AN INVESTIGATION OF THE INTERRELATIONSHIPS EXISTING BETWEEN THE PLANT'S CAPACITY AND PRODUCTION, INVENTORY, WORK-FORCE LEVELS AND PROMOTIONAL EFFORTS

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1. INTRODUCTION

It is well known that the estimation of a linear inverse demand-price relationship is a relatively easy job provided that the necessary data are available. But how could we estimate the coefficients of the demand function when data is scanty?

In particular, if we were scheduling aggregate production in connection with a dynamic pricing policy, how could we estimate the values of the coefficient «a» which appears in the relationship (see equation (1) below) and the price elasticity indicators when dealing with seasonally demanded products?

There are at least three ways in theory of evaluating the coefficients of the inverse form of the demand curve

$$O_t = a - b_t p_t \quad (1)$$

where:

O_t = the forecasted order quantity for time t.

p_t = selling price per unit of production during period t.

a = a constant.

b_t = the measure of change in demand per unit change in price.

The three most commonly used methods involve:

(a) Time series analysis

(b) Simulation

(c) Quantification of subjective data.

(a) Evaluation by time series analysis

This is the wellknown classical method of evaluating the coefficients «a» and bt. The method is based on the assumption that historical data regarding the product will provide insight into customer and competitor actions in the future.

Other methods which are closely related to the above are the measuring of price elasticity of demand using correlation analysis, Dalrymple (3), Dixon (4), Kunrenther and Schrage (9); the error components model, the random parameter regression model and the variable parameter regression model, Johansson (8), Holdren (5), Cowen and Green (2).

(b) Evaluation by a simulation procedure

Prospective customer responses to alternative marketing strategies can be estimated by this method. The main purpose is to provide data for accurate estimates of the relationship existing between price and demand. These data may be provided by the use of data-generating methods.

(c) Evaluation by quantification of subjective data

In this method knowledgeable people are repeatedly asked questions about their subjective evaluation of the order of quantities that may be sold at each of a series of prices. The main purpose of questioning is to draw out the reasoning behind the respondents' replies, the factors they consider relevant to the problem, and their own estimate of these factors. Finally they are asked to try to specify the kind of data they feel would enable them to arrive at a better appraisal of these factors.

J. J. Finetry (6) presenting and discussing the above mentioned three metnods says:

«... Each of the foregoing methods for generating price/demand relationships has merits that can be exploited. In many cases the methods may be used in parallel, allowing each to corroborate the other. The most significant feature of each method is that it provides a systematic procedure whereby one is directed, step-by-step, to a repeatable conclusion. If the conclusion is invalid, it will be due to uncertainty about future influencing events use the information that was available to us ..

2. AN OPERATIONAL RESEARCH APPROACH

2.1. But how could we estimate the coefficients «a» and b_t if there were no data available? We may attempt to obtain reasonable estimation of the coefficient b_t by using the following indirect and somewhat intuitive approach and assuming for simplicity reasons that we are dealing with a monopoly. The coefficient «a» can be defined as a market and capacity constrained constant.

In referring to «a» as a market constrained constant we mean that «a» must not take a value greater than the maximum quantity that could be sold by reducing the selling price to a very low level in a certain market for a certain time period. But in referring to «a» as a production capacity constrained constant also, we mean that «a» could not take values greater than the maximum quantity that could be produced during the planning horizon, under the assumption that the level of investment remains constant. Thus the maximum capacity of the plant or the maximum market demand, whichever is the smaller, would set an upper bound to the possible values of «a».

But how can we estimate a lower bound for «a»? In our work on the Variable Price Model (VPM) for production, inventory and price control (see reference (10) we have found that as the value of «a» decreases so the product becomes more and more inelastic and therefore the difference between the maximum and minimum values of selling prices suggested by the aforementioned model, will increase. These variations could be presented in the following form :

$$\Delta P_t = \left| P_c - P_t \right| \qquad \begin{array}{l} 1.\ \text{when } p_c > p_t \text{ then } p_c - p_t \\ 2.\ \text{when } p_c = p_t \text{ then } 0 \\ 3.\ \text{when } p_c < p_t \text{ then } p_t - p_c \end{array}$$

where :

p_c is defined as the constant selling price which could have been obtained by the firm.

p_t is defined as the selling price suggested by the VPM for time t.

Since variations in prices of as much as up to 40 % could happen these days we feel that this maximum variation should not be more than 40 % of a constant selling price. Therefore the minimum value that «a» could take, is a relationship of Δp_t and the lower bound of «a» could be defined as follows :

«a» takes the minimum value when : $\Delta p_t = 0{,}4\ p_c$ (see graph. 2.1.).

Having defined the lower and upper bounds of «a» we can utilise sensitive analyses to observe the behaviour of the decision variables of the model as we vary «a» within those bounds. Having made out decision as far as «a» and p_c are concerned we can estimate the values of coeficient b_t by using equation (1) with p_c replacing p_t in the form :

$$b_t = \frac{a - O_t}{p_c} \tag{2}$$

We have carried out such sensitive analyses using the Variable Price Model (VPM) of Kioulafas (10), (11), (12), (13), with data presented by Holt et al. (7), Peterson (15), Taubert (16) and Leitch (14), as input.

The VPM extends the HMMS model by introducing dynamic pricing policy in conjunction with marketing policies. The model has been developed under the assumption that the firm we are dealing with is a monopolist. The model in its complete form has the following objective function.

$$C_T = \sum_{t=1}^{T} \{ (c_1 - c_6) w_t + c_2 (w_t - w_{t-1} - c_{11})^2 + c_3 (X_t - c_4 w_t)^2 + c_5 X_t + c_7 [(I_t - c_8 - c_9 (a - b_t p_t + Z_{1t} - Z_{2t})]^2 - p_t (a - b_t p_t + Z_{1t} - Z_{2t}) + d_t Z_{1t}^2 + e_t Z_{2t}^2 \} + QP_c \tag{3}$$

which has to be minimised subject to the constraint :

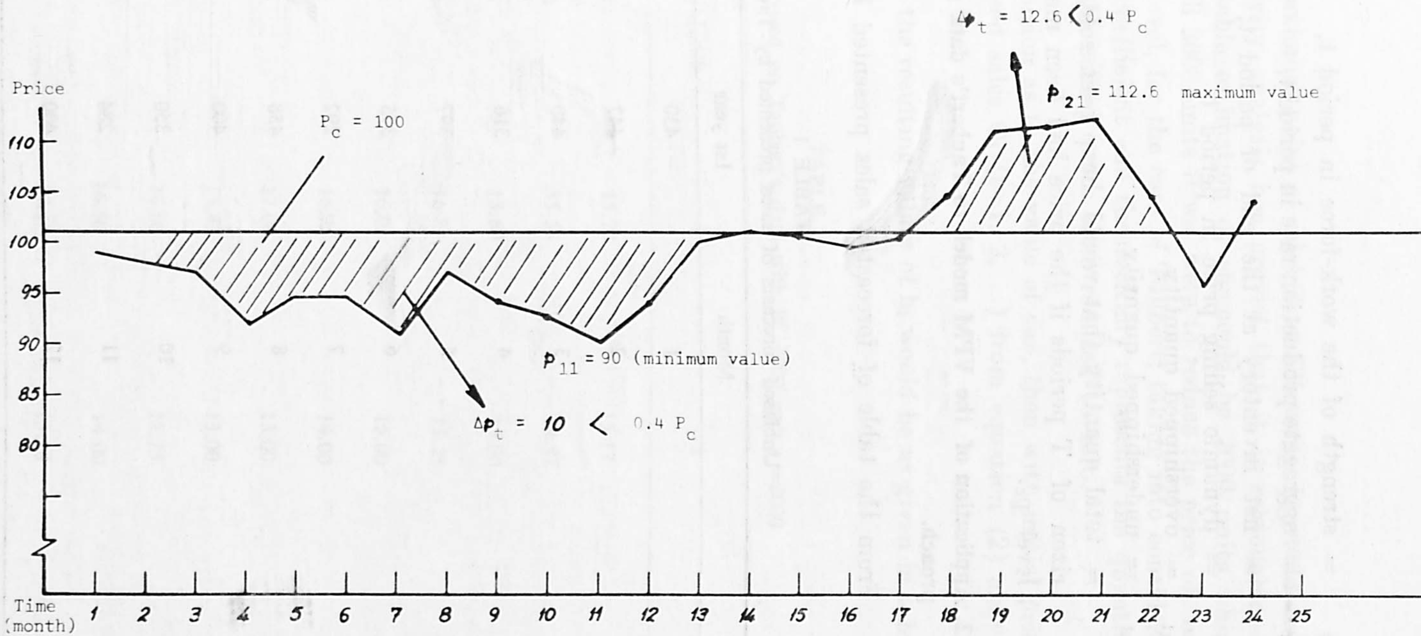
$$X_t + b_t p_t - Z_{1t} + Z_{2t} - I_t + I_{t-1} = a \tag{4}$$

where :

Regular payroll cost	$= c_1 w_t$
Hiring and lay-off costs	$= c_2 (w_t - w_{t-1} - c_{11})^2$
Overtime and idle time costs	$= c_3 (X_t - c_4 w_t)^2 + c_5 X_t$
Inventory related costs	$= c_7 [I_t - c_8 - c_9 (a - b_t p_t + Z_{1t} - Z_{2t}]^2$
Opportunity cost	$= Qp_c - p_t (a - b_t p_t + Z_{1t} - Z_{2t})$
Overshipment cost	$= d_t Z_{1t}^2$
Undershipment cost	$= e_t z_{2t}^2$

Rough sketch for Graph 2.1. (see folio 6)

Graph 2.1.
Selling prices obtained by using the VPM based on Taubert's data for a = 2000



w_t = strength of the work-force in period t,
X_t = aggregate production rate in period t,
I_t = net inventory at the end of period t,
p_t = dynamic selling price in period t,
Z_{1t} = overshipped quantity,
Z_{2t} = undershipped quantity,
Q = total quantity that would have been sold over the planning horizon of T periods if the price had been maintained at a constant level p_c.

2.2. Application of the VPM model to Taubert's data (16) using the above approach.

From the table of forecasted sales presented below (Table 1).

TABLE 1
Unbiased forecasts of sales presented by Taubert

Month	1st year	2nd year
1	430	483
2	447	509
3	440	500
4	316	475
5	397	500
6	375	600
7	292	700
8	458	700
9	400	725
10	350	600
11	284	432
12	400	615
	4,589	6,839

We see that the maximum point of sales is at 725 units. It would seem reasonable—and sensitivity analysis has proved that it is reasonable—to suggest that an acceptable estimation of «a» could be 2000 units, which means that we could sell 2000 units if we were to reduce the price of our product to a very low level, for the case of Taubert, taking into consideration the fact that the coefficient «a» theoretically represents the quantity that would be sold if the selling price were reduced to zero.

Having made a decision as to the value of «a», then with initial price level p_c and the forecased sales O_t ($t=1, 2, \dots$) from equation (2) above, if, for example, we use :

$p_c=100$, and «a»$=2000$, the resulting values of b_t would be as given in table 2 below.

TABLE 2

Coefficients bt based on Taubert's data with Pc = 100 and a = 2000

Year /Month	1	2
1	15.70	15.17
2	15.53	14.91
3	15.60	15.00
4	16.84	15.25
5	16.03	15.00
6	16.25	14.00
7	17.08	13.00
8	15.42	13.00
9	16.00	12.75
10	16.50	14.00
11	17.16	15.58
12	16.00	13.85

We have no estimate of the maximum capacity of the plant to which Taubert refers but it would be reasonable to suppcse that the maximum capacity of the firm could not be more than three times the maximum forecased level of production.

Taubert presents as maximum quantity produced 661 units per unit of time. Assuming that this quantity has been produced by the firm with the use of 80 % of normal capacity we estimate the maximum value of «a» as :

$$\frac{661 \times 3 \times 10}{8} = 2488 \text{ say } 2500$$

where 3 is the number of shifts.

By making sensitivity analysis we have found, using data presented by Taubert, that the VPM gives reasonable results for the following range of values «a» :

$$1500 \leqslant a \leqslant 2500$$

For values greater than 2500 the VPM gives generally better smoothed and more profitable results than those presented by the HMMS and Taubert and Peterson models, but a value of «a» greater than 2500 is an unrealistic value. For values less than 1500 the VPM gives better smoothed and more profitable results than those of Taubrert's model but it gives an unrealistic increase of selling price because 'of the inelastic situation. Therefore for values of «a» less than 1500 the values Δp_t takes values greater than 0.4 p_c.

2.3 Perfoming sensitivity analysis in terms of «a»

By increasing the value of «a» we assume an increase in both the firm's production capacity and the maximum demand of the market, but meanwhile we keep the initial value of work-force at 81 (value given by HMMS-Taubert).

This would imply a substantial amount of unused capacity within the factory and a large increase in demand would, of necessity, lead to a large increase in work-force. Hence production increases steadily with «a» as also does profit (see Table 3).

TABLE 3

Relationship existing between the numerical value of the parameter «a» and predicted average values for upper and lower extreme levels of the variables Production, Workforce, Sales, Selling Price, Investigation and overshipment, hy using the «VPM».

«a» / variables	1300 max	1300 min	1500 maw	1500 min	2000 max	2000 min	2500 max	2500 min
Production	505	425	515	440	570	452	680	500
Work-force	80	74	82	78	101	79	118	79
Sales	340	180	400	260	430	270	540	305
Selling Prices	145	100	132	98	122	96	115	95
Inventory	370	295	360	370	350	265	360	255
Oveěsöipment	382	225	393	240	400	255	385	285

Note : It has peen used data presented by the HMMS model and for $p_c = 100$

As «a» increases, variation in price becomes less, irrespective of price per unit (see Table 3). For lower values of «a», variation in price, when there are no other marketing efforts, is less than or approximately equal to that when there are other marketing efforts. Finally, inventory decreases with an increasing «a», and fluctuations in overshipments decrease.

So, by varying «a» within its maximum and minimum bounds we can carry out sensitivity analyses in order to study the interrelationships between the plant's maximum capacity and production, inventory, workforce, selling price levels and promotional efforts.

AMOTHER BASIS FOR DETERMINING THE PARAMETER «A» (1)

The productive capacity demands upon many factors such as available manpower, capital invested, the industrial efficiency of the firm, etc. None of the authors, HMMS (7), Peterson (15) or Taubert (16). have anything tangible to offer as to the possible order of magnitude of any of these factors, other than manpower, for the paint factory whose demand data they all use. Nor does Tuite(17) quantify such thing, again with the exception of manpower for the hypothetical firm. However, HMMS(7) use the following expression for the expected cost for overtime :

$$\text{Expected overtime cost} = c_3(X_t - c_4 w_t)^2 + c_5 X_t - c_6 w_t + c_{12} X_t w_t \qquad (5)$$

The quantity $c_4 w_t$ represents the regular-time normal production level expected from work-force of size w_t: and c_4 must therefore represent normal productivity per worker.

We would suggest that the value of «a» may be determined from a form such as

a= normal value of labour productivity x initial level of work-force x possible maximum shift ratio x V,

or

$$a = c_4 w_{10} \times N \times V \qquad (6)$$

where :

$$N = \frac{\text{number of shifts possible per day}}{\text{number of shifts worked per day}}$$

V= a factor to compensate for unknown components in the productive capacity and for any large forecasted demands in the interval t-1 to t-12.

w_{i0} = initial work-force in the ith planning horizon.

There are two facets of form (6) which may need further comments: (i) the factor V is arbitary and may need to be altered within the range $(1.1 < V < 1.35)$ in particular situations, but seems to work reasonably well in applications we have already attempted, and (ii) the use of the symbol w_{10} implies that «a» would remain constant during each planning horizon —taken consistently in this work to be of 12 months' duration—but may vary as between different horizons. So, in a 5-year run there could be five different values for «a».

Using formula (6) on Taubert's data for the paint factory and assuming that the factory, though capable of working a three-shift system is at present using single-shift working, we have :

$$a = c_4 x w_{i0} \times 3 \times V = 1791.1 \,(= 1800, \text{ say})$$
$$\text{where } c_4 = 5.67,\ w_{i0} = 81 \text{ and } V = 1.30.$$

Having determined a value for «a» we may evaluate b_t from the formula (2) which is derived directly from (1) above with p_c replacing p_t.

If in (2) $O_t > a$, i.e. the forecasted demand in month t is greater than the estimated value «a», b_t would assume a negative value. Should this occur the computer programmes have been designed to give an error signal and to stop running. However, with «a» estimated as in (6) the possibility of such an occurrence is extremelly remote.

4. SOME OBSERVATIONS ON THE INFLUENCE OF THE MAXIMUM PLANT CAPACITY IN THE WORK-FORCE, PRODUCTION RATES, SALES AND SELLING PRICE AS INDICATED BY THE MVPM.

The Modified Variable Price Model (MVPM) (1) is an extension of Dr. Kioulafas' (VPM) Model (10), (11) and differs from the VPM model in that the facility for undershipment is dropped and certain safeguards are written into the computer programmes which ensure reasonable changes in price levels and reasonable amounts of overshipment.

To obtain general knowledge of the influence of the parameter «a» on the decision variables we have used the 12 months' sales data presented by HMMS model have been used as input to the MVPM's programme together with the following values of the basic parameters and Initial Values also used by HMMS (see Table 4).

The programme was run with the parameter «a» being increased from 750 in steps of 250 to 2500 and the main results are reported below.

(i) Work-force size

The effect of the predictions of work - force as a result of changing the value of "a" is illustrated in Table 5 below, where the average upper and lower extreme values of work - force are plotted for various values of "a". We note that both the upper and lower values would seen to be levelling out at a = 2500. However, there is a definite minimum range of work - force in the range a = 1500 to a = 2000. A value of "a" somewhat less than 2000 would therefore be optimal if stability of work - force were to be a major consideration as, surely, it would generally have to be (see Table 5).

TABLE 4

Basic Parameters and Initial Values for the HMMS Paint Factory

$c_1 = 310.0$	$c_7 = 0.0825$
$c_2 = 64.3$	$c_8 = 320.0$
$c_3 = 0.2$	$c_9 = 0.0$
$c_4 = 5.67$	$I_0 = 263.0$
$c_5 = 51.2$	$W_0 = 81$ men
$c_6 = 281.0$	

TABLE 5

Relationship existing between the numerical value of the parameter «a» and the predicted values of upper and lower levels of the variables, Production, Work-force, Price and Sales by using the «MVPM».

«a» / variables	1300		1500		2000		2500	
	max	min	max	min	max	min	max	min
Production	440	360	460	410	490	470	560	500
Work-force	72	62	76	67	78	70	88	70
Selling Price	108	98	105	97	103	96	102	96
Sales	400	350	440	400	506	440	550	450

Note: It has peen used the data presented by HMMS and Pc = 100.

The general upward trend of work - force with increasing "a" is in accord with our definition of the parameter in terms of the company's producing capacity. Obviously, the greater the number of personnel employed, the greater the productive capacity. But, as previously mentioned, we must aim for stability in the work - force and we believe our formula for the parameter "a" as given in (1) above will achieve this stability.

Theoretically, one should be able to control variations in work - force by altering the HMMS parameter C_2, since the cost of hiring is given by :

$$\text{Hiring and firing costs} = c_2 \, (w_t - w_{t-1} - c_{11})^2 \qquad (7)$$

However, our sensitivity analyses have shown that the fluctuations in work - force are quite insensitive to changes in c_2, even the original value was (a) halved , and (b) doubled. Changes in parameter "a", however, were most effective in controlling work - force fluctuations.

(ii). Production levels

As was to be expected, production levels are affected by the magnitude of "a". In table 5 above we presented the average values for upper and lower extreme values of predicted production levels x_t, for the various values of "a" used. Again there is a general increase of both upper and lower values, and therefore with increasing "a" and, as was the case for work - force, levels, the minimum range occurs when «a» takes values in the 1500 to 2000 tange. Since the formula for x_t is a linear combination of w_{t-1}, w_t and w_{t+1} (see equation 7 above) we would expect the production levels to follow closely the pattern displayed by work - force levels. Stable production levels imply stable work-force fevels and with this stability would come a substandial reduction in costs to the firm and also good morale could be maintained in a work - force which was not constantly changing due to hirings and firings.

(iii) Sales Levels and Overshipment Quantities

In Table 5 above, we illustrate the behaviour of the average upper and lower levels of sales rate for the various values of "a" chosen. Again there is a general upward trend in both series as "a" increases with, again, the minimum range occuring between a = 1500 and a = 2000.

It would appear that , generally, different values of "a" would produce minimum fluctuations in work - force and production levels on the one hand and in levels of sales on the other. It is our considered opinion that priority should be given to obtaining stability in the two former variables rather than the latter, since the production and work - force smoothing would lead to considerable reduction in the firm's costs whilst sales smoothing would affect only inventory - related costs which, in any ca se. are reduced to quite small amounts in virtue of the use of the overshipment facility built into the model.

(iv) Price Levels

In table 5 above, we display the relationship between the parameter "a" and the average upper and lower predicted values of selling price. As "a" increases, both upper and lower levels of prices decrease and the range of the prices also decreases to reach a minimum at or about the value a = 2000. This is in economic accord with the relarionship between "a" and predicted production levels shown in the Table 5 above. We would expect a price decrease to coincide with an increase in producrtion to meet the anticipated increase in demand.

5. CONCLUDING REMARKS

The research reported here leads us ro the following conclusions :

(a) In cases where the data necessary for the application of econometric methods is not available to us we may use the indirect methods suggested above to obtain estimates of the coefficients in a firm's demand functions.

(b) Having used these indirect approaches to establish upper and lower bounds for the coefficient "a" we may then use the VPM or MVPM to study the effect of changes in "a" on the independent variables of these models.

(c) It would seem that the two models, VPM ans MVPM, yield similar results generally when using the same data as input.

(d) Taking the coefficient "a" as a function of the firm's productive capacity we may readilly obtain estimates of the price elasticity indicators provided that we have available a sufficient number of forecasted sales.

(e) Finally and most importantly, we note that the two models may be used in sensitivity analyses to study the influence of changes in a firm's productive capacity on such variables as work - force, inventory, levels of sales and selling price.

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