against changes in their standard deviation. Positive changes in $E(\Pi)$ are conventionally assumed to imply increased utility. Similar changes however in the uncertainty accompanying changes in the firm's activity structure imply negative changes in utility. Furthermore it is assumed that for a given increase in $\Delta\sigma_\Pi$, the change (increase) in $\Delta E(\Pi)$ required for the utility level to remain unaffected, is greater the greater the $\Delta\sigma_{\Pi}$ associated with the firm's already effected diversification. These assumptions imply for the firm a type of attitude towards risk defined as 'risk-aversion'. A risk-averter's indifference curves, between $\Delta E(\Pi)$ and $\Delta\sigma_{\Pi}$, each of them indicating a series of alternative combinations for the two variables providing the same utility level, are of positive slope and convex from below. A set of such indifference curves is shown in Figure 4.

By superimposing the indifference curve map on the $\{\Delta E(\Pi), \Delta\sigma_{\Pi}\}$ two-dimensional space, in which the 'possibility locus' has already been constructed, we easily determine, following the conventional rule of tangency between the two curves, the optimum combination of the two variables and, by working clockwise, we eventually identify the exact extent of diversification required in order to achieve this optimum combination.

II. A MODEL FOR DIVERSIFICATION DECISION MAKING

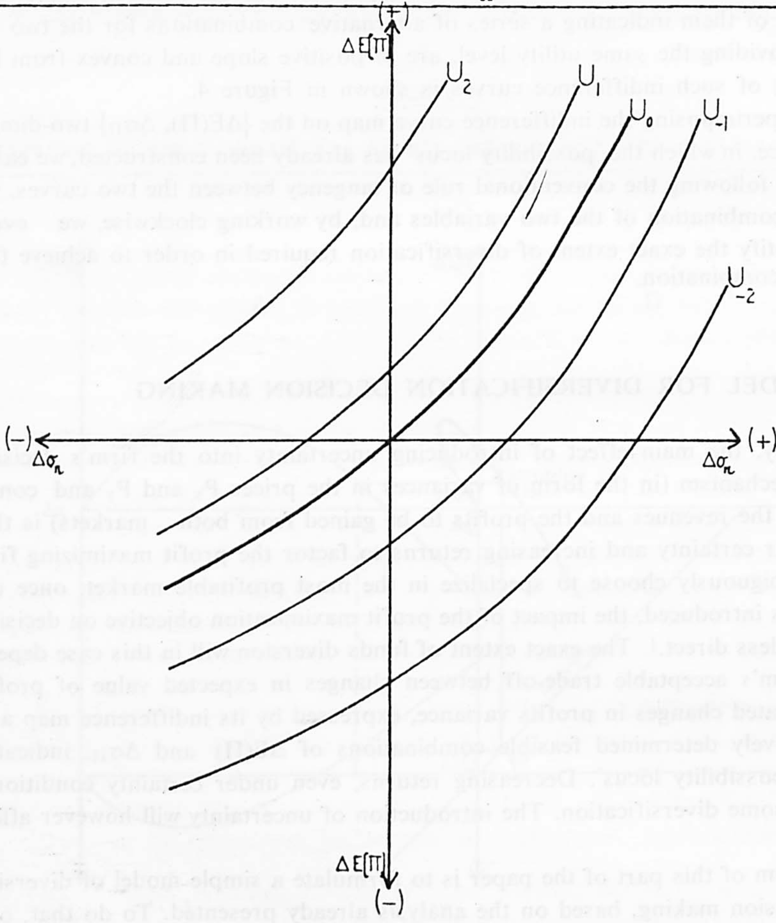
Clearly, the main effect of introducing uncertainty into the firm's decision making mechanism (in the form of variances in the prices P_x and P_y and consequently in the revenues and the profits to be gained from both markets) is that while under certainty and increasing returns to factor the profit maximizing firm will unambiguously choose to specialize in the most profitable market, once uncertainty is introduced, the impact of the profit maximization objective on decision making is less direct.[1] The exact extent of funds diversion will in this case depend on the firm's acceptable trade-off between changes in expected value of profits and associated changes in profits variance, expressed by its indifference map and the objectively determined feasible combinations of $\Delta E(\Pi)$ and $\Delta\sigma_{\Pi}$, indicated by their 'possibility locus'. Decreasing returns, even under certainty conditions, allow for some diversification. The introduction of uncertainty will however affect its extent.

The aim of this part of the paper is to formulate a simple model of diversification decision making, based on the analysis already presented. To do that, one must begin by isolating those factors to be regarded as the firm's focus of interest, in the sense that changes in their values will influence its diversification decisions. We have already identified the variables likely to affect the course of $\Delta E(\Pi)$ and

1. Assuming F to be fixed and totally absorbed for production purposes, maximization of expected value of profits becomes equivalent to maximization of expected value of total sales and therefore the analysis applies equally for both motivational assumptions.

FIGURE 4

The Diversifying Firm's $\{\Delta E(\Pi),\ \Delta\sigma_{\Pi}\}$ Indifference Curves Map



Indifference curves U_{-2}, U_{-1}, U_0, U_1, U_2 indicate successively higher levels of utility for the firm. Notice, however, that although the map covers the whole two-dimensional space (since $\Delta E(\Pi)$, $\Delta\sigma_{\Pi}<0$ are also relevant considerations), the frontier of the relevant section for the diversifying from is determined by the indifference curve U_0, associated with the level of utility obtained under complete specialization. All indifference curves above U_0 imply that diversification results in increased utility.

$\Delta\sigma_{\Pi}$ as we gradually divert funds from X to Y-production[1]. Changes in the values of some or all of those variables will affect the course of changes in expected profits and /or that of changes in profits variance and consequently the shape of the $\{\Delta E(\Pi), \Delta\sigma_{\Pi}\}$ 'possibility locus' will be altered. With an unchanged indifferenc-curves map, this will also affect the location of the optimum $[\Delta E(\Pi), \Delta\sigma_{\Pi}]$ combie nation which, in its turn, will determine a new optimum rate of funds diversion.

Assume that the nature of the production frontier is predetermined for the particular firm by technological conditions, and that the characteristics of the X-market (specialized activity area) are known to the firm from its own past experience[1]. An increase in the expected price $E(P_y)$ implies a decrease in the ratio $\frac{E(P_x)^*}{E(P_y)}$ and this will obviously result in a new set of flatter iso-$E(\Pi)$ lines which will cause an upwards rotation of the $\Delta E(\Pi)$ path and consequently a similar rotation of the $\{\Delta E(\Pi), \Delta\sigma_{\Pi}\}$ 'posibility locus'. This sequence of events translated in diversification theory terminology, suggests that whenever a profit-maximizing firm is considering two alternative diversification proposals, both promising a higher (but less certain) expected value of profits than its specialized activity and which, although identical with respect to the degree of uncertainty they differ in the level of $E(\Pi)$ to be gained, then the more profitable proposal will be the obvious choice.

Similarly, with given $E(P_y)$ a reduction in the uncertainty involved in the new activity area (σ_y), will result (under both extreme values for r_{xy}) in a new set of steeper iso-σ_{Π} lines, a downwards rotation of the $\Delta\sigma_{\Pi}$ path and, as a result, in a rotation of the 'possibility locus' which will lead to more diversification being undertaken. The inference from this is that whenever there is a choice between two diversification projects, equally promising in terms of $E(\Pi)$ but differing in the degree of certainty for the actual level of profits to be gained, the less uncertain one will always be preferred.

Furthermore, suppose that the firm is faced with a single diversification proposal. In analysing its decision making mechanism and especially in considering the value of σ_y^2 to be taken into account, we assumed that this was known to the firm together with the expected price of Y. Since however Y represents an activity area in which the particular firm is not presently active, we have to further assume that the value of σ_y^2 is taken as exogenously determined and in fact as equal to that which other firms, already producing Y, have experienced. We define this

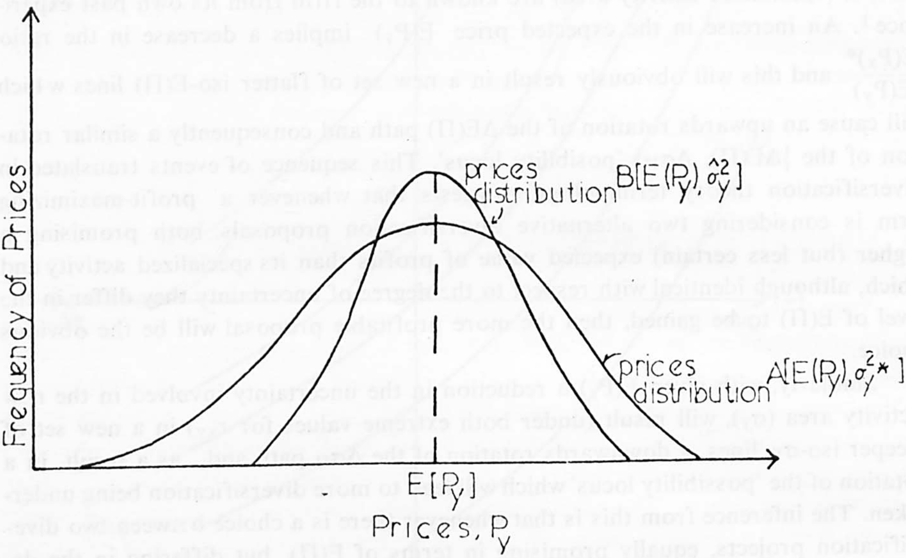
1. See pages 5 and 8.

2. To indicate the 'known' values for the two variables, we denote : $E(P_x) = E(P_x)^*$ and $\sigma_x^2 = \sigma_x^{2*}$.

FIGURE 5

The Diversifying Firm's 'Observed' and 'Estimated' Price Variance in its Secondary Activity Area

$(\hat{\sigma}_y^2 < \sigma_y^{2*})$



value of σ_y^2 as the 'observed' variance σ_y^{2*}. It is quite likely however, that the particular firm, in considering its own prospects in the new market, might have a different estimate for the same variable—a value defined as 'estimated' variance $\hat{\sigma}_y^2$. The firm may, for example, think that after entering the new Y market, in spite of being a new (and 'late') entrant, it will have various comparative advantages —in production, finance or marketing—which, although unsufficient to allow charging higher prices they will allow it to get, for its 'brand' of Y, prices which will lie closer to the established mean one, thus reducing the variance of its profits in the new market. On the other hand, these advantages may be more than offset by comparative disadvantages resulting from being a new entrant and therefore, almost by definition, not familiar with the Y market to the same extent as its already established rivals-to-be. The distinction between 'observed' and one of the possible values that 'estimated' variance may take it shown in Figure 5 abore. Both prices distribution A and B reflect the way in which actual prices for good Y are distributed around their mean. Distribution A, indicating the range of variation being experienced by established producers, is associated with the 'observed' value for the price variance (σ_y^{2*}) while distribution B, and its 'estimated' variance ($\hat{\sigma}_y^2$ in this case is taken as being lower than σ_y^{2*}) indicates the situation which the diversifying firm expects to face. It follows that, although a first approach to diversification project appraisal based on the variance of prices and profits experienced by other firms could legitimately be made, the crucial variance value is not necessarily this 'observed' value but instead a subjective estimate of it. The two values are not necessarily equal to each other. On the contrary 'estimated' variance could be greater or lower than the 'observed one.

In a mathematical form :

$$0 \leqq \hat{\sigma}_y^2 \gtreqless \sigma_y^{2*}$$

and $$\hat{\sigma}_y^2 = \sigma_y^{2*}\left(1 + \frac{\Delta_y}{\sigma_y^{2*}}\right) = \sigma_y^{2*}(1 + \delta_y)$$

where $\hat{\sigma}_y^2$ is the 'estimated' value for price variance in the Y market,

σ_y^{2*} is the 'observed' variance (equal to that experienced by already established firms in the Y market),

and Δ_y is the net change in σ_y^{2*} possible for the diversifying firm[1]. Finally, δ_y measures Δ_y as a proportion of σ_y^{2*}.

It, therefore, follows that :

$\hat{\sigma}_y^2 \gtreqless \sigma_y^{2*}$ according to whether $\delta_y \gtreqless 0$.

The introduction of this new element in the appraisal procedure requires some

---

1. $|-\Delta_y| \geqq \sigma_y^{2*}$. Since negative values for variances are impossible, net reductions of variance cannot exceed its 'observed' value.

qualification in the prediction made earlier that whenever two alternative diversification proposals equally promising in terms of expected profits but differing in the degree of inherent uncertainty are considered the more certain one (the one with lower 'observed' variance will be preferable. With 'estimated' variance substituted as the crucial variable, the firm chooses to enter the market for which its 'estimated' variance is lower and not necessarily the one which is regarded by other firms are less uncertain. Since the relation between the two values for the variance ultimately depends on the net change in the 'observed' value possible for a particular firm of given technological, financial, managerial and marketing characteristics, one has to specify separately the factors likely to result in negative change and those resulting in positive ones and then balance the two categories. Alternatively, if the distinction is retained, diversification will depend on two, instead of one, sets of explanatory factors.

There is still another point, related to the variance considerations, which has not been discussed yet. In the analysis of the $\Delta\sigma_\Pi$ path it was shown that this will be fundamentally different when r_{xy} (the coefficient of correlation between the prices of the two products) equals $(+1)$, from that with r_{xy} taken as equal to (-1). In particular, we showed that with $r_{xy} = (-1)$, that is with prices completely negatively correlated, the reductions in σ_Π resulting from early diversions of funds originally being allocated exclusively to X production, are likely to be greater than under $r_{xy} = (+1)$. Similarly, as funds diversion continues, $\Delta\sigma_\Pi$ increases by less than if the prices were completely possitively correlated. In a sense, choosing to diversify in a market whose price fluctuations can exactly offset the fluctuations in the primary activity area, will have the same effect on total profits stability as that of diversifying in a market with a lower price variance and therefore the analysis already presented still applies. The only modification is the additional prediction that risk-averting firms will prefer to combine activities whose price fluctuations are negatively, rather than positively correlated.

There are two final points to be made, both related to what has been described as 'entry barrier' or 'critical mass'. Both versions of the concept imply that certain markets into which entry is considered, may require the new entrant, if he is going to be at all viable, to achieve some minimum level of production. In terms of the present analysis this means that although $\Delta E(\Pi)$ and $\Delta\sigma_\Pi$ could be studied by gradually increasing investment in Y-production, increases in expected profits and profits standard deviation will not be realized unless the final decision on optimal diversification implies a level of Y production at least equal to the minimum required for 'safe' entry. While the analysis has so far treated a tangency between one of the firm's indifference curves and the $\{\Delta E(\Pi), \Delta\sigma_\Pi\}$ 'possibility locus' as a sufficient condition for optimum diversion of funds, the possible existence of 'entry barriers' qualifies it as a necessary but not sufficient one. This suggests that some firms having gone through the appraisal procedure described here and in spite of accepting in principle the virtues of the diversification in hand, they will not proceed to its implementation because the optimum Y-production suggested falls short of some critical value. They may instead start considering alternative directions for diversification or choose to remain specialized, although they recognize that reinvesting in their primary activity constitutes a suboptimal use of funds.

If $\bar{Y}$ represents the 'entry barrier' to be overcome, the necessary condition for the diversification decision to be implemented is:

$$\left(\frac{F_y}{w_y}\right)^{\beta} \geqq \bar{Y}$$

where $F_y = F - F_x$, is the absolute amount of funds diverted to Y-production. The share of the total amount of F to be allocated in each of the two types of activity depends, as we have seen, on various factors. If however all these factors remain constant, the absolute amount F_y will increase if F itself is increased. This implies that whether the analysis will suggest an optimum funds diversion providing for a level of Y production larger or equal to $\bar{Y}$, depends crucially on the total amount of funds available. This, in turn, suggests that certain large and financially powerful firms will be able to implement diversification decisions which other smaller firms will, of necessity, avoid.

Furthermore, the specification of the critical value of Y:

$$\left(\frac{F_y}{w_y}\right)^{\beta} \geqq \bar{Y}$$

brings within the boundaries of the present analysis an argument advanced in modern theories of the firm, refering to the types of inputs required for Y-production. It has been argued that from the whole spectrum of inputs necessary for the discovery of possible directions for diversification and thereafter for the investigation, appraisal, planning and successful implementation of diversification decisions, a substantial part, allegedly the most important one, consists of inputs generated within the firm. Special emphasis has been given to the generation and improvement of the services of the team of managers whose efficiency depends on the combination of their individual talents and on their association, as a group, with the company, as an institution. Furthermore, each of these managers, by being a member of a team rather than a talent in isolation, has for the firm a value much in excess of his market value. The complete Penrosian Theory of the Growth of the Firm and in particular the arguments related to the internal supply of managerial serlvices, has been formally incorporated into a diversification decision making mode by P. H. Rubin [2]. The same could easily be done in the present formulation by assuming that whatever can be regarded as a 'particular' factor ot production for a firm who possesses it, constitutes, at the same time, an input in very short supply for other firms who do not. In our terminology, if we substitute a vector ot inputs for the single inputs L_x and L_y in the X and Y production function respectively, we would be able to express $\bar{Y}$ in terms of the amount of funds diverted (F_y) and the prices of each of these inputs. Clearly, an input price w_y which is lower for a particular firm, could offset some financial weakness and still allow it to overcome the entry barrier without increasing its total expenditure. On the contrary, if a firm does not already possess such inputs, it would have to overcom the constraint imposed by their supply by sacrificing a large amount of funds in order to purchase the source of supply itself, which could mean acquiring the whole managerial team or even an entire company, thus 'buying its way' into the newemarket.

Constraints on implementation of diversification proposale approved in principle are not however imposed exclusively by requirements for minimum Y-production. The amount of funds F_y to be diverted does not only need to be sufficient to overcome 'entry barriesr' it also has to be a 'surplus' one, in the sense that it can only be diverted if such a decision does not jeopardize the firm's performance in its primary activity area. This implies that a minimum level of reinvestment may be required to secure a level of performance consistent with the firm's overall objectives. This minimum level of reinvestment, defined here as 'constraint to exit', will be equal to :

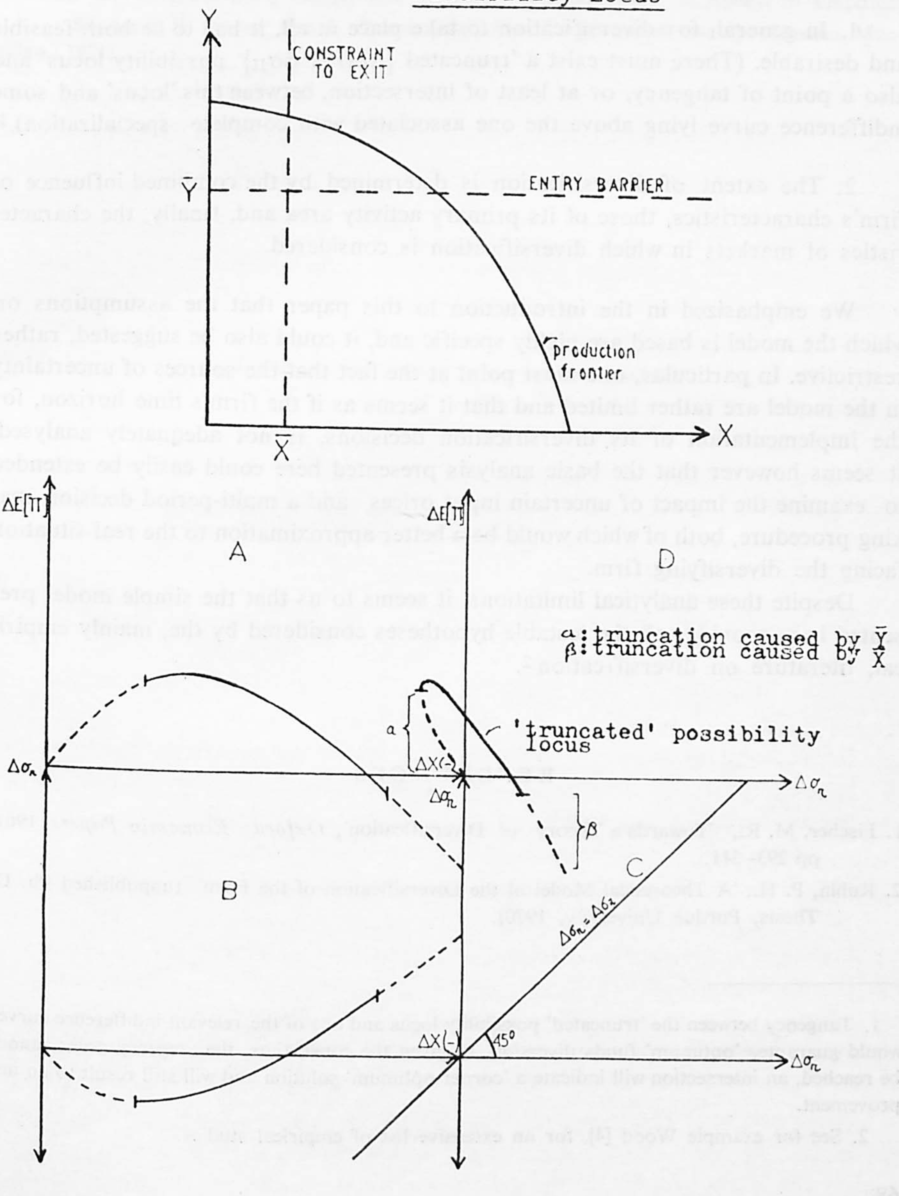
$$\left(\frac{F_x}{w_x} \right)^{\alpha} \geqq \bar{X}$$

The same arguments used earlier to indicate the factors enabling the firm to overcome 'entry barriers' (general financial power and the availability to the firm of 'particular' resources at relatively low prices) can be repeated in the present context.

The possible existence of both 'entry barriers' and 'constraints to exit' have a significant effect on the described $\{\Delta E(\Pi), \Delta\sigma\Pi\}$ 'possibility locus'. What the two concepts imply is that the firm may find early diversions of funds being irrelevant, since they may be insufficient to secure a safe entry into the new Y market, and at the same time avoid very high rates of diversion if, under the fixed F assumption, this implies insufficient funds left for the required minimum reinvestent in X production. The result of these considerations is that the original 'possibility locus' is truncated from both ends. Needless to say, it is possible that truncation is so drastic that there is no relevant 'locus' left, in which case diversification beco-

FIGURE 6

Derivation of the Diversifying Firm's {ΔE(Π), $\Delta\sigma_{\Pi}$} 'Truncated Possibility Locus'



mes impossible. Figure 6 abowe shows how the 'possibility locus', constructed in Figure 3, is transformed, as a result of 'entry barriers' and 'constraints to exit' existing for Y and X markets respectively.

There are two general predictions resulting from the analysis presented :

1. In general, for diversification to take place at all, it has to be both feasible and desirable. (There must exist a 'truncated $\{\Delta E(\Pi), \Delta\sigma_{\Pi}\}$ possibility locus' and also a point of tangency, or at least of intersection, between this 'locus' and some indifference curve lying above the one associated with complete specialization) [1].

2. The extent of diversification is determined by the combined influence of firm's characteristics, those of its primary activity area and, finally, the characteristics of markets in which diversification is considered.

We emphasized in the introduction to this paper that the assumptions on which the model is based are highly specific and, it could also be suggested, rather restrictive. In particular, one must point at the fact that the sources of uncertainty in the model are rather limited and that it seems as if the firm's time horizon, for the implementation of its diversification decisions, is not adequately analysed. It seems however that the basic analysis presented here could easily be extended to examine the impact of uncertain input prices and a multi-period decision making procedure, both of which would be a better approximation to the real situation facing the diversifying firm.

Despite these analytical limitations, it seems to us that the simple model presented here provides all the testable hypotheses considered by the, mainly empirical, literature on diversification [2].

REFERENCES

1. Fischer, M. R., 'Towards a Theory of Diversification', *Oxford Economic Papers*, 1961, pp 293- 311.
2. Rubin, P. H., 'A Theoretical Model of the Diversification of the Firm' (unpublished Ph. D. Thesis, Purdue University, 1970).

---

1. Tangency between the 'truncated' possibility locus and one of the relevant indifference curves would guarantee 'optimum' funds diversion. If, given the constraints, the tangency point cannot be reached, an intersection will indicate a 'corner-optimum' solution and will still result to an improvement.

2. See for example Wood [4], for an extensive list of empirical studies.

3. Hassid, J., 'Patterns and Trends of Diversification in the U. K. Manufacturing Industry' (unpu- blished Ph. D. Thesis, University of Manchester, 1976).

4. Wood, A., 'Diversification, Merger and Research Expenditures : A Review of Empirical Studies in R. Marris and A. Wood (eds), *The Corporate Economy* (London, Macmillan, 1971)