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International Positive Production Externalities Under a Transfer Payment Scheme – the Case for Cooperation

By

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Abstract

In the present work we try to find out whether the existence of positive international externalities generates an incentive for cooperation between governments and if the adoption of a transfer payments scheme moderates that intensive. We adopt a simple economic model incorporating the international linkage of national economies. Utility proves always to be higher when countries cooperate than when they play Nash to each other. We then add a transfer payment scheme and prove it intensifies the intensive to cooperate, since a moral hazard problem arises on the top of the free riding problem.

JEL Classifications: H23, F35, F42.
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1. Introduction

The Chinese government announced on May 2004 it is planning to invest more than $ 5 billion in Brazilian ports and railways, during a visit of Brazilian President, Luiz Inacio da Silva, in Beijing[1]. By that time China was Brazil's fourth largest trading partner[2]. Investing in Brazil's infrastructure will secure the delivery of necessary raw materials from the Latin American country to the large Asian booming economy. This is a typical case where public spending in one country affects positively economic activity elsewhere in the world. One may identify several similar episodes of positive production spillovers between countries: building roads or ports in one country may help companies in other countries to export their products. Developing a modern telecommunication network may be of equal importance. Even education or promotion of liberal democratic regimes may be recognized as such cases of international public goods.

The existence of international externalities should generate an incentive for international cooperation between different governments: according to the literature, in the presence of positive spillovers, players' actions increase when switching from an uncoordinated to a coordinated equilibrium[3]. As long as governments stay on the rising part of the Laffer curve, such an increase implies a welfare improvement, providing governments with an incentive to cooperate. Philippopoulos and Economides (2003) develop an endogenous growth model with public goods, comparing optimal taxes in the Nash equilibrium and in the case of a cooperative solution. Epifani and Gancia (2008) recently studied the effect of trade openness on the size of governments. Hatfield (2006) shows that federalism leads to higher economic growth, in a framework of endogenous growth with government services.

A solution one might suggest is to adopt an international transfer payment scheme[4]. It's rather common for governments to adopt such schemes[5]. According to official data, the total budget for the Structural Funds and the Cohesion Funds during the period 2007-2013 will amount € 347 billion[6]. Transfer payments, however, may generate a moral hazard problem, where poorer economies prefer to lag behind and receive transfer payments rather than growing faster and have to donate transfer payments themselves. Recently, Economides, Kalyvitis and Philippopoulos (2008) presented a framework where transfers allow the financing of infrastructure, but they also induce rent-seeking competition by self-interested individuals. They find evidence that aid has a direct positive effect on growth, which is however significantly mitigated by the adverse indirect effects of associated rent-seeking activities.

In the present work, we establish a simple economic model incorporating the international linkage of national economies in order to check whether an incentive for cooperation in setting national policies exists. A technical approach within a pure neoclassical growth framework will be adopted. In particular we assume there is some public good, financed by tax revenues, affecting production positively both at home and abroad. We then add an international transfer payment scheme to check how incentives are affected.

First (section 2), we try to describe the model, calculate a Competitive Decentralized Equilibrium. In order to incorporate the international linkage of national economies, we adopt a form presented by Alesina and Wacziarg (1999)[7], considering it as more applicable to a standard Cobb – Douglas function. Alternatively one may use an additive function for the public good. This, however, results in symmetry to per capita income depending directly on the number of economies sharing the public good[8], while this is not the case when using the product of national tax revenues[9]. Obviously, by using logarithms one form may be transformed to the other. Alesina and Wacziarg (1999) let: $Y_j = AL_j^{1-\alpha} K_j^{\alpha} \prod_{i=1}^{n} \left[\left(\omega_i G_i \right)^{\frac{1-\alpha}{n}} \right]$. In the present work a more general form will be adopted, where weights ω_i are used as exponents – an assumption in compliance with the need of logarithmic transformation. We assume there is no tax evasion, i.e. the public good is financed by the 100% of the tax income, τ^*Y_j. Corruption, wrong tax incentives and other malfunctions reduce that percentage in most countries. We choose, however, to ignore this effect and focus on the effect the international public good has on cooperative incentives.

We then (section 3) solve governments' optimisation problem when they play Nash to each other and when they cooperate, adopting a common tax rate (tax harmonization). In both cases, we find the long run equilibrium values and study dynamics around equilibrium. Finally, we compare the results of both cases and identify the factors that determine the difference between Nash and cooperative policy instruments and the associated growth rates.

In section 4, an international transfer payments scheme is added to the above framework. Again we start with the Nash case and then study the case governments decide to cooperate. Finally (section 5), we summarize main results and try to derive some conclusions.

2. The environment

Consider a continuous time model, where a given finite number of countries, n, indexed by i=1,2,...,n, exist. Each country is populated by 'immortal' (i.e. with infinite planning horizon) identical private agents, who get utility by consuming a single good. For individual i it will be:

$$U_i=\ln(C_i) \tag{1}$$

Since only one good exists, there are no prices. All markets work perfectly competitive. Private agents in each economy possess amounts of capital, K, and labour, L, through which the single good is produced. Production also depends on a public production factor, G, that is non-rival and non-excludable for all n economies. The extent of the externality goes beyond the frontiers of each country, namely, there are cross-country externalities, so that each country benefits from public goods produced in the rest of the world. G may describe the general level of knowledge or technology known in the specific part of the world or common social values shared by all nations and helping production (such as democracy, personal rights, social state e.tc.) or simply common public infrastructures (transport networks, harbours, airports e.tc.). Production function in country j will, hence, be: $Y_j=A_jK_j^{\alpha}L_j^{1-\alpha}G_j^{1-\alpha}$ or

$$y_j=A_jk_j^{\alpha}G_j^{1-\alpha} \tag{2}$$

where $0<\alpha<1$, small letters denote per capita values, Y_j the only good produced and Aj the level of technology in country j[10]. There is no trade between countries and private agents cannot invest or work abroad. For simplicity labour force is normalized to unity at economy's level and assumed to be constant over time (i.e. each agent owns one unit of labour at any point of time)[11].

In each country there is also a benevolent government, that taxes domestic production to finance the provision of the public production factor. At any point in time, the level of the public production factor depends on public spending in all n economies:

$$G_j=\prod_{i=1}^{n}\left(g_i^{\omega_{i,j}}\right) = \prod_{i=1}^{n}\left(\tau_i y_i\right)^{\omega_{i,j}} \tag{3}$$

where $0\leq\tau_j\leq1$ is the tax rate imposed by government in economy j on domestic per capita income, $0<\omega_{i,j}<1\,\forall$ i, and $\omega_{i,j}\leq\omega_{j,j}\ \forall\ i\neq j$. The weight $\omega_{i,j}$ represents the extent to which public spending in country i affects the level of the public factor in country j. The sequence of moves is as follows: first national governments choose their tax policy; then private agents make their consumption – investment choices[12]. Policy responsibilities and the sequence of moves are taken as given.

A representative private agent, h, in country j wishes to maximize utility intertemporally, choosing a consumption level under the restriction of his income.

Private agent's maximization problem will, hence, be: $\max\limits_{c_{hj}}\left\{\int_0^{\infty}\left[\ln(c_{hj})e^{-\rho t}\right]dt\right\}$, s.t. $w_j+r_jb_{hj}=c_{hj}+\dot{b}_{hj}$

where the parameter $\varrho>0$ denotes the rate of time preference, b_{hj} agent's h assets, c_{hj} the chosen consumption level, w_j wages and r_j interest rates in economy j. Prices, public goods and tax policy are taken as given. At country level maximization yields to:

$$\frac{\dot{c}_j}{c_j}=r_j-\varrho \tag{4}$$

Given the absence of a bond market, capital market is clearing at any point in time: $b_j=k_j$; thus, country j's budget constraint asks for:

$$\dot{k}_j=w_j+r_jk_j-c_j \tag{5}$$

There is also a transversality condition, which states that the value of assets in terms of current utility should approach zero as time approaches infinity:

$$\lim_{t\to\infty}\left(\lambda_j b_j e^{-\rho t}\right)=0 \tag{6}$$

A representative firm h in country j tries to maximize profits choosing the ratio of capital to labour employed[13] and taking prices, public goods and tax policy as given. The maximization problem will, hence, be: $\max_{k_{hj}}$ $\{L_{hj}[(1-\tau_j)A_j k_{hj}^{\alpha} G_j^{1-\alpha}-w_j-(r_j+\delta)k_{hj}]\}$, where the parameter $\delta\geq 0$ is the rate of capital depreciation. At country level profit maximization yields to:

$$r_j=\alpha(1-\tau_j)A_j G_j^{1-\alpha} k_j^{\alpha-1}-\delta \tag{7}$$

Given constant returns to scale there will be no profits at any point of time:

$$w_j=(1-\alpha)(1-\tau_j)A_j k_j^{\alpha} G_j^{1-\alpha} \tag{8}$$

In each country, government should set the tax rate in order to finance the provision of public production factor. Given the absence of any bond market, governments' budget should be balanced at each point of time in all economies, i.e. at any point of time equation ③ should hold.

A World Competitive Decentralized Equilibrium[14] (WCDE) can now be characterized. This is for any feasible national fiscal policies as summarized by the national tax rates, $\tau_i\ \forall\ i=1, \ldots n$.

Definition 1

In the World Competitive Decentralized Equilibrium and for any feasible national tax rate: (i) all private agents maximize utility; (ii) all constraints are satisfied; (iii) all markets clear.

In country j there are five unknown variables (G_j, k_j, c_j, r_j and w_j), which will be determined by equations ③, ④, ⑤, ⑦ and ⑧. Equation ③, however, includes also k_i and G_i $\forall\ i\neq j$, which will be determined by the corresponding system of equations in country i. Thus, the problem entails 5*n equations (i.e. the above set of equations for all n countries) that will determine values for 5*n variables [i.e. G_j, k_j, c_j, r_j and w_j $\forall\ j\in(1, n)$]. In the symmetric case these equations give closed-form analytical solutions for equilibrium allocations as functions of national tax rates, τ_j.

3. Determination of national tax policies

In this section national policies will be indigenised. Initially, national tax rates, $\tau_j \ \forall \ j=1,...,n$, will be determined by a Nash game among benevolent national governments, which try to intertemporally maximize utility in their country. In choosing τ_j national government in country j takes into account the constraints of its economy in a WCDE (specified above). We, then, treat the case where governments cooperate to choose a common tax rate, τ, in order to intertemporally maximize global utility. Alternatively one may assume *ex-ante* symmetric economies and solve for the problem of a representative economy.

3.1 Non-cooperative (Nash) national policies

Government in country j chooses τ_j to maximize utility subject to its own constraints in a WCDE. In doing so, it takes into account the behavior of domestic agents, it takes as given τ_i, c_i, k_i (where $i \neq j$) and their shadow prices and plays Stackelberg vis-à-vis private agents. Government in country j tries to:

$$\max_{\tau_j} \int_0^\infty \left[\ln(c_j) e^{-\rho t} \right] dt$$, s.t. equations ② - ⑧ above hold.

Without loss of generality, the focus will be on Symmetric Nash Equilibria (SNE) in national policy rules[15]. Symmetry implies countries may differ *ex-ante*, but after determining policy strategies they become identical, i.e. they will be symmetric *ex-post*. Thus, in equilibrium, $\tau_j=\tau_i=\tau$, and hence $c_j=c_i=c$, $k_j=k_i=k$, e.tc., where $i \neq j$. The extent of the contribution of domestic infrastructure will be common for all countries, i.e. $\omega_{j,j}=\omega \ \forall \ j \in (1, ..., n)$. The extent of the external contribution of domestic infrastructure in foreign production functions will also be common across countries, i.e. $\omega_{i,j}=\omega_{f,j}=\frac{1-\omega}{n-1} \ \forall \ i, f \neq j$. An obvious realistic assumption would be that $\omega>\frac{1-\omega}{n-1} \Leftrightarrow \omega>\frac{1}{n}$, i.e. domestic infrastructure affects stronger the economy than foreign does. Invoking symmetry into the first order conditions resulting from government's maximization problem yields to result 1, below.

Result 1

A Symmetric Nash Equilibrium (SNE) in national tax policies and the associated World Competitive Decentralized Equilibrium is summarized by equations (9a) – (9f) below. Equation (9a) determines the Nash tax rate, $0<\tau^*<1$, which is unique.

Proof: See Appendix A

$$\tau^* = (1\text{-}\alpha)\frac{\alpha\omega(n\text{-}1)+(1\text{-}\alpha)(1\text{-}\omega)}{\alpha(n\text{-}1)+(1\text{-}\alpha)(1\text{-}\omega)} \quad (9a)$$

$$\frac{\dot{k}}{k} = (1\text{-}\tau)\frac{y}{k} - \delta - \frac{c}{k} \quad (9b)$$

$$\frac{\dot{c}}{c} = (1\text{-}\tau)\alpha\frac{y}{k} - \delta - \varrho \quad (9c)$$

$$\frac{\dot{v}}{v} = \varrho + \delta + \{\alpha(1\text{-}\tau)(1\text{-}\alpha)(1\text{-}\omega)\frac{y}{k}\frac{\mu}{v}\frac{c}{k} - (1\text{-}\tau)\frac{y}{k}[(1\text{-}\alpha)\frac{1-\omega}{n\text{-}1} + \alpha]\}[1\text{-}(1\text{-}\alpha)(\omega\text{-}\frac{1-\omega}{n\text{-}1})]^{-1} \quad (9d)$$

$$\frac{\dot{\mu}}{\mu} = \frac{v}{\mu} - \frac{1}{c\mu} - \alpha(1\text{-}\tau)\frac{y}{k} + \delta + 2\varrho \quad (9e)$$

$$\lim_{t\to\infty}\left(vke^{-\rho t}\right) = 0 \quad (9f)$$

Notice that the Nash tax rate in equation (9a) decreases with the number of countries, n and increases with the weight public spending at home has on the public factor, ω. Moreover, the Nash tax rate is non-state contingent, i.e. it doesn't change over time. This result is not universal, coming from the specific form of the production function (Cobb-Douglas) and economy's budget constraint (linear).

Definition 2

Define the long run equilibrium as a steady – state where both capital and consumption grow at the same constant positive rate.

Let denote the steady state values in the Symmetric Long-run Nash Equilibrium (SLNE) of (τ, k, c) by (τ*, k*, c*). In symmetry it will be: $\frac{y}{k} = (\tau^{1-\alpha}A)^{1/\alpha}$.

Let $\varphi \equiv (1\text{-}\tau)\frac{y}{k}$. For τ=τ*, $\varphi^* = (1\text{-}\tau^*)\left(\frac{y}{k}\right)^*$ will be a constant. Equations (9c)

& (9d) can now be written as: $c=c_0 e^{(\alpha\varphi^*-\delta-\varrho)t}$ and $k=k_0 e^{(\varphi^*-\delta)t} - \frac{c_0}{(1-\alpha)\varphi^*+\rho}$ $[e^{(\varphi^*-\delta)t} - e^{(\alpha\varphi^*-\delta-\varrho)t}]$. According to the definition of the long-run equilibrium, at the steady state consumption, c, and capital, k, grow at the same constant positive rate, i.e. $\frac{\dot{k}}{k} = \frac{\dot{c}}{c}$. In order to reach the steady state, for t=0 it should be: $c_0=[(1-\alpha)\varphi^*+\varrho]k_0$. One ends up with:

$$k=k_0 e^{(\alpha\varphi^*-\delta-\varrho)t} \tag{10a}$$

$$c=[(1-\alpha)\varphi^*+\varrho]k_0 e^{(\alpha\varphi^*-\delta-\varrho)t} \tag{10b}$$

Now let's check whether the resulting steady state solution is well defined. A well defined steady state requires: i) c* and k* to be positive and grow at the same rate, ii) the economy to grow, iii) $0<\tau^*<1$ and iv) transversality condition to hold.

Result 2

Condition $\alpha^2 \frac{(n-1)+(1-\alpha)(1-\omega n)}{\alpha\omega(n-1)+(1-\alpha)(1-\omega)} \left[A(1-\alpha)^{1-\alpha} \frac{\alpha\omega(n-1)+(1-\alpha)(1-\omega)}{\alpha(n-1)+(1-\alpha)(1-\omega)} \right]^{1/\alpha}$ $>\varrho+\delta$ *is necessary and sufficient for the economies to grow at a Symmetric Long-run Nash Equilibrium (SLNE) in national policies. This will be summarized by equations (10a) & (10b) and the tax rate that solves equation (9a). This tax rate supports a unique, well-defined steady state in which capital and consumption grow at the same constant positive rate, described by equation (11a) below.*

Proof: See Appendix B

At the long run equilibrium both consumption and capital grow at the same rate:

$$g^*=\alpha(1-\tau^*)[A(\tau^*)^{1-\alpha}]^{1/\alpha}-\varrho-\delta \tag{11a}$$

Intertemporal utility for any agent will be described by:

$$U^*=\frac{1}{\rho^2}[\varrho\ln(c_0)+g^*]+U_0 \tag{11b}$$

In symmetry the production function becomes a typical version of the 'AK family'. As in all AK models there are no transitional dynamics for 'actual' variables (i.e. for c, k): economies should either start at the steady state [i.e. $\frac{c_0}{k_0}$ $= (1\text{-}\alpha)\varphi^* + \varrho$] or 'jump' to it. Changes in the underlying parameters can affect levels and growth rates of 'real' variables[16].

Result 3

At the Symmetric Long-run Nash Equilibrium (SLNE) the optimal tax rate depends negatively on the external contribution of foreign country infrastructure to the domestic economy, 1-ω, and negatively on the number of economies, n, sharing the externality. The associated long-run growth rate depends negatively on the external contribution of foreign country infrastructure, 1-ω, and on the number of economies, n, sharing the externality.

Proof: One easily proves that: $\frac{\partial\tau^*}{\partial n} = -\alpha\left[\frac{(1\text{-}\alpha)(1\text{-}\omega)}{\alpha(n\text{-}1)+(1\text{-}\alpha)(1\text{-}\omega)}\right]^2 < 0$ and $\frac{\partial\tau^*}{\partial(1-\omega)}$ $= -(1\text{-}\alpha)\left[\frac{\alpha(n\text{-}1)}{\alpha(n\text{-}1)+(1\text{-}\alpha)(1\text{-}\omega)}\right]^2 < 0$. For the optimal tax rate it will be: $\frac{\partial g^*}{\partial\tau^*} = \frac{1\text{-}\alpha\text{-}\tau^*}{\tau^*}$ $\left[A(\tau^*)^{1\text{-}\alpha}\right]^{1/\alpha} > 0$, $\frac{\partial g^*}{\partial n} = \frac{\partial g^*}{\partial\tau^*}\frac{\partial\tau^*}{\partial n} < 0$ and $= \frac{\partial g^*}{\partial(1-\omega)} = \frac{\partial g^*}{\partial\tau^*}\frac{\partial\tau^*}{\partial(1-\omega)} < 0$.

Since public spending abroad affects the level of the public production factor at home, an externality between economies comes up. This is a typical case of the 'free riding problem': governments are tempted to make disproportionate use of the public good. Nash tax rate alternatively may be written as: $\tau_j^* = (1\text{-}\alpha)$ $\frac{\alpha\omega_{j,j}+(1-\alpha)\omega_{j,i}}{\alpha+(1-\alpha)\omega_{j,i}}$, where $\omega_{j,i,} = \frac{1-\omega}{n-1}$. One may check that τ_j^* depends positively on $\omega_{j,i,}$ which in turn is negatively affected by n. As the number of economies sharing the externality, n, increases, the effect of domestic public spending on the foreign production function, $\omega_{j,i,}$ diminishes, affecting negatively the Nash tax rate. On the other hand, as the external contribution of foreign country infrastructure to the domestic economy, 1-ω, decreases, governments may rely less on foreign public spending and the free riding problem becomes less important.

One may, hence, conclude that τ^* will decrease as the free riding problem intensifies, i.e. for higher n and higher 1-ω values. The growth rate, on the other hand, is not directly affected either through n or through ω (check equation 11a). It depends positively on the tax rate, as long as the economy stays on the rising part of the Laffer curve (i.e. the tax rate is less than 1-α), which is actually the case for τ^*. Thus, the growth rate will indirectly depend positively on n and negatively on ω.

3.2 Cooperative solution

Consider now the benchmark case, where governments jointly choose a common tax rate, in order to maximize the sum of individual countries' welfare. The maximization problem will be: $\max_{\tau}\left\langle \int\left\{\sum_{j=1}^{n}\left[\ln(c_j)e^{-\rho t}\right]\right\}dt \right\rangle$, given economies' budget constraints and decisions of private sector in all economies: $\dot{k}_j = k_j r_j + w_j - c_j$ & $\dot{c}_j = c_j(r_j - \varrho)$, $r_j = \alpha(1-\tau)A_j k_j^{\alpha-1} G^{1-\alpha} - \delta$ & $w_j = (1-\alpha)(1-\tau)A_j k_j^{\alpha} G^{1-\alpha}$, where j=1, ..., n. The focus will be on Symmetric Cooperative Solutions in national policy rules. That is, economies may differ *ex ante* but they become identical *ex post*. Invoking symmetry into the FOC's, after some algebra (shown in Appendix C) one ends up with the below result.

Result 4

A Symmetric Cooperative Solution (SCS) in national tax policies is summarized by equations (12a)-(12f) below. Equation (12a) determines a unique tax rate, denoted as $0<\tilde{\tau}<1$.

Proof: See Appendix C

$$\tau = 1-\alpha \tag{12a}$$

$$\frac{\dot{k}}{k} = (1-\tau)\frac{y}{k} - \delta - \frac{c}{k} \tag{12b}$$

$$\frac{\dot{c}}{c} = \alpha(1-\tau)\frac{y}{k} - \delta - \varrho \tag{12c}$$

$$\frac{\dot{v}}{v} = \varrho + \delta - (1-\tau)\frac{y}{k} \tag{12d}$$

$$\frac{\dot{\mu}}{\mu} = \frac{v}{\mu} - \frac{1}{c\mu} - \alpha(1-\tau)\frac{y}{k} + \delta + 2\varrho \tag{12e}$$

$$\lim_{t \to \infty}\left(kve^{-\rho t}\right) = 0 \tag{12f}$$

In this case, the optimal tax rate is independent of the number of countries, n, and the external contribution of foreign country infrastructure to the domestic economy, 1-ω, being equal to the rate of public factor's productivity, 1-α. This is actually Barro's (1990) well-known solution for the tax rate. The optimal tax rate is non-state contingent, i.e. it doesn't change over time.

Let us denote the steady state values in the Symmetric Long-run Cooperative Solution (SLCS) of (τ, c, k) by $(\tilde{\tau}, \tilde{c}, \tilde{k})$. According to Definition 2 a well-defined steady state requires: i) $\tilde{c}$ & $\tilde{k}$ be positive and grow at the same rate, ii) economy grow, iii) $0<\tilde{\tau}<1$ and iv) transversality condition hold. After some algebra (see Appendix D), the dynamic system described above generates the following equations:

$$c = k_0\left[(1-\alpha)(1-\tau)\frac{y}{k} + \rho\right] e^{\left[\alpha(1-\tau)y/k - \delta - \rho\right]t} \tag{13a}$$

$$k = k_0 e^{\left[\alpha(1-\tau)y/k - \delta - \rho\right]t} \tag{13b}$$

As it is also shown in Appendix D, for this system it will always be: $\frac{\tilde{c}}{\tilde{k}} = (1-\alpha)$ $(1-\tilde{\tau})\frac{\tilde{y}}{\tilde{k}} + \varrho$, i.e. there will be no transitional dynamics, as in all AK models. Economies either start on the steady state or 'jump' onto it.

Result 5

Condition $\alpha^2 A^{1/\alpha}(1-\alpha)^{1/\alpha} > \delta + \varrho$ is necessary and sufficient to determine a Symmetric Long-run Cooperative Solution (SLCS) in national policies. This will be summarized by equations (13a) and (13b) and the tax rate solving equation (12a). This tax rate supports a unique, well-defined balanced growth path, in which capital and consumption grow at a constant positive rate, described by equation (14a) below.

Proof: See Appendix D

Now turn to the comparative static properties of the optimal tax rate and the associated growth rate:

$$\tilde{g} = \alpha^2 A^{1/\alpha}(1-\alpha)^{1-\alpha/\alpha} - \varrho - \delta \tag{14a}$$

Result 6

At the Symmetric Long-run Cooperative Solution (SLCS) the optimal tax rate equals the productivity of the public factor, 1-α, and does not depend on the number of the economies sharing the public factor, n, and the external contribution of foreign country' infrastructure to the domestic economy, 1-ω.

Proof: Check equation (12a) above

When governments cooperate, they internalise the externality created by the effect public spending abroad has in economy j's production function. Thus, there will be no free riding problem. As discussed in the next section, this is actually the incentive governments have to cooperate. The cooperative solution corresponds to the natural efficiency condition, $\frac{\partial Y}{\partial G}=1$, i.e. the level where the social cost of a unit of G (which is 1) equals its social benefit (which is $\frac{\partial Y}{\partial G}$): $\frac{\partial Y}{\partial G}=(1-\alpha)\frac{Y}{G}$ $=(1-\alpha)\frac{Y}{G}=1$. Governments do not choose, however, the first best solution, since decisions are still decentralized at private agent's level[17]. Combing equations (14a) and ① one gets:

$$\tilde{U} = \frac{1}{\rho^2}\,[\varrho \ln(c_0)+\tilde{g}\,]+U_0 \tag{14b}$$

3.3 Discussion

According to Cooper and John (1980, p. 448), "in the presence of positive spillovers, there will be a tendency to insufficient action in Nash equilibrium". In such a case, players' actions increase when they switch from an uncoordinated (i.e. Nash) to a coordinated equilibrium (i.e. cooperative). This should also happen in the present model, given that the existence of the common public production factor generates positive spillovers. Therefore, one should expect that, when governments play Nash to each other they should end up with a lower tax rate than the one chosen when they cooperate. Indeed, equations (9a) and (12a) lead to Result 7, below. The difference between optimal tax rates of the two cases will be given by:

$$\tilde{\tau}\text{-}\tau^* = \frac{\alpha(n-1)(1-\alpha)(1-\omega)}{\alpha(n-1)+(1-\alpha)(1-\omega)} \tag{15}$$

Result 7

The optimal tax rate in SLCS will be higher than the optimal tax rate in SLNE: $\tilde{\tau}>\tau^*$. This difference will increase with the number, n, of the economies sharing the public production factor and as the external contribution of foreign country infrastructure to the domestic economy, 1-ω, intensifies.

Proof: Given that $0<\alpha<1$, $n\geq 2$ and $0<\omega<1$, equation (15) results to $\tilde{\tau}>\tau^*$. It will, also, be: $\frac{\partial(\tilde{\tau}-\tau^*)}{\partial n}=\alpha\left[\frac{(1-\alpha)(1-\omega)}{\alpha(n-1)+(1-\alpha)(1-\omega)}\right]^2>0$ and $\frac{\partial(\tilde{\tau}-\tau^*)}{\partial(1-\omega)}=$ $(1-\alpha)\left[\frac{\alpha(n-1)}{\alpha(n-1)+(1-\alpha)(1-\omega)}\right]^2>0$.

The difference in optimal tax rates, $\tilde{\tau}\text{-}\tau^*$, increases with n. Note that actually n influences τ^* through $\omega_{j,i}$, which depends negatively on n due to the symmetry assumption[18]. Given that $\tilde{\tau}$ does not depend on $\omega_{j,i}$, the difference between the two tax rates will increase. The difference in optimal tax rates, $\tilde{\tau}\text{-}\tau^*$, will also increase together with the external contribution of foreign country infrastructure, 1-ω.

In the present context, a coordinated increase in the strategies of all agents would be welfare improving[19]. Hence the Nash equilibrium should prove to be 'inferior' to the cooperative equilibrium and governments should choose to adopt a common tax policy. Indeed, this is what happens: in both cases, total utility in the long run will be given by: $\int_0^\infty [\ln(c)e^{-\rho t}]dt=\frac{1}{\rho^2}(\rho\ln c_0+g)>0$. The difference in utility will, hence, depend positively on the growth rate, therefore utility will be higher in cooperation and governments will always have an incentive to cooperate.

Following equations (11a-b) and (14a-b), the difference in growth rates and utility ratios will be described by:

$$\tilde{g}\text{-}g^*=\alpha^2A^{1/\alpha}(1-\alpha)^{1-\alpha/\alpha}\{1-\frac{\alpha\omega(n-1)+(n-\alpha)(1-\omega)}{\alpha\omega(n-1)+(1-\alpha)(1-\omega)}\left[\frac{\alpha\omega(n-1)+(1-\alpha)(1-\omega)}{\alpha(n-1)+(1-\alpha)(1-\omega)}\right]^{1/\alpha}\} \tag{15a}$$

$$\widetilde{U}-U^*=\frac{1}{\rho^2}(\widetilde{g}-g^*) \tag{15b}$$

Result 8

At the Long-run Symmetric Cooperative Solution (SLCS) the growth rate and utility attained will always be higher than in the case of Long-run Symmetric Nash Equilibrium (SLNE). This difference will increase with the number, n, of the economies sharing the public production factor and as the external contribution of foreign country infrastructure to the domestic economy, 1-ω, intensifies.

Proof: See Appendix E

No matter whether governments cooperate or not, assuming symmetry implies that the growth rate in the long run will be described by: $g=\alpha(1-\tau)\frac{y}{k}-\delta-\varrho$. Thus, differences in the chosen tax rate chosen result to differences in growth rates attained by economies. When governments cooperate, higher tax rate increases $\frac{y}{k}$ [remember in symmetry it equals $(A\tau^{1-\alpha})^{1/\alpha}$, i.e. it depends positively on τ], which affects positively the interest rate; hence, the growth rate increases. On the other hand, higher tax rate means lower after tax income, resulting to lower interest rate, which affects negatively the growth rate. The aggregate effect, however, will be positive as long as $\tau<1-\alpha$, i.e. growth rate will be higher when governments cooperate. The optimal tax rate under cooperation does not depend on n or ω, since there is no free riding problem. In contrast, the Nash tax rate depends on both of them, which implies that economies growth rate will also depend on them (result 3). As n or 1-ω increase, g^* will decrease while $\widetilde{g}$ will not be affected at all, resulting to an increase in $\widetilde{g}-g^*$.

FIGURE 1

The effect of changes in the size of the group

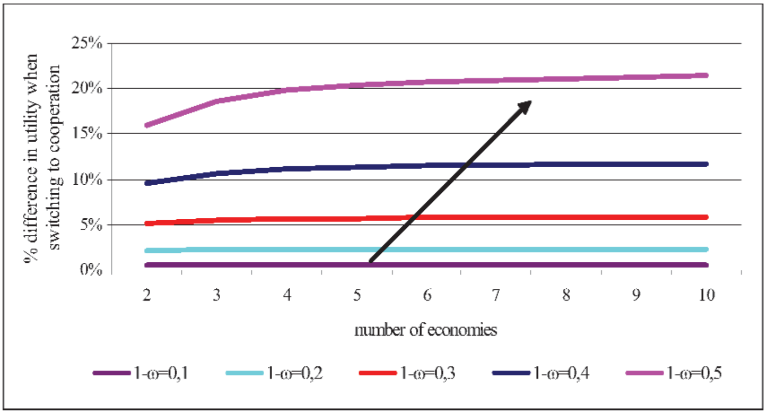


Figure 1 presents how changes in the number of economies affected by the public production factor, n, result to utility differences between the Nash equilibrium and the cooperative solution[20]. Five different cases for the external contribution of foreign country infrastructure to the domestic economy, 1-ω, are examined. As one may claim, increasing n produces substantial gains in utility for relatively high values of 1-ω. After all, the effect of n on the results should be attributed to the existence of the spillover between economies. The stronger spillover is (i.e. the higher 1-ω), the stronger will also be the effect of n on the Nash tax rate and through that on the growth rate and utility ratio. For higher n values, the positive effect of a further increase in n becomes smaller. The reason is that while $\left.\frac{\partial \tau^*}{\partial \omega_{i,j}}\right|_{\omega=\text{given}} > 0$, i.e. Nash tax rate depends positively on $\omega_{i,j}$,

$\left.\frac{\partial^2 \tau^*}{(\partial \omega_{i,j})^2}\right|_{\omega=\text{given}} < 0$, i.e. this relationship 'weakens' as $\omega_{i,j}$ increases.

FIGURE 2

Changes in the importance of public spending abroad

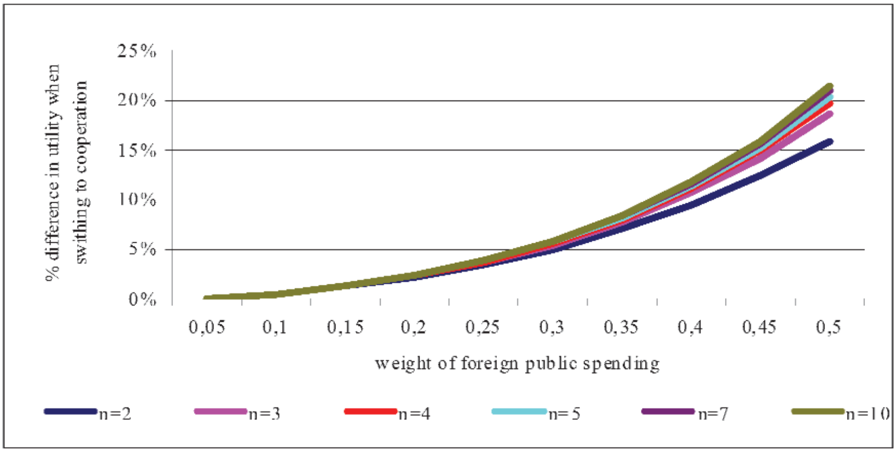


Figure 2 depicts the way the gain percentage when deciding to cooperate, changes as the weight of public spending abroad (1-ω) increases. This time five different cases for the size of the group (n) are presented, while the rest parameter values are the same as above (see footnote 20). In Figure 2 one may distinguish a clearly positive relationship, which does not depend importantly on the number of economies sharing the spillover [though there is some slight positive effect, given that $\frac{\partial\tau^*}{\partial(1-\omega)}$ depends negatively on $\omega_{i,j}$, which decreases for higher n values].

As 1-ω increases, the gains in terms of utility from cooperating increase at an accelerating rate. The reason is that an increase in 1-ω will affect the Nash tax rate negatively through the equivalent decrease in ω and positively through the increase in $\omega_{i,j}$. However, while $\left.\frac{\partial^2\tau^*}{(\partial\omega)^2}\right|_{\omega_{i,j}=\text{constant}}=0$, it will be: $\left.\frac{\partial^2\tau^*}{(\partial\omega_{i,j})^2}\right|_{\omega=\text{constant}}>0$. Given that $\frac{\partial\tau^*}{\partial(1-\omega)}=\frac{1}{n-1}\left.\frac{\partial\tau^*}{\partial\omega_{i,j}}\right|_{\omega=\text{constant}} - \left.\frac{\partial\tau^*}{\partial\omega}\right|_{\omega_{i,j}=\text{constant}}$, one may conclude that $\frac{\partial\tau^*}{\partial(1-\omega)}$ will increase for higher 1-ω values.

Summing up, governments' incentive to cooperate should be attributed to the presence of the positive spillover in present model's setup. As n or 1-ω increases, cooperation implies higher benefits in terms of intertemporal utility. The larger the number of economies sharing the externality is and the more important foreign public spending becomes in the production function, the higher the cooperation benefits.

4. Transfer payments

An international, state-contingent, transfer payments scheme will now be added to the framework developed above. This may generate a moral hazard problem, where poorer economies prefer to lag behind and receive transfer payments rather than growing faster and have to donate transfer payments themselves. State-contingent transfers exacerbate the Nash – type problems (grounded on the existence of international public services), increasing the difference between cooperative and Nash policies. The rationale is that a 'moral hazard' problem comes on top of the already existing 'free riding' problem. The two problems do not work in opposite ways; thus, a transfer payment scheme will not work as a substitute (even a partial one) to cooperation – it will make it even more attractive.

4.1 The environment

Again the model setup and sequence of agents' moves presented above will be adopted. However, in this case the public good in each country is financed both through tax revenues and some transfer payments. Transfer payments are state-contingent and redistribute from the rich to the poor economies. Production in j, will be described by:

$$y_j = A_j k_j^{\alpha} G_j^{1-\alpha} \tag{2}$$

$$G_j = \prod_{\lambda=1}^{n} \left(\tau_\lambda y_\lambda + Z_\lambda\right)^{\omega_{\lambda,j}} \tag{3}$$

where Z_λ denotes transfer payments paid to / by economy λ. We shall adopt the rule $Z_\lambda = z(\bar{y} - y_\lambda)$, where z is a given redistribution constant parameter and $\bar{y}$ the average per capita income of both economies. This is a commonly used policy rule[21]. For analytical convenience we assume there are only two economies (i and j). It will be: $\bar{y} = \dfrac{y_j + y_i}{2}$ and the transfer payments rule will become:

$F_j = \frac{z}{2}(y_i - y_j)$. Thus, we get: $G_j = [\tau_j y_j + \frac{z}{2}(y_i - y_j)]^{\omega_j}[\tau_i y_i - \frac{z}{2}(y_i - y_j)]^{1-\omega_j}$ (3a)

Similarly, production in economy i will be described by:

$$y_i = A_i k_i^{\alpha} G_i^{1-\alpha} \quad (2a)$$

$$G_i = [\tau_i y_i - \frac{z}{2}(y_i - y_j)]^{\omega_i}[\tau_j y_j + \frac{z}{2}(y_i - y_j)]^{1-\omega_i} \quad (3b)$$

The sequence of moves remains unchanged: first national governments choose their tax policy[22]; then private agents make their consumption – investment choices. Policy responsibilities and the sequence of moves is taken as given. Households' or firms' maximization problem doesn't change, yielding, at country level in economy j:

$$\dot{c}_j = c_j(r_j - \varrho) \quad (4)$$

$$\dot{k}_j = w_j + r_j k_j - c_j \quad (5)$$

$$\lim_{t\to\infty}\left(\lambda_j b_j e^{-\rho t}\right) = 0 \quad (6)$$

$$r_j = \alpha(1-\tau_j)A_j k_j^{\alpha-1} G_j^{1-\alpha} - \delta \quad (7)$$

$$w_j = (1-\alpha)(1-\tau_j)A_j k_j^{\alpha} G_j^{1-\alpha} \quad (9)$$

Similar equations hold for economy i. In each country, the government should set the tax rate in order to finance the provision of public production factor. Given the absence of any bond market, governments' budget should be balanced at each point of time, i.e. equation (3a) should hold. A WCDE can now be characterized. In economy j there will be five endogenous variables (G_j, c_j, k_j, r_j and w_j) that will be determined by equations (3a), ④, ⑤, ⑦ and ⑧. Given that equation (3a) involves also y_i, one may not get equilibrium values for economy j without dealing with economy i as well. Thus, the WCDE will be described by a system of 5*n equations and 5*n variables[23]. National governments, taking this into account, decide on their tax policy, either by playing Nash to each other or by cooperating with each other.

4.2 Non-cooperative (Nash) national policies

The government in economy j chooses τ_j to maximize its own household utility function subject to its own constraints in a WCDE. In doing so, it takes τ_i, c_i, k_i (where $i \neq j$) and their shadow prices as given and plays Stackelberg vis-à-vis private agents. Government in country j tries to:

$$\max_{\tau_j} \int_0^\infty \left[\ln(c_j)e^{-\rho t} \right] dt$$, s.t. equations ② - ⑧ above,

Again, the focus will be on Symmetric Nash Equilibria (SNE) in national policy rules, i.e. while countries may differ *ex-ante*, they will be symmetric *ex-post* (thus, in equilibrium, $\tau_j=\tau_i=\tau$, and hence $c_j=c_i=c$, $k_j=k_i=k$, e.tc., where $i\neq j$). Let $\omega=\omega_{j,j}=\omega_{i,i}$, where $0.5\leq\omega\leq1$. As shown (in Appendix F) FOC's yield to (from now on, one can omit country subscripts) Result 9, below.

Result 9

A Symmetric Nash Equilibrium (SNE) in national tax policies and the associated World Competitive Decentralized Equilibrium is summarized by equations (16a) – (16f) below. Equation (16a) determines the Nash tax rate, denoted as $0<\tau^N<1$, which is unique and lower than the one chosen in the absence of transfer payments.

Proof: See Appendix F

$$\tau^N=\frac{1-\omega-(2\omega-1)\left[\frac{z}{2}(1+\alpha)-\alpha\right]+\sqrt{\left\{1-\omega-(2\omega-1)\left[\frac{z}{2}(1+\alpha)-\alpha\right]\right\}^2+4\left[1-(1-\alpha)\omega\right](2\omega-1)\frac{z}{2}}}{2\left(\frac{1}{1-\alpha}-\omega\right)} \tag{16a}$$

$$\frac{\dot{k}}{k}=(1-\tau)\frac{y}{k}-\delta-\frac{c}{k} \tag{16b}$$

$$\frac{\dot{c}}{c}=(1-\tau)\alpha\frac{y}{k}-\delta-\varrho \tag{16c}$$

$$\frac{\dot{v}}{v}=\delta+\varrho-(1-\tau)\frac{y}{k}+(1-\tau)(1-\alpha)\frac{y}{k}(1+\alpha\frac{\mu}{v}\frac{c}{k})[1-\omega-\frac{1}{\tau}\frac{z}{2}(1-2\omega)][1+(1-\alpha)(1-2\omega)\frac{1}{\tau}(\tau-z)]^{-1} \tag{16d}$$

$$\frac{\dot{\mu}}{\mu}=\frac{v}{\mu}-\frac{1}{c\mu}-\alpha(1-\tau)\frac{y}{k}+\delta+2\varrho \tag{16e}$$

$$\lim_{t\to\infty}\left(vke^{-\rho t}\right) = 0 \tag{16f}$$

The Nash tax rate will be non-state contingent. In the presence of transfer payments it proves to be lower than in the absence of them. One may conclude that a transfer payment scheme brings about a 'moral hazard' problem that comes in addition to the 'free riding' problem, already existing in this setting.

Let denote the steady state values in the Symmetric Long-run Nash Equilibrium (SLNE) of (τ, k, c) by (τ^N, k^N, c^N). The long run equilibrium will still be defined according to definition 2. In symmetry it will be: $\frac{G}{k} = (\tau A)^{1/\alpha}$, hence it should also be: $\frac{y}{k} = (\tau^{1-\alpha}A)^{1/\alpha}$. Let $\varphi \equiv (1-\tau)\frac{y}{k}$. For $\tau=\tau^N$, $\varphi^N=(1-\tau^N)\left(\frac{y}{k}\right)^N$ will be a constant. Equation (16c) now becomes: $c=c_0 e^{(\alpha\varphi^N-\delta-\rho)t}$. Hence, for $\tau=\tau^N$ (16b) becomes: $\dot{k} = (\varphi^N-\delta)k - c_0 e^{(\alpha\varphi^N-\delta-\rho)t}$, which yields to: $k = e^{(\varphi^N-\delta)t} k_0 - \frac{c_0}{\varphi^N+\rho}\left[e^{(\varphi^N-\delta)t} - e^{(\alpha\varphi^N-\delta-\rho)t}\right]$. According to the definition of the longrun equilibrium, at the steady state consumption, c, and capital, k, grow at the same constant positive rates, i.e. $\frac{\dot{k}}{k} = \frac{\dot{c}}{c}$. In this case, equations (16b) and (16c) imply that:

$$c^N = [(1-\alpha)\varphi^N + \varrho]k^N \tag{17}$$

Hence, in order to reach the steady state, for $t=0$ it should be: $c_0 = [(1-\alpha)\pi^N + \varrho]k_0$. One ends up with:

$$k = k_0 e^{(\alpha\varphi^N-\delta-\rho)t} \tag{17a}$$

$$c = [(1-\alpha)\varphi^N + \varrho]k_0 e^{(\alpha\varphi^N-\delta-\rho)t} \tag{17b}$$

Now let's check whether the resulting steady state solution is well-defined. A well-defined steady state requires: i) c^N and k^N be positive and grow at the same constant rate, ii) the economy grow, iii) $0<\tau^N<1$ and iv) transversality condition hold. In Appendix G it is proved that:

Result 10

A Symmetric Long-run Nash Equilibrium (SLNE) in national tax policies exists and is unique, being summarized by equations (17a)-(17b) and the tax rate described in (16a). Condition (18a) below is necessary and sufficient for economies to grow at a SLNE in national tax policies.

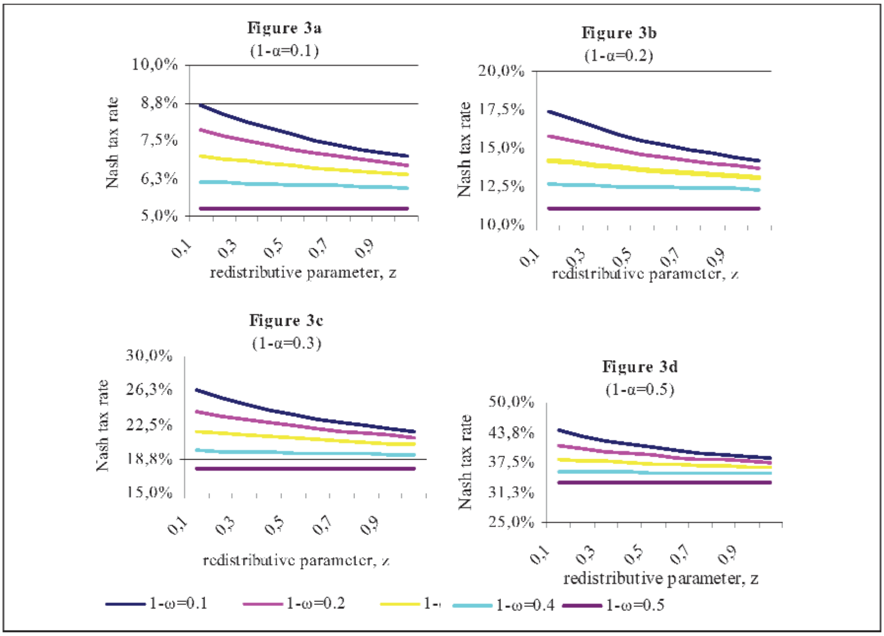
$$g^N=\alpha\varphi^N-\delta-\varrho>0 \tag{18}$$

where $\varphi^N=(1-\tau^N)[A(\tau^N)^{1-\alpha}]^{1/\alpha}$ and τ^N is described in equation (16a).

Proof: See Appendix G

Note that in this system always: $\frac{c^N}{k^N}=(1-\alpha)(1-\tau^N)\frac{y^N}{k^N}+\varrho$, i.e. there are no transitional dynamics, as in all AK models. Economies either start on the steady state or 'jump' onto it immediately. Again, changes in the underlying parameters can affect levels and growth rates of 'real' variables. We shall focus on the effects international positive production externalities have, studying how the strength of the externality (i.e. the weight foreign infrastructure in domestic production function has, 1-ω) and redistribution parameter, z, affect the Nash tax rate, τ^N, and the resulting growth rate, g^N (determined by equations (16a) and (18) respectively).

Figures (3a)-(3d) depict a simple numerical illustration of the Nash tax rate for several valid parameter values ($0.5\leq\alpha<1$, $0<1-\omega\leq0.5$, $0<z\leq1$)[24]. As redistribution parameter, z, increases, Nash tax rate, τ^N, decreases.



The negative relationship holds for all different values of public factor's productivity, 1-α[25]. On the other hand, one may notice that this negative relationship, becomes weaker for higher 1-ω values, while for 1-ω=ω=0.5 one may notice that the Nash tax rate does not change for different z values. The existence of transfer payments brings about a 'moral hazard' problem: government in economy j considers that net transfer revenues will decrease as its income increases. As a result it chooses a tax rate lower than the one chosen in the absence of a transfer payments scheme. That moral hazard problem intensifies, as transfer payments are associated to a higher portion of the income difference, resulting to a lower Nash tax rate. Moreover, for lower 1-ω values, governments choose a higher tax rate and the effects of changes in z intensify: as production abroad becomes less important, the moral hazard problem intensifies.

Numerical Remark 1

Under a scheme of international transfer payments, the Nash tax rate, τ^N, depends negatively on the redistribution parameter, z, and on the extent of the external contribution of foreign infrastructure in the domestic economy, 1-ω. As the latter increases, the negative impact of z on τ^N vanishes.

Equation (18), above, determines the growth rate of the economy. Changes in parameter values 1-ω and z will affect the growth rate through their effects on the tax rate[26]. It will be: $\frac{\partial g^N}{\partial \tau^N} = \frac{1-\alpha-\tau^N}{\tau^N}[A(\tau^N)^{1-\alpha}]^{\frac{1}{\alpha}} > 0$, given that $\tau^N < \tau^* < 1-\alpha$. Thus, increases in the optimal tax rate will affect positively the growth rate: g^N will increase for lower 1-ω and z values. Moreover, intertemporal utility attained will be given by: $U = \frac{1}{\rho^2}(\varrho \ln c_0 + g^L)$, i.e. utility will depend positively on the growth rate: lower values of 1-ω and z will result to higher utility.

Numerical Remark 2

Under international transfers, the growth rate when governments play Nash to each other, g^N, decreases as the redistribution parameter, z, increases or as the extent of the external contribution of foreign infrastructure in the domestic economy, 1-ω, increases. In the symmetric case, intertemporal utility attained by any representative agent will also depend negatively on z and 1-ω.

We stress, at this point, the importance that the economies be symmetric *ex post*. Because of this assumption, there will be no transfer payments *ex post*[27]. Therefore, increasing z will not increase the utility in lower income economies, since there is no such economy. Dropping the symmetry assumption, the above numerical remark may not hold, since higher transfer payments may raise intertemporal utility in a relatively poor economy. Moreover, note that the transfer payment scheme is adopted *per se* and not as a remedy to 'fix' some existing inefficiency.

4.3 Cooperative solution

Consider now the benchmark case, where all governments cooperate to choose a common tax rate, τ, in order to intertemporally maximize utility for all countries, given the individual budget constraints and decisions of the private sector. The maximization problem will be: $\max_{\tau} \int_0^{\infty} \left[\ln(c_j)e^{-\rho t} + \ln(c_i)e^{-\rho t}\right] dt$, given budget constraints and private sector's decisions in all economies and the constraint that net transfer payments of j should equal net transfer revenues in i. The focus will be on Symmetric Cooperative Solutions in national policy rules. Because of symmetry in FOC, one gets the system in Appendix H, which is exactly the same with the one found in the case of no transfer payment scheme.

Result 11

Cooperative national policies are not affected at all by the existence of a transfer payment scheme in an ex-post symmetric framework.

Proof: See Appendix H and compare the results with the ones found in Appendix C

When governments cooperate to set the tax rate, they find it optimal to keep the tax rate flat over time. This is a tax smoothing effect, which is the least distorting policy.

4.4 Discussion

Adopting a transfer payment scheme results to lower Nash tax rate (result 9) than the one found in the absence of them: $\tau^*-\tau^N>0$. The moral hazard problem (being present only when a transfer payments scheme exists) comes on top of the free riding problem, resulting to governments choosing even lower tax rate. This effect is stronger, as the redistribution parameter, z, increases and as the extent of the external contribution of foreign infrastructure in the domestic economy, 1-ω, decreases.

On the other hand, according to result 11, cooperative national policies are not affected at all by the presence of such a scheme. Given that even without transfer payments the Nash tax rate is lower than the cooperative tax rate, one concludes that also in the presence of transfer payments, the Nash tax rate will be lower than the tax rate chosen when governments cooperate, i.e. it is: $\tilde{\tau} > \tau^* > \tau^N$. Given that the tax rate chosen in cooperation does not depend on 1-ω or z, one may use numerical Remark 1 to find out that the difference $\tilde{\tau} - \tau^N$ will depend positively on them.

Numerical Remark 3

The Nash tax rate chosen when a transfer payments scheme exists, τ^N, will be lower than the Nash tax rate in the absence of such a scheme, τ^*. It will also be lower than the tax rate chosen when governments cooperate, $\tilde{\tau}$. For higher values of the redistributive parameter, z, those differences will increase. As the extent of the external contribution of foreign infrastructure in the domestic economy, 1-ω, increases, the difference $\tilde{\tau} - \tau^N$ will increase while the difference $\tau^* - \tau^N$ will decrease.

Numerical Remark 2 established that the growth rate when governments play Nash to each other, g^N, depends through τ^N negatively on z and 1-ω. Given numerical Remark 3 one may conclude that, the difference in Nash growth rates when there is no transfer payment scheme and when there is such a scheme, g^*-g^N, will increase for higher z and lower 1-ω values, while the difference between the growth rate attained in cooperation and the Nash growth rate, $\tilde{g}$ -g^N depends positively on z and 1-ω. Intertemporal utility attained in all cases will be: $\int_0^\infty [\ln(c)e^{-\rho t}]dt = \frac{1}{\rho^2}$ ($\varrho \ln c_0 + g$). Changes in the growth rate will result to analogous alterations in utility.

Numerical Remark 4

Under a transfer payment scheme, intertemporal utility when governments cooperate will be higher than utility attained when they play Nash to each other. Thus governments have an incentive to cooperate. This difference in intertemporal utility will increase for higher redistributive parameter values, z and stronger contribution of foreign infrastructure in the domestic economy, 1-ω. Under the assumption of ex post symmetry, in case governments decide not to cooperate, utility under a transfer payment scheme will be lower than utility in the absence of it, with the difference depending positively on z and negatively on 1-ω.

In the previous section it was shown that, when economies are symmetric *ex-post*, utility attained is always higher when governments cooperate than when they play Nash to each other. Adopting a transfer payment scheme, not only does not dissolve the incentive to cooperate, but it makes it even stronger. State-contingent distorting transfers exacerbate the Nash – type problems, increasing the difference between cooperative and Nash policies. The reason is that a moral hazard problem will now be present, on the top of the free riding problem already existing in the present set up.

5. Conclusion

In the present work we tried to find out whether the existence of positive international externalities generate an incentive for cooperation between different governments and if the adoption of a transfer payments scheme moderates that intensive. We adopted a simple economic model incorporating the international linkage of national economies. A well-defined Symmetric Long Run Nash Equilibrium exists, while the related tax rate proves to be non-state contingent. On the other hand, the cooperative solution is identical to Barro's (1990) well-known solution for the tax rate, financing some public factor. Utility proves always to be higher when countries cooperate than when they play Nash to each other, with the difference depending positively on the number of economies sharing the public good and the importance of foreign public spending in domestic production.

We then added a transfer payment scheme. Again, a Symmetric Nash Equilibrium in national policies exists. It proves to be unique and lower than the one chosen in the absence of transfer payments, indicating the existence of a moral hazard problem. The Nash tax rate depends negatively on the redistribution parameter, z, and on the extent of contribution of foreign infrastructure in the domestic economy, 1-ω. The cooperative solution is not affected at all by the existence of a transfer payment scheme in an *ex-post* symmetric framework. Thus, the Nash tax rate is lower than the cooperative solution, with the difference depending positively on z and 1-ω.

The adoption of a transfer payments scheme not only does not weaken the intensive to cooperate, but it also intensifies it, since a moral hazard problem arises on the top of the free riding problem. It is important to note, however, that this result may change in case one drops the symmetry assumption. The moral hazard problem will still be present, but there will also be a positive income effect (due to the transfer) resulting to higher utility for the specific economy. A government of a relatively rich country may decide to transfer funds to a poor country with higher marginal product of capital, expecting a partial benefit through higher levels of the foreign-financed public good. As a matter of fact, this could explain the existence of such schemes, without adopting altruistic motivations. One possible extension of the model may thus be to study the case of asymmetric economies, both *ex-ante* and *ex-post*. An alternative extension may be to allow for international capital movement.

Appendix A.

Utility maximization by the government in country j results to a current value Hamiltonian of the form: $J_j=\ln(c_j)+v_j(k_jr_j+w_j-c_j)+\mu_jc_j(r_j-\varrho)$, where μ_j and v_j are multipliers associated with equations ④ and ⑤, respectively. The first-order conditions (FOC's) with respect to τ_j, k_j and c_j may now be derived.

According to equation ③ it is: $G_k=\prod_{i=1}^{n}(\tau_i y_i)^{\omega_{i,k}}$. Governments take $\frac{\partial\tau_i}{\partial\tau_j}=0$, $\forall\ i\neq j$. Optimization

asks for: $$\frac{\partial J_j}{\partial\tau_j}=0 \Leftrightarrow y_j(v_j+\alpha\mu_j\frac{c_j}{k_j})[(1-\tau_j)\,(1-\alpha)\frac{\partial G_j}{\partial\tau_j}\frac{1}{G_j}-1]=0 \qquad \text{(I)}$$

By definition it is: $y_j(v_j+\alpha\mu_j\frac{c_j}{k_j})>0$, hence it should be: $(1-\tau_j)(1-\alpha)\frac{\partial G_j}{\partial\tau_j}=G_j$.

In symmetry one gets: $\frac{\partial G_j}{\partial \tau_j}$ $=[\omega-(1-\alpha)(\omega-\frac{1-\omega}{n-1})]\frac{G}{\tau}\frac{1}{\alpha}[1-(1-\alpha)(\omega-\frac{1-\omega}{n-1})]^{-1}$ and eq. (I) yields to: $\tau=(1-\alpha)\frac{\alpha\omega(n-1)+(1-\alpha)(1-\omega)}{\alpha(n-1)+(1-\alpha)(1-\omega)}$ ❶.

Denote the tax rate solving equation ❶ as τ^*. It will obviously be unique. Given that, by definition, $0<\omega<1$, it will be $\frac{\alpha\omega(n-1)+(1-\alpha)(1-\omega)}{\alpha(n-1)+(1-\alpha)(1-\omega)}<1$ and since $1-\alpha<1$ as well, one may conclude that $0<\tau^*<1-\alpha$. Moreover, it is: $1-\tau^*=\alpha\frac{(n-1)+(1-\alpha)(1-\omega n)}{\alpha(n-1)+(1-\alpha)(1-\omega)}>0 \Leftrightarrow \tau^*<1$.

It will be: $\frac{\partial \tau_i}{\partial k_j}=\frac{\partial k_i}{\partial k_j}=0,\ \forall\ i\neq j$. It should be: $\frac{\partial J_j}{\partial k_j}=\varrho v_j-\dot{v}_j$. In symmetry it will be: $\frac{\partial G_j}{\partial k_j}=[\omega-(1-\alpha)(\omega-\frac{1-\omega}{n-1})]\frac{G}{k}[1-(1-\alpha)(\omega-\frac{1-\omega}{n-1})]^{-1}$.

Finally, it should be: $\frac{\partial J_j}{\partial c_j}=\varrho\mu_j-\dot{\mu}_j$.

Thus, one ends up with: $\frac{\dot{k}}{k}=(1-\tau)\frac{y}{k}-\delta-\frac{c}{k}$, $\frac{\dot{c}}{c}=(1-\tau)\alpha\frac{y}{k}-\delta-\varrho$, $\frac{\dot{v}}{v}=\varrho+\delta+\{\alpha(1-\tau)(1-\alpha)(1-\omega)\frac{y}{k}\frac{\mu}{v}\frac{c}{k}-(1-\tau)\frac{y}{k}[(1-\alpha\frac{1-\omega}{n-1}+\alpha]\}[1-(1-\alpha)(\omega-\frac{1-\omega}{n-1})]^{-1}$, $\frac{\dot{\mu}}{\mu}=\frac{v}{\mu}-\frac{1}{c\mu}-\alpha(1-\tau)\frac{y}{k}+\delta+2\varrho$ and $\lim_{t\to\infty}\left(vke^{-\rho t}\right)=0$.

Appendix B.

It will be: $\varphi^*=\frac{\alpha}{1-\alpha}[(1-\alpha)A]^{1/\alpha}\frac{(n-1)+(1-\alpha)(1-\omega n)}{\alpha\omega(n-1)+(1-\alpha)(1-\omega)}$

$$\left[\frac{\alpha\omega(n-1)+(1-\alpha)(1-\omega)}{\alpha(n-1)+(1-\alpha)(1-\omega)}\right]^{1/\alpha} > 0.$$

At the long run equilibrium it should be: $\frac{\dot{k}}{k} = \frac{\dot{c}}{c}$, which yields to: $\left(\frac{c}{k}\right)^{*}$ $=(1-\alpha)\varphi^{*}+\varrho$. At the long run equilibrium will, hence, be: $\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \alpha\varphi^{*}-\varrho-\delta$. For this growth rate to be positive it should be: $\alpha\varphi^{*}>\varrho+\delta$. Since τ^{*} is unique, φ^{*} and the steady state will also be unique. From equations (10a) and (10b) one may easily check that c^{*} and k^{*} are positive and grow at the same rate.

Now let: $\sigma_1 \equiv \delta+\varrho-\varphi^{*}[\alpha+(1-\alpha)\frac{1-\omega}{n-1}][1-(1-\alpha)(\omega-\frac{1-\omega}{n-1})]^{-1}$ and $\sigma_2 \equiv \alpha[(1-\alpha)\varphi^{*}+\varrho](1-\alpha)\varphi^{*}(1-\omega)[1-(1-\alpha)(\omega-\frac{1-\omega}{n-1})]^{-1}$, so as to write eq. (9d) as: $\dot{v}=\sigma_1 v+\sigma_2\mu$. Both σ_1 and σ_2 will be constants.

Also let: $\sigma_3 \equiv \delta+2\varrho-\alpha\varphi^{*}$ and $\sigma_4 \equiv \frac{1}{k_0}\frac{1}{(1-\alpha)\varphi^{*}+\rho}$ (again both will be constants) to get from eq. (9e): $\dot{\mu}=v+\sigma_3\mu-\sigma_4 e^{(\sigma_3-\rho)t}$.

Taking into account the transversality condition (9f) one gets after some Algebra: $\mu=(\mu_0-B)e^{\lambda_2 t}+Be^{(\sigma_3-\rho)t}$ and $v=(\lambda_2-\sigma_3)(\mu_0-B)e^{\lambda_2 t}-\sigma_2\sigma_4[\varrho^2+\varrho(\sigma_1-\sigma_3)-\sigma_2]^{-1} e^{(\delta+\rho-\alpha\varphi^{*})t}$, where $B=\sigma_4(\varrho-\sigma_3+\sigma_1)(\varrho-\sigma_3+\lambda_2)^{-1}(\varrho-\sigma_3+\lambda_1)^{-1}$ and $\lambda_2= \frac{\sigma_1+\sigma_3-\sqrt{(\sigma_1-\sigma_3)^2+4\sigma_2}}{}$. Check that the transversality condition $\lim_{t\to\infty}\left(vke^{-\rho t}\right)=(\lambda_2-\sigma_3)k_0(\mu_0-B)\lim_{t\to\infty}\left[e^{(\lambda_2-\sigma_3)t}\right]-k_0\sigma_2\sigma_4[\varrho^2+\varrho(\sigma_1-\sigma_3)-\sigma_2]^{-1}\lim_{t\to\infty}\left(e^{-\rho t}\right)$ $=0$, indeed holds, since $\lambda_2-\sigma_3<0$ and $\varrho>0$.

Appendix C.

The current value Hamiltonian will be:

$$J=\sum_{i=1}^{n}[\ln(c_i)]+\sum_{i=1}^{n}[v_i(k_i r_i+w_i-c_i)]+\sum_{i=1}^{n}[c_i\mu_i(r_i-\rho)],$$

where v_i and μ_i are multipliers associated with ④ and ⑤, respectively.

For the case of a common tax rate, equation ③ becomes: $G_j=\prod_{i=1}^{n}(\tau y_i)^{\omega_{i,j}}$.

Optimization asks for: $\frac{\partial J}{\partial \tau}=0 \Leftrightarrow \sum_{i=1}^{n}\left[v_i\left(k_i\frac{\partial r_i}{\partial \tau}+\frac{\partial w_i}{\partial \tau}\right)\right]+\sum_{i=1}^{n}\left(c_i\mu_i\frac{\partial r_i}{\partial \tau}\right)$ $=0 \Leftrightarrow \frac{y_j}{G_j}(v_j+\alpha\mu_j\frac{c_j}{k_j})[(1-\alpha)(1-\tau)\frac{\partial G_j}{\partial \tau}-G_j]=0$. In symmetry this reduces to: $\frac{y}{G}$ $[(1-\alpha)(1-\tau)\frac{\partial G}{\partial \tau}-G](v+\alpha\mu\frac{c}{k})=0$. By definition $\frac{y}{G}(v+\alpha\mu\frac{c}{k})\neq 0$; hence: $(1-\tau)(1-\alpha)$ $\frac{\partial G}{\partial \tau}=G$. It will be: $\frac{\partial G}{\partial \tau}=\frac{G}{\tau}+(1-\alpha)\frac{\partial G}{\partial \tau} \Leftrightarrow \frac{\partial G}{\partial \tau}=\frac{1}{\alpha}\frac{G}{\tau}$ and optimality condition becomes: $(1-\alpha)(1-\tau)\frac{1}{\alpha}\frac{G}{\tau}=G \Leftrightarrow \tau=1-\alpha$. Denote the optimal tax rate $\tilde{\tau}$. It will obviously be unique and $0<\tilde{\tau}<1$.

It should be: $\frac{\partial J}{\partial k_j}=\varrho v_j-\dot{v}_j \Leftrightarrow \frac{\dot{v}_j}{v_j}=\varrho-r_j-\frac{1}{v_j}\sum_{i=1}^{n}\left[v_i\left(k_i\frac{\partial r_i}{\partial k_j}+\frac{\partial w_i}{\partial k_j}\right)+c_i\mu_i\frac{\partial r_i}{\partial k_j}\right]$, where $j=1, ..., n$. Finally, it should be: $\frac{\partial J}{\partial c_j}=\varrho\mu_j-\dot{\mu}_j$. Imposing symmetry, results to: $\tau=1-\alpha$, $\frac{\dot{k}}{k}=(1-\tau)\frac{y}{k}-\delta-\frac{c}{k}$, $\frac{\dot{c}}{c}=\alpha(1-\tau)\frac{y}{k}-\delta-\varrho$, $\frac{\dot{v}}{v}=\varrho+\delta-(1-\tau)\frac{y}{k}$, $\frac{\dot{\mu}}{\mu}=\frac{v}{\mu}-\frac{1}{c\mu}-$ $\alpha(1-\tau)\frac{y}{k}+\delta+2\varrho$ and $\lim_{t\to\infty}\left(kve^{-\rho t}\right)=0$.

Appendix D.

Given that equation (12a) assures optimal tax rate is constant over time, equation (13c) yields to: $c=c_0e^{\left[\alpha(1-\tau)y/k-\delta-\rho\right]t}$, which transforms eq. (12b) to: $\frac{\dot{k}}{k}=$

$(1-\tau)\quad \frac{y}{k}-\delta-\frac{c_0}{k} \quad e^{\left[\alpha(1-\tau)y/k-\delta-\rho\right]t} \quad \Leftrightarrow \quad k=\frac{c_0 e^{\left[\alpha(1-\tau)y/k-\delta-\rho\right]t}}{(1-\alpha)(1-\tau)y/k+\rho}+e^{\left[(1-\tau)y/k-\delta\right]t}$

$(k_0-\frac{c_0}{(1-\alpha)(1-\tau)y/k+\rho})$.

Equation (12d) yields to: $v=v_0 e^{\left[\rho+\delta-(1-\tau)y/k\right]t}$. Thus, transversality condition requires: $\lim_{t\to\infty}\left(kve^{-\rho t}\right)=0 \Leftrightarrow k_0=\frac{c_0}{(1-\alpha)(1-\tau)y/k+\rho}$. In symmetry it will be: $\frac{y}{k}=(A\tau 1-\alpha)^{\frac{1}{\alpha}}$. For a constant tax rate, the term $(1-\alpha)(1-\tau)\frac{y}{k}+\varrho$ will also be constant. The transversality condition will be satisfied for: $k_0=\frac{c_0}{(1-\alpha)(1-\tau)y/k+\rho}$.

At the long run equilibrium it should be:

$\frac{\dot{k}}{k}=\frac{\dot{c}}{c}\Leftrightarrow\frac{\tilde{c}}{\tilde{k}}=[(1-\alpha)(1-\tilde{\tau})\frac{\tilde{y}}{\tilde{k}}+\varrho]$,

which in this case either holds straight from the beginning

[i.e. $k_0=\frac{c_0}{(1-\alpha)(1-\tau)y/k+\rho}$] or doesn't hold at all.

The steady state described by $\tilde{\tau}=1-\alpha$, $\tilde{c}=k_0[(1-\alpha)(1-\tilde{\tau})\frac{\tilde{y}}{\tilde{k}}+\varrho]e^{\left[\alpha(1-\tilde{\tau})\tilde{y}/\tilde{k}-\rho-\delta\right]t}$ and $\tilde{k}=k_0 e^{\left[\alpha(1-\tilde{\tau})\tilde{y}/\tilde{k}-\rho-\delta\right]t}$, where $\frac{\tilde{y}}{\tilde{k}}=(A\,\tilde{\tau}\,1-\alpha)^{\frac{1}{\alpha}}$, will be unique, determined only by predetermined values (A, α, ϱ, δ and k_0).

For $\alpha(1-\tilde{\tau})\frac{\tilde{y}}{\tilde{k}}>\delta+\varrho\Leftrightarrow\alpha^2[A(1-\alpha)1-\alpha]^{\frac{1}{\alpha}}>\delta+\varrho$ it will be $\frac{\dot{k}}{k}=\frac{\dot{c}}{c}>0$, thus 'real' variables will grow at the same, constant, positive rate.

Appendix E.

Assuming symmetry, in the long run it will always be: $g=\alpha(1-\tau)A^{1/\alpha}\tau^{1-\alpha/\alpha}-\delta-\varrho$.

It is $\frac{\partial g}{\partial \tau}=\alpha(A\tau^{1-\alpha})^{1/\alpha}(\frac{1-\alpha}{\alpha}\frac{1-\tau}{\tau}-1)$, which will be positive for $\tau<1-\alpha$. Remember that $\widetilde{\tau}=1-\alpha>\tau^*$. Thus, one may conclude that $\widetilde{g}>g^*$ and $\widetilde{U}-U^*=\frac{1}{\rho^2}(\widetilde{g}-g^*)>0$.

It is $\widetilde{g}=\alpha^2 A^{1/\alpha}(1-\alpha)^{1-\alpha/\alpha}-\delta-\varrho$, which obviously does not depend on n or ω. On the contrary, according to result 3, it will be: $\frac{\partial g^*}{\partial n}<0$ and $\frac{\partial g^*}{\partial \omega}>0$. Thus, it will be: $\frac{\partial(\tilde{g}-g^*)}{\partial n}=-\frac{\partial g^*}{\partial n}>0$ and $\frac{\partial(\tilde{g}-g^*)}{\partial(1-\omega)}=-\frac{\partial(\tilde{g}-g^*)}{\partial \omega}=-\frac{\partial g^*}{\partial \omega}<0$. Applying this result into equation (15b), one easily concludes that: $\frac{\partial(\widetilde{U}-U^*)}{\partial n}=\frac{1}{\rho^2}\frac{\partial(\widetilde{g}-g^*)}{\partial n}>0$ and $\frac{\partial(\widetilde{U}-U^*)}{\partial(1-\omega)}=\frac{1}{\rho^2}\frac{\partial(\widetilde{g}-g^*)}{\partial(1-\omega)}>0$.

Appendix F.

Utility maximization by the government in country j results to a current value Hamiltonian of the form: $J_j=\ln(c_j)+v_j(k_j r_j+w_j-c_j)+\mu_j c_j(r_j-\varrho)$, where μ_j and v_j are multipliers associated with equations ④ and ⑤, respectively.

Optimization asks for: $\frac{\partial J_j}{\partial \tau_j}=0 \Leftrightarrow (v_j+\alpha\mu_j\frac{c_j}{k_j})[(1-\tau_j)\frac{\partial y_j}{\partial \tau_j}-y_j]=0$ (I)

Given that $v_j+\alpha\mu_j\frac{c_j}{k_j}\neq 0$ we get: $\frac{\partial y_j}{\partial \tau_j}(1-\tau_j)=y_j$. In symmetry one gets: $\frac{\partial y_j}{\partial \tau_j}=\frac{1-\alpha}{\alpha}y[\omega+\frac{y}{G}(1-\alpha)(1-2\omega)(\tau-\frac{z}{2})][\tau+(1-\alpha)(1-2\omega)(\tau-z)]^{-1}$ and eq. (I) yields to: $(\frac{1}{1-\alpha}-\omega)\tau^2-\{1-\omega+(1-2\omega)[\frac{z}{2}(1+\alpha)-\alpha]\}\tau+(1-\alpha)(1-2\omega)\frac{z}{2}=0$. The two roots of this quadratic equation are given by $\tau_{1,2}=\dfrac{1-\omega+(1-2\omega)\left[z/2(1+\alpha)-\alpha\right]\pm\sqrt{\Delta}}{2\left(1/_{1-\alpha}-\omega\right)}$,

where $\Delta=\{1\text{-}\omega+(1\text{-}2\omega)[\frac{z}{2}(1+\alpha)\text{-}\alpha]\}^2\text{-}4[1\text{-}(1\text{-}\alpha)\omega](1\text{-}2\omega)\frac{z}{2}$. For $\omega\geq 0.5$ it will be $1\text{-}2\omega\leq 0 \Leftrightarrow 4[1\text{-}(1\text{-}\alpha)\omega](1\text{-}2\omega)\frac{z}{2}\leq 0$, which results to $\Delta\geq 0$. Let $a\equiv \frac{1}{1-\alpha}\text{-}\omega$, $b\equiv\text{-}\{1\text{-}\omega+(1\text{-}2\omega)[\frac{z}{2}(1+\alpha)\text{-}\alpha]\}$ and $c\equiv(1\text{-}2\omega)(1\text{-}\alpha)\frac{z}{2}$ so as to get: $\tau_{1,2}=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$. It should be $\tau_1\tau_2=\frac{c}{a}$. By definition $0<\alpha<1$ & $\omega<1$, thus $(1\text{-}\alpha)\omega<1 \Leftrightarrow \frac{1}{1-\alpha}>\omega \Leftrightarrow a>0$. Hence, for $\omega>0.5 \Leftrightarrow c<0$ it will be: $\tau_1\tau_2<0$, i.e. there will be only one positive (valid) solution. Given that $\sqrt{\Delta}>0$, one easily concludes that: $\tau_1>\tau_2$, i.e. $\tau_1=\dfrac{-b+\sqrt{\Delta}}{2a}>0$ and $\tau_2=\dfrac{-b-\sqrt{\Delta}}{2a}<0$.

Finally, $a+b+c=\frac{1}{1-\alpha}\text{-}\omega\text{-}1+\omega\text{-}(1\text{-}2\omega)[\frac{z}{2}(1+\alpha)\text{-}\alpha]+(1\text{-}2\omega)(1\text{-}\alpha)\frac{z}{2}=\frac{\alpha}{1-\alpha}[1\text{-}(1\text{-}\alpha)(2\omega\text{-}1)(1\text{-}z)]>0$, given that $(1\text{-}\alpha)(2\omega\text{-}1)(1\text{-}z)<1$. So it will be: $a(a+b+c)>0 \Leftrightarrow (2a+b)^2>b^2\text{-}4ac$. Since $\Delta>0$ and $2a+b=(2\omega\text{-}1)[1\text{-}\alpha+\frac{z}{2}(1+\alpha)]+\frac{1}{1-\alpha}\text{-}\omega+\frac{\alpha}{1-\alpha}>0$, it will be: $2a+b>\sqrt{\Delta} \Leftrightarrow \tau_1<1$, i.e. τ_1 will be the unique valid solution. Denote it as τ^N.

It will be: $\dfrac{\partial\tau_i}{\partial k_j}=\dfrac{\partial k_i}{\partial k_j}=0$, $\forall$ $i\neq j$. It should be: $\dfrac{\partial J_j}{\partial k_j}=\varrho v_j\text{-}\dot{v}_j$. In symmetry it will be: $\dfrac{\partial y_j}{\partial k_j}=\dfrac{y}{k}\{\alpha+(1\text{-}\alpha)\frac{1}{\tau}[(\tau\text{-}z)(1\text{-}\omega)+\frac{z}{2}]\}[1\text{-}(1\text{-}\alpha)(1\text{-}2\omega)\frac{1}{\tau}(z\text{-}\tau)]^{-1}$. Finally, it should be: $\dfrac{\partial J_j}{\partial c_j}=\varrho\mu_j\text{-}\dot{\mu}_j$.

Thus, one ends up with the following equations:

$$\tau^N = \frac{1-\omega-(2\omega\text{-}1)\left[\frac{z}{2}(1+\alpha)-\alpha\right]+\sqrt{\left\{1-\omega-(2\omega\text{-}1)\left[\frac{z}{2}(1+\alpha)-\alpha\right]\right\}^2+4\left[1-(1-\alpha)\omega\right](2\omega\text{-}1)\frac{z}{2}}}{2\left(\frac{1}{1-\alpha}-\omega\right)} \tag{16a}$$

$$\frac{\dot{k}}{k}=(1\text{-}\tau)\,\frac{y}{k}\,\text{-}\delta\text{-}\,\frac{c}{k} \tag{16b}$$

$$\frac{\dot{c}}{c}=(1\text{-}\tau)\alpha\frac{y}{k}\,\text{-}\delta\text{-}\varrho \tag{16c}$$

$$\frac{\dot{v}}{v}=\delta+\varrho\text{-}(1\text{-}\tau)\frac{y}{k}+(1\text{-}\tau)(1\text{-}\alpha)\,\frac{y}{k}(1+\alpha\frac{\mu}{v}\,\frac{c}{k}\,)[1\text{-}\omega\text{-}\frac{1}{\tau}\,\frac{z}{2}\,(1\text{-}2\omega)][1+(1\text{-}\alpha)(1\text{-}2\omega)\frac{1}{\tau}(\tau\text{-}z)]^{-1} \tag{16d}$$

$$\frac{\dot{\mu}}{\mu}=\frac{v}{\mu}\,\text{-}\frac{1}{c\mu}\,\text{-}\alpha(1\text{-}\tau)\frac{y}{k}\,+\delta+2\varrho \tag{16e}$$

$$\lim_{t\to\infty}\left(vke^{-\rho t}\right)=0 \tag{16f}$$

For n=2 the optimal tax rate chosen in the Symmetric Long-run Nash Equilibrium in the absence of transfer payments will be: $\tau^*(n=2)=(1\text{-}\alpha)\dfrac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)(1-\omega)}$.

Note that $\tau^N\text{-}\tau^*(n=2)=(1\text{-}\alpha)\dfrac{\sqrt{\Delta}-(1-\omega)-(2\omega-1)\left[\alpha+\frac{z}{2}(1+\alpha)\right]}{2\left[1-\omega(1-\alpha)\right]}$. Given that $\alpha^2(2\omega\text{-}1)>0$ one may prove that: $1\text{-}\omega+(2\omega\text{-}1)[\alpha+(1+\alpha)\frac{z}{2}]>\sqrt{\Delta}$. Thus it will be: $\tau^N<\tau^*$.

Appendix G.

For $\frac{\dot{k}}{k}=\frac{\dot{c}}{c}=\alpha\varphi^N\text{-}\delta\text{-}\varrho>0$, it should be: $\alpha\varphi^N>\dfrac{\delta+\rho}{\alpha}$. Equations (17a) and (17b) will end up to positive values for k, c in case $k_0>0$. In Appendix F it is proved that $0<\tau^N<1$.

Turn to equation (16d) and let $\sigma_1 \equiv \delta+\varrho-\varphi^N+(1-\alpha)\varphi^N[1-\omega-\frac{z}{2}\frac{1}{\tau^N}(1-2\omega)][1+(1-\alpha)(1-2\omega)\frac{1}{\tau^N}(\tau^N-z)]^{-1}$ and $\sigma_2 \equiv \alpha[(1-\alpha)\varphi^N+\varrho](1-\alpha)\varphi^N[1-\omega-\frac{z}{2}\frac{1}{\tau^N}(1-2\omega)][1+(1-\alpha)(1-2\omega)\frac{1}{\tau^N}(\tau^N-z)]^{-1}$, so as to get: $\dot{v}^N=\sigma_1 v^N+\sigma_2\mu^N$. Note that both σ_1 and σ_2 will be constants. Also let $\sigma_3 \equiv \delta+2\varrho-\alpha\varphi^N$ and $\sigma_4 \equiv \frac{1}{k_0}\frac{1}{(1-\alpha)\varphi^N+\rho}$ (again both will be constants) so as to get: $\dot{\mu}^N=v^N+\sigma_3\mu^N-\sigma_4 e^{(\sigma_3-\rho)t}$.

Solving the resulting system of differential equations and taking into account the transversality condition, one ends up with: $\mu=Be^{(\sigma_3-\rho)t}-(\mu_0-B)e^{\lambda_2 t}$ and $v=(\sigma_4-\varrho B)e^{(\sigma_3-\rho)t}-(\sigma_3-\lambda_2)(\mu_0-B)e^{\lambda_2 t}$, where $B=\sigma_4(\varrho-\sigma_3+\sigma_1)(\varrho-\sigma_3+\lambda_2)^{-1}(\varrho-\sigma_3+\lambda_1)^{-1}$ and $\lambda_{1,2}=\frac{\sigma_1+\sigma_3\pm\sqrt{(\sigma_1-\sigma_3)^2+4\sigma_2}}{2}$.

Appendix H.

The current value Hamiltonian will be: $J=\ln(c_j)+\ln(c_i)+v_j(k_j r_j+w_j-c_j)+\mu_j c_j(r_j-\varrho)+v_i(k_i r_i+w_i-c_i)+\mu_i c_i(r_i-\varrho)$, where v_j, v_i, μ_i and μi are multipliers associated with the budget constraint and the Euler equation in each economy. Optimization asks for: $\frac{\partial J}{\partial \tau}=0 \Leftrightarrow v_j(k_j\frac{\partial r_j}{\partial \tau}+\frac{\partial w_j}{\partial \tau})+\mu_j c_j\frac{\partial r_j}{\partial \tau}-v_i(k_i\frac{\partial r_i}{\partial \tau}+\frac{\partial w_i}{\partial \tau})+\mu_i c_i\frac{\partial r_i}{\partial \tau}=0$ (a)

It is: $\frac{1}{G_j}\frac{\partial G_j}{\partial \tau}=\omega_j[y_j+(\tau-\frac{z}{2})\frac{\partial y_j}{\partial \tau}+\frac{z}{2}\frac{\partial y_i}{\partial \tau}][\tau y_j+\frac{z}{2}(y_i-y_j)]^{-1}+(1-\omega_j)[y_i+(\tau-\frac{z}{2})\frac{\partial y_i}{\partial \tau}+\frac{z}{2}\frac{\partial y_j}{\partial \tau}][\tau y_i-\frac{z}{2}(y_i-y_j)]^{-1}$, $\frac{1}{G_i}\frac{\partial G_i}{\partial \tau}=(1-\omega_i)[y_j+(\tau-\frac{z}{2})\frac{\partial y_j}{\partial \tau}+\frac{z}{2}\frac{\partial y_i}{\partial \tau}][\tau y_j+\frac{z}{2}(y_i-y_j)]^{-1}+\omega_i[y_i+(\tau-\frac{z}{2})\frac{\partial y_i}{\partial \tau}+\frac{z}{2}\frac{\partial y_j}{\partial \tau}][\tau Y_i-\frac{z}{2}(y_i-y_j)]^{-1}$, $\frac{\partial y_j}{\partial \tau}=(1-\alpha)\frac{y_j}{G_j}\frac{\partial G_j}{\partial \tau}$ and $\frac{\partial y_i}{\partial \tau}=(1-\alpha)\frac{y_i}{G_i}\frac{\partial G_i}{\partial \tau}$. In symmetry one, finally, gets: $\frac{\partial y_j}{\partial \tau}=\frac{1-\alpha}{\alpha}$

$\frac{y}{\tau}$. Hence, equation (a) becomes: $2y(\frac{1-\alpha}{\alpha}\frac{1-\tau}{\tau}-1)(v+\alpha\mu\frac{c}{k})=0 \Leftrightarrow \frac{1-\alpha}{\alpha}\frac{1-\tau}{\tau}=1 \Leftrightarrow \tau=1-\alpha$, since $y(v+\alpha\mu\frac{c}{k})>0$ by definition.

It will be: $\frac{\partial\tau_i}{\partial k_j}=\frac{\partial k_i}{\partial k_j}=0$, $\forall$ $i\neq j$. It should be: $\frac{\partial J_j}{\partial k_j}=\varrho v_j-\dot{v}_j$ and $\frac{\partial J}{\partial k_i}=\varrho v_i-\dot{v}_i$.

In symmetry it will be: $\frac{\dot{v}}{v}=\varrho+\delta-(1-\tau)(\frac{\partial y_j}{\partial k_j}+\frac{\partial y_i}{\partial k_j})-\alpha(1-\tau)\frac{\mu}{v}\frac{c}{k}(\frac{\partial y_j}{\partial k_j}+\frac{\partial y_i}{\partial k_j}-\frac{y}{k})$. It is $\frac{\partial y_j}{\partial k_j}+\frac{\partial y_i}{\partial k_j}=\frac{y}{k}$, thus we finally end up with: $\frac{\dot{v}}{v}=\varrho+\delta-(1-\tau)\frac{y}{k}$. Finally, it should be: $\frac{\partial J_j}{\partial c_j}=\varrho\mu_j-\dot{\mu}_j$ and $\frac{\partial J}{\partial c_i}=\varrho\mu_i-\dot{\mu}_i$, which in symmetry become: $\frac{\dot{\mu}}{\mu}=\frac{v}{\mu}-\frac{1}{c}\frac{1}{\mu}-(r-2\varrho)$.

Thus, one ends up with the following equations:

$$\tau=1-\alpha \tag{19a}$$

$$\frac{\dot{k}}{k}=(1-\tau)\frac{y}{k}-\delta-\frac{c}{k} \tag{19b}$$

$$\frac{\dot{c}}{c}=\alpha(1-\tau)\frac{y}{k}-\delta-\varrho \tag{19c}$$

$$\frac{\dot{v}}{v}=\varrho+\delta-(1-\tau)\frac{y}{k} \tag{19d}$$

$$\frac{\dot{\mu}}{\mu}=\frac{v}{\mu}-\frac{1}{c\mu}-\alpha(1-\tau)\frac{y}{k}+\delta+2\varrho \tag{19e}$$

$$\lim_{t\to\infty}\left(kve^{-\rho t}\right)=0 \tag{19f}$$

These are exactly the same equations with the ones found in the absence of a transfer payment scheme.

Notes

1. Agencia Estado reported on 11/5/2004 that "China is ready to invest $ 5 billion in Brazilian infrastructure, Brazilian Mercantile and Futures Exchange president Manoel Felix Cunha said", while according to Gazeta Mercantil "Chinese entrepreneurs want to invest in infrastructure in Brazil to help carry products out via the Pacific Ocean".

2. See the statistical section in the website of Ministry of Commerce of the People's Republic of China.

3. For example, see the seminal paper of Cooper and John (1988).

4. For example, Caplan *et al.* (2000) argue that the adoption of a redistribution scheme may help to internalise spill over related to an international public good.

5. Literature suggests several arguments justifying transfer payments. Pure altruism may provide an explanation: people care about others' utility. 'Thinking about the neighbor' or even being concerned about national prosperity may be a convincing argument. Alternatively, arguments related to risk-sharing, political concerns or the need to create new markets, have been proposed as possible explanations.

6. Data found from Regional Policy Directorate of the European Commission. Amounts are in 1999 prices.

7. Alesina and Wacziarg (1999) present a model with cross-country externalities, where output depends on labor, private capital and the weighted product of public services across the world. Governments impose a proportional tax on output to finance the provision of the public service.

8. In symmetry it turns to be: $G=(n\tau A)^{1/\alpha}k$, which results to: $y=[(n\tau)^{1-\alpha}A]^{1/\alpha}k$.

9. In the latter case, the Nash tax rate proves to depend on the number of economies sharing the externality. This, however, may be explained as a typical result of a free riding problem.

10. The formulation in equation ② assumes that the flow of government purchases, G, enters into the production function. Including a stock of accumulated public capital may be more realistic. For simplicity reasons, however, the focus will be on the case of 'flow' government purchases.

11. This is possible due to constant returns to scale. See also Barro and Sala-i-Martin, ch. 2.

12. Because of the timing, there is no credibility problem vis-a-vis the private sector in the choice of the capital tax rate. Persson and Tabellini (1999) provide an extensive discussion of these credibility problems and of how the timing assumed here could be enforced through the design of political institutions that delegate policymaking to an elected official.

13. Remember that labour force is normalized to unity at economy's level.

14. The term 'competitive' suggest that prices are taken as given, while the term 'decentralized' that private agents may not internalize externalities. For a more detailed presentation see Blanchard and Fischer (1989, p. 76).

15. This is a standard practice in the literature. For example see Kehoe (1987, p. 361).

16. The effect of changes in the technology level, A, the depreciation rate, δ and the rate of time preference, ϱ, will not be examined, given that they do not affect the tax rate and that their effect on the growth rate is trivial: As in all models of the AK family, changes in A affect the growth rate positively, higher δ wipes out capital faster, while a lower ϱ means that people care more about the future, hence they decide to consume less and invest more. Moreover, the effect of changes in private capital's productivity, α, is also not studied, in order to focus on the free riding problem, brought about by the existence of the 'international' public factor, the effect of which is described by ω.

17. In cooperation the model turns to be a version of Barro's model.

18. Alternatively, equation (15) may be written as: $\tilde{\tau}-\tau^* = \frac{\alpha(1-\alpha)(1-\omega_{j,j})}{\alpha+(1-\alpha)\omega_{j,i}}$, where $\omega_{j,j}=\omega$ and $\omega_{j,\iota}= \frac{1-\omega}{n-1}$.

19. Note that *ex-post* symmetry is a crucial assumption to end up with such a result. In case economies are allowed to differ both *ex-ante* and *ex-post*, cooperation may not prove to be welfare improving for some countries.

20. The vertical axe depicts changes percentage when switching from the Nash equilibrium to cooperation, i.e. $\frac{\tilde{U}-U^*}{U^*}$. One may easily prove that $\frac{\tilde{U}-U^*}{U^*} = \frac{\tilde{g}-g^*}{g^*}$, i.e. the vertical axe also represents gains percentage in the growth rate. The growth rate under cooperation is assumed to be 3%, the productivity of private capital 0.8 and it is assumed there is no utility attained at t=0. In particular, it is assumed that: $\alpha=0.8$, $A=0.275$, $\delta=\varrho=3\%$ and $c_0=1$.

21. Aghion and Howitt (1998), for example, when presenting the case of redistribution (p. 284-286) adopt a similar policy rule.

22. They do so taking as given the predetermined redistribution parameter, z.

23. Under the rule $Z_\lambda=z(\bar{y}-y_\lambda)$, the budget constraint in transfer payments, $Z_j+Z_i=0$, always holds.

24. In each figure, 1-α equals some constant (the most reasonable case would be the one depicted in figure 3b, where $1-\alpha=0.2$ - cases where $1-\alpha>0.5$ are not depicted as not realistic) and five lines are drawn for different values of 1-ω (the most reasonable case would be the one depicted by the purple line, where $1-\omega=0.2$ - cases where $1-\omega>0.5$ are not presented, because it is not realistic to assume the foreign-financed public factor to be more productive than the domestically financed public factor), each one depicting the relationship between z and τ^N.

25. For higher values of 1-α the lines become steeper, as the moral hazard problem intensifies. This is not obvious in the figures, due to the different values in the vertical axis in each figure. The specific presentation is chosen in order to underline the negative relationship in every case.

26. We do not study how that difference reacts for different 1-α values, because there is no interesting rationale behind this relationship. Using numerical examples one may check that for higher 1-α values the difference increases, reaches some maximum point, and then starts to decrease.

27. The transfer payment scheme, however, still influences private agents' incentives.

6. Bibliography

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