# FREE DEPRECIATION AND THE INTERNAL RATE OF RETURN (IRR)

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This note owes its existence to a very curious note published in the *Journal of Finance* by Hodges and brealey (H + B)[1]. H + B 'proved' that the after tax IRR on new projects accepted by British companies has been of the order of minus 0.48.

Although, for some inexplicable reason, the H + B article was not meant to be pedagogic it is questionable whether everybody was able to see the point they were making. The objective of this note is twofold : firstly to show that the H + B result was financially meaningles. and secondly to show that in a world as postulated by H + B corporate taxes do not affect the decision to invest. H + B arrived at the above conclusion by solving an after tax IRR equation, based on the following before tax cash flows :

$$-X_0, X_1, X_2 \ldots X_n$$

where $X_0$ is the capital outlay and $X_1 \ldots X_n$ represent net cash flows at times t = 1 to n. Assuming 100 % year a capital allowances (otherwise known as free depreciation) and a one year tax time lag, the after tax cash flows take the following form (Table 1) where T is the corporate tax rate.

Such a project has an internal rate of return equal to (T — 1). (See H + B op. cit.). Assuming T equals 0.52 the after tax internal rate of return will equal −0.48.

Of course H + B considered only the mathematically correct but finan-

TABLE I

| End of year | 0 | 1 | 2 | ... | n | n+1 |
|---|---|---|---|---|---|---|
| Cash flows before tax | $-X_0$ | $X_1$ | $X_2$ | ... | $X_n$ | |
| Tax effects | | $TX_0$ | $-TX_1$ | ... | $-TX_{n-1}$ | $-TX_n$ |
| Total | $-X_0$ | $X_1+TX_0$ | $X_2-TX_1$ | | $X_n-TX_{n-1}$ | $-TX_n$ |

cially incorrect negative value of the internal rate of return in preference to the mathematically and finncially correct positive value of the internal rate of return.

To begin with let us recall that an internal rate of return equation is a polynomial of degree N that has N roots. As is well known, for projects with conventional cash flows there will be N roots and hence N values for the internal rate of return. However, all but one of the values will be either negative or imaginary. Thus, if an equation solves for —0.50 and 0.10 the —0.50 is ignored on the grounds that it has to financial meaning. The analyst is then left with the 0.10 as the relevant discount rate with which to appraise the project.

For projects with nonconventional cash flows it is possible that there will be more than one positive root and thus there m a y be more than one positive solution for the internal rate of return itself. Descarte's rule of sign states that there may be as many positive roots for the internal rate of return as there are changes in the sign of the cash flows. Now, a one year tax time lag will convert a conventional before tax set of cash flows into a nonconventional set of cash flows. The analyst may therefore be wondering whether on an after tax basis the project will yield two positive solutions. Mathematically, the mere existence of a negative case flow at the end of the project's life does not necessarily mean two positive internal rates of return, as this also depends on the size and position of the negative cash flow in the equation.

In point of fact, as it will be shown immediately below, the tax system assumed by H + B cannot possibly have any material effect on post tax internal rates of return. This can be shown quite easily. Let us firstly specify the IRR equation of the pretax casn flows, i.e. :

$$X_0 = \frac{X_1}{(1+r)} + \frac{X_2}{(1+r)^2} + \ldots + \frac{X_n}{(1+r)^n} \quad (1)$$

Where r is the pretax internal rate of return. Our after tax IRR equation can be written as :

$$X_0 = \frac{X_1 + T_0 X}{(1+r_t)} = \frac{X_2 - TX_1}{(1+r)^2_t} + \ldots + \frac{X_n - TX_{n-1}}{(1+r_t)^n}$$

$$\frac{-Txn}{(1+r_t)^{n+t}} \quad (2)$$

Equation (2) can also be written as

$$X_0 \left[ 1 - \frac{T}{(1+r_t)} \right] = \left[ 1 - \frac{T}{1+r_t} \right] \left[ \frac{X_1}{1+r_t} + \frac{X_2}{(1+r_t)^2} + \ldots \frac{X_n}{(1+r_t)^n} \right] \quad (3)$$

By cancelling the terms $1 - T/(1/r_t)$, which denote each lagged tax effect, the cash flows of equation (1) are effectively each reduced proportionately, and equation (3) becomes equation (1) such that $r_t = r$; ie the after tax IRR is exactly the same as the before tax IRR. In fact the same holds under the Capital Asset Pricing Model where risk is explicity taken into account as well[2].

The above can also be seen with the aid of a simple numerical illustra-

TABLE II

| End of year | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Expenditure | —2486 | | | | |
| Pretax inflows | 1000 | 1000 | 1000 | | |
| Capital allowance benefit | | 1243 | | | |
| Tax on inflows | | | —500 | —500 | —500 |
| Total | —2486 | 2243 | 500 | 500 | —500 |

tion.[3] Assume before tax cash flows of: $X_0 = 2486$; $X_1 = 1000$; $X_2 = 1000$; $X_3 = 1000$ and $n=3$. The above project has one IRR of 10 %. The derivation of the after tax cash flows is shown in Table II assuming a 50 % tax rate.

There are two after--tax IRRs of 10 % and minus 50 %, but the negative value should of course be ignored.

Hence we deduce that the British Tax System as modelled by H+B does not adverseiy affect the decision to invest. If a project is acceptable on a before-tax basis it will also be acceptable on an after-tax basis. Effectively the government becomes a business partner sharing in the investment, by the fact that capital allowances reduce future tax payable, and in the future cash flows to the same extent, leaving the remaining proportion available for other investors. Hence the firm earns the same IRR as before tax but on a smaller investment base due to the depreciation tax shield. Equally the government earns the same IRR as tne firm on its share of the investment. Distortions may be caused by (a) allowances of less than 100 %, (b) different marginal tax rates in different time periods, (c) insufficient taxable profits against which to offset tax allowances and, (d) discrepancies between net operating cash flows and taxable profits, a result due to the periodic changes in accounts payable and receivabbe.

## REFERENCES


1. S. D. Hodges and R. A. Brealey, «The Rate of Return on New Investment in the UK», The Journal of Finance, Vol. XXXV, No 6, June 1980.
2. J. Pointon, «Investment and Risk: the Effect of Capital Allowances», Accounting and Business Research, Autumn 1980.
3. J. Pointon, «Towards a Corporate Cash Flow Tax System». Accountancy, July 1978.