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*SPOUDAI*
*Journal of Economics and Business*
Σπουδαί
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# The Sensitivity of Value-at-Risk Estimates Using Monte Carlo Approach

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**Abstract**

This study examines the sensitivity of VaR estimates obtained with Monte Carlo technique using the data set of Benninga and Wiener (1998) and applies the Kupiec test either by assuming large sample properties or by obtaining *p-values* through simulation process.

**JEL Classification:** G32.
**Keywords:** VaR, Monte Carlo method, Kupiec test.

## 1. Introduction

The concept of measuring the risk of a portfolio is well determined by the Value at Risk (VaR) method which determines the worst expected loss over a given horizon under normal market conditions at a given confidence level. VaR provides users with a summary measure of market risk, which is expressed by a single number defining firm's exposure to market risk as well as the probability of an adverse move.

Approaches to VaR analysis can be classified basically into two groups: a) with local valuation and b) with full valuation. Local valuation methods measure risk by valuing the portfolio once, at the initial position and using local derivatives to infer possible movements. Full valuation methods measure risk by fully reprising the portfolio over a range of scenarios and it is implemented either by the historical simulation method or by the Monte Carlo simulation method. The pros and cons of local versus fully valuation methods are well presented and discussed in literature by Keith *et al.* (2006). However, a full valuation method, and, particularly, the Monte Carlo simulation method, is strongly preferred in practice since it covers a wide range of possible values in financial variables taking into account correlations.

This paper examines the sensitivity of VaR estimates using the Monte Carlo technique based on the data set of Benninga and Wiener (1998). Furthermore,

these VaR estimates are used to apply the Kupiec test, as also suggested by Veiga and McAleer (2008), either by assuming large sample properties or by obtaining *p-values* through a simulation process as a backtesting procedure to verify the accuracy of the model, see also Campbell (2005), where both types of tests were properly formed in accordance with Costello *et al.* (2008). The whole simulation process was implemented using *Mathematica* following Shaw (2011).

## 2. Monte Carlo approach

The Monte Carlo simulation technique is by far the most powerful method used to obtain VaR estimates of a portfolio, since it can increase the accuracy of determining VaR. Based on some information about the parameters of a portfolio a large number of scenarios is produced. For each scenario the value of the portfolio is calculated and the entire probability density function is well defined from which VaR arises as the lowest $q$-quantile of this distribution. Therefore, it will be very interesting to evaluate this procedure and to examine how VaR is going to be affected by the number of simulations.

Using the algorithm and the data set of Benninga and Wiener (1998) for 1,000, 5,000 and 10,000 trials, as well as for 1%, 5%, 10% and 20% levels of significance the VaR of portfolio is computed and it is reported on Table 1. Actually, as the number of trials increases, one expects to get more accurate results of VaR. However, as it turns out from Table 1, VaR is not significantly affected by the number of trials, as it is affected by the level of significance. For example, for 5% level of significance the expected percentage losses remain at the level of 8.55% regardless of the number of simulations. Only for the 20% level of significance VaR chances slightly from -3.76% to -4.04% using 1,000 and 5,000 trials respectively and it remains at that level even for 10,000 trials, a change that it is reasonably expectable due to the chosen high level of significance.

**TABLE 1**

Estimation of VaR

| | **Number of trials** | | |
|---|---|---|---|
| **Level of significance** | **1,000** | **5,000** | **10,000** |
| **1%** | -12.0836 | -12.1625 | -12.3708 |
| **5%** | -8.5352 | -8.6202 | -8.5475 |
| **10%** | -6.5284 | -6.5411 | -6.4594 |
| **20%** | -3.7636 | -4.0453 | -4.0311 |

On the other hand, for given number of trials the expected percentage loss increases as the level of significance rises. For example, for 5,000 trials the percentage losses range from 4% to 12% approximately for 20% to 1% levels of significance respectively. Therefore, it is very interesting to note that for this data set VaR estimates are only determined by the level of significance and not by the number of trials.

## 3. The Kupiec test

Kupiec (1995) has developed a test to verify the accuracy of the model used to determine the risk of a portfolio based on the proportion of times VaR is exceeded in a given sample. For level of significance, $p$, chosen for determining VaR, the test is based on the following null hypothesis:

$$H_0: \; N/T = p \tag{1}$$

against the alternative:

$$H_1: \; N/T \neq p \tag{2}$$

where $N$ is the number of times in which the loss of a portfolio exceeds VaR for sample size of $T$ observations and $N/T$ is called the failure rate. The test is implemented using a log-likelihood ratio test and the $LR$ statistic is defined as:

$$LR = -2\ln\left[(1-p)^{T-N}\, p^{N}\right] + 2\ln\left\{\left[1-(N/T)\right]^{T-N}(N/T)^{N}\right\} \tag{3}$$

where $LR$ follows asymptotically a chi-squared distribution with one degree of freedom, i.e., $LR \sim X^2_1$, under the null hypothesis that $p$ is the true probability. The null hypothesis will be rejected if the value of $LR$ statistic is greater than the critical value obtained by the chi-squared distribution with one degree of freedom for given level of significance.

Using the algorithm and the data set of Benninga and Wiener (1998) the estimates of VaR are used to calculate values of the $LR$ statistic which are found to be 0.63979, 0.25947 and 0.59056 for levels of significance 5%, 10% and 20% respectively. Hence, the null hypothesis will be accepted since the values of the $LR$ statistic are smaller than the critical value obtained from the chi-squared distribution, i.e., for 5% level of significance the critical value is $X^2_{1,0.05} = 3.84$.

In addition, during this process one can obtain *p-values* and number of times in which the loss of a portfolio exceeds VaR. In particular, based on the values of the $LR$ statistic the *p-values* are equal to 0.4422, 0.6105 and 0.4238 and the number of times in which the loss of a portfolio exceeds VaR is equal to ten, five

and one for 20%, 10% and 5% levels of significance used to compute VaR respectively. Also, it is interesting to indicate that the value of the LR statistic, and hence the *p-value* of the test, has not been affected by the number of trials, since the estimate of VaR remained almost unchanged.

The backtesting procedure described above depends critically on the sample size. However, since the number of observations for this data set is very small, it is better to apply the test based on Monte Carlo simulated *p-values* rather than those obtained from the $X^2$ distribution. As indicated by Christoffersen, P. F. (2003), the simulated *p-values* for this test can be calculated by first generating 999 samples of random *i.i.d.* Bernoulli*(p)* variables, since each exception follows a Bernoulli distribution with parameter *p.* Based on those artificial samples, the 999 simulated values of the *LR* statistic, called them $\left\{\widetilde{LR}(i)\right\}_{i=1}^{999}$ , can be calculated. Then, the simulated *p-value* occurs as the percentage of the simulated $\widetilde{LR}$ values that are greater than the true value of *LR* and it is computed as follows:

$$p-value=\frac{1}{1000}\left\{1+\sum_{i=1}^{999}1\left(\widetilde{LR}(i)>LR\right)\right\} \tag{4}$$

where 1(•) takes on the value of one if the argument, $\widetilde{LR}(i)>LR$ , is true and zero otherwise.

The simulated *p-values* for the backtesting procedure of the Kupiec test based on the previously obtained estimated values of VaR are reported on Table 2 and they are defining the minimum probability of accepting the null hypothesis for given level of significance and for given number of trials. These values are all significantly greater than 5%, indicating that the null hypothesis cannot be rejected and therefore the model used to compute VaR is considered to be very reliable. For example, for 20% level of significance and for 1,000 simulations the minimum probability of accepting $H_0$ is equal to 0.799.

**TABLE 2**

Simulated *p-values*

| | **Number of simulations** | | |
|---|---|---|---|
| **Level of significance** | **1,000** | **5,000** | **10,000** |
| **5%** | 0.268 | 0.536 | 0.543 |
| **10%** | 0.319 | 0.537 | 0.557 |
| **20%** | 0.799 | 0.863 | 0.863 |

Moreover, it is interesting to note that the minimum probability of accepting $H_0$ increases as the number of trials increases, for given level of significance of computing VaR. For example, for 5% level of significance the *p-value* increases from 0.268 to 0.536 using 1,000 and 5,000 simulations respectively. Also, as the level of significance used to compute VaR increases the minimum probability of accepting $H_0$ increases for given number of trials. For example, for 5,000 trials the *p-value* of the test becomes from 0.536, for 5% level of significance, to 0.863, for 20% level of significance.

## 4. Concluding remarks

The Value-at-Risk method is a very powerful tool which gives the ability to risk managers to understand and measure risk. This study examined the performance of VaR using the Monte Carlo technique based on the data set of Benninga and Wiener (1998) and it finds that the determination of VaR it is not sensitive to the number of trials used to calculate its value. In addition, the Kupiec test, as a backtesting procedure to verify the accuracy of VaR, is applied to the data set either by assuming large sample properties or by obtaining *p-values* through a simulation process and in both cases the result was to accept the null hypothesis that the model estimates risk properly.

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