CASH MANAGEMENT AND THE DEMAND FOR MONEY WITH NON - LINEAR TRANSFER AND OPPORTUNITY COSTS

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I. INTRODUCTION

Statistical models of money demand, whether of the types proposed by Friedman (5) and Meltzer (10) or by Baumol (1) and Tobin (21), assume implicitly that the costs of both holding and exchanging money balances for interest - bearing assets are linear. This assumption simplifies the computation and interpretation of elasticities of the demand for money, but its validity is highly questionable. Standard estimates of interest and transactions elasticities of the demand for money are therefore likely to be biased and, in principle, inherently unstable. The present paper seeks to elucidate these possibilities as well as investigate the implications that emerge when the linearity premise is relaxed. This is done by combining the deterministic transactions approach with the postulate that the costs of portfolio adjustment are essentially non - linear. Moreover, although the mathematical forms of the non - linear costs assumed herein are specific, they are sufficiently general to indicate the importance of extending the state of the art in this respect.

The analysis derives several results. On the methodological plane it shows that the predictions one can obtain from the transactions model about the elasticity of the demand for cash with respect to interest rate and value of transactions are not necessarily some fixed parameters. Thus the drawback of the original or later versions[1] of the model, predicting that interest and transactions elasticities must be equal precisely to a given number, is easily dealt with. Proceeding along the same lines it is also suggested that, since the interest and transactions elasticities turn out to be functions rather than parameters, most conventional log - linear models are misspecified. On the somewhat more practical plane, the revised model is used

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1. See, for example, the studies by Eppen and Fama (3), Orr (16) and Weitzman (24).

to indicate that some of the observed intra - sectoral differences in the estimated elasticities can be successfully reconciled. Last, but not least, non - linear specifications of the transfer and opportunity costs are found to be consistent with the available, although admittedly not overpowering, evidence on «brokerage fees» and «rates of return».

Since the deterministic inventory model with linear costs can serve as the point of departure for the results to be presented, section II summarizes its main features, drawing from Baumol's work (1). In section III, the assumption of linearity of transfer and opportunity costs is dropped and some nonlinear forms are investigated instead. Section IV examines the implications of the non - linearity premise for the estimates of interest and transactions elasticities. Finally, the conclusions and some evidence concerning the non-linearity of transfer and opportunity costs are contained in section V and the Appendix, respectively.

II. INVENTORY - THEORETIC DEMAND FOR MONEY : A SUMMARY.

Taking cash to be a non - interest - bearing asset, individuals and firms would like to keep their liquid balances continuously in income - earning assets. Yet, they hold some of it in cash to meet current expenditures. This is due to the fact that every transfer of cash from and to an interest-bearing asset involves a cost, however small. In this framework, the logical question raised is : If an economic agent acts to minimize the costs of using cash, how much should he hold in order to be able to make payments without delay?

In order to delimit the problem, the following assumptions are set forth :

Assumption 1 There are only two financial assets : a) an earning asset such as government securities which bears interest at the rate of i dollars per dollar per period, and b) cash with which payments are made.

Assumption 2 The decision - maker pays out a predetermined amount of T dollars per year, for example, in a steady stream.

Assumption 3 Cash is obtained by liquidating the earning asset in lots of C dollars spaced evenly throughout the year.

Assumption 4 a) Every transfer of funds between the two assets involves some «brokerage fee» or transfer cost of X dollars; b) this charge is the same for buying and for selling of securities but it may be either constant or an increasing linear function of C.

Assumption 5 Transfer of funds between the two assets can take place instantaneously, i.e., the lead time is short enough to be ignored.

The analysis then proceeds to determine the expression describing the total costs the decision - maker must bear for the use of cash.

These costs consist, of course, of the interest lost in keeping money idle and the amount paid for cash transfers. The interest cost equals $\frac{iC}{2}$, because if the C dollars are spent in a steady stream, the average cash holding is $\frac{C}{2}$ dollars. If we let X = b, and there are $\frac{T}{C}$ transfers during the year (see assumptions 2 and 3), the transfer costs amount to $\frac{bT}{C}$. Thus the expression sought is given by

$$(1) \qquad M = \frac{bT}{C} + \frac{iC}{2},$$

where M stands for total user cost of cash [2].

Minimization of (1) with respect to C yields

$$(2) \qquad C = \left[\frac{2bT}{i} \right]^{1/2},$$

the familiar «square root rule» of the problem of optimal lot size in inventory theory. Consequently an individual or a firm must hold cash in an average amount of [3]

$$(3) \qquad C^* = \frac{C}{2} = \left[\frac{bT}{2i} \right]^{1/2}$$

in order to cover a total value of transactions T.

The effect of the interest rate and the value of transactions on the demand for money can now be determined from the resulting solution. Letting n_i and n_T be the interest and transactions elasticities respectively, it follows from (2) that

$$(4.a) \qquad n_i = \frac{i}{C} \frac{\partial C}{\partial i} = \frac{-1}{2} \quad \text{and}$$

$$(4.b) \qquad n_T = \frac{T}{C} \frac{\partial C}{\partial T} = \frac{+1}{2}.$$

That is, the model predicts constant interest and transactions elasticities of iden-

2. It is easy to show that in case X = b + aC, where b and a are both constants, the results will remain unchanged.

3. Formula (3), of course, gives the long-run demand for cash.

4. This implication is indeed very restrictive because it presumes that a small investor is as efficient in cash management as a big investor and that changes in the level of interest do not affect one's behavior towards the holding of cash.

tical magnitude but opposite sign. These elasticity values have been criticized on the grounds that a) they imply no variability in the cash-holding behavior of individuals or firms in response to changes in the interest rate and/or value of transactions [4], and b) the values consistently encountered in various empirical investigations of the corresponding elasticities are nowhere near $|^1/_2|$. Moreover, the prediction that the absolute magnitude of the two elasticities will be identical is in itself troublesome [5]. Thus, as formulated by Baumol, the model has some features that limit substantially its predictive ability.

III. DETERMINISTIC CASH FLOWS WITH NON-LINEAR TRANSFER AND OPPORTUNITY COST

This section shows that the weaknesses of the model which have just been discussed can be overcome by dropping the assumption that transfer costs are linear. We start with the alternative postulate that

(5) $$X = AC^n \text{ where } 0 \leqslant n \leqslant 1,$$

where $\underline{A}$ and $\underline{n}$ are positive parameters. Observe that the assumption of constant transfer costs is a special case of (5) corresponding to $n = 0$. On the other hand, n is restricted from being equal to 1, for this would suggest holding zero cash balances, as one would thereby save on interest costs. Thus (5) suggests that transfer costs increase but at a declining rate.

Ignoring for the moment the empirical validity of the postulated form, if (5) is used in connection with (1) one obtains

(6) $$M = AC^n \left(\frac{T}{C}\right) + \frac{iC}{2}.$$

The demand for money then becomes

(7) $$C = \left[\frac{2(1-n)AT}{i}\right]^{\frac{1}{2-n}},$$

from which we obtain

(8.a) $$n_i = -\frac{1}{2-n} \quad \text{and}$$

(8.b) $$n_i = +\frac{1}{2-n}.$$

5. Most empirical studies find money demand elasticities on the order of + 1, with respect to the transactions variable, and nearer to 0, with respect to the interest rate.

These elasticity expressions clearly are variable, with their possible values ranging from $|^1/_2|$ almost to $|1|$. In addition, (8.a) and (8.b) indicate most clearly the direct dependence of the elasticity figures on the structure of transfer costs.

At this point we should like to suggest that the transfer cost does not depend solely on the optimal size of cash withdrawals C. Some influence is also exercised by the value of transactions T. This suggestion rests on the view that the middleman may give large customers, i.e., customers with a large value of transactions T, preferential treatment compared with smaller customers. Indeed, as suggested in the Appendix, the practice of giving volume discounts from transfer costs is the rule rather than the exception among dealers and brokers. On these grounds, and for reasons to become obvious presently, we propose to amend (5) to read

$$X = AC^n e^{-\frac{m}{2}\left(ln \frac{T}{T_0}\right)^2}, \tag{9}$$

where $\underline{e}$ represents the basis of natural logarithms $\underline{ln}$, $\underline{m}$ is a parameter, and T_0 is a minimum level of transactions below which, for every possible optimal lot C, the transfer costs exceed the earnings that would result from investing surplus funds so that it pays to keep them in cash. Repeating the preceding steps after replacing (5) by (9), the cost expression becomes

$$M = AC^n e^{-\frac{m}{2}\left(ln \frac{T}{T_0}\right)^2}\left(\frac{T}{C}\right) + \frac{iC}{2} \tag{10}$$

yielding

$$C = \left[2\,(1-n)\,ATe^{-\frac{m}{2}\left(ln \frac{T}{T_0}\right)^2}\right]^{\frac{1}{2-n}} \tag{11}$$

$$n_i = -\frac{1}{2-n}, \quad \text{and} \tag{12.a}$$

$$n_T = \frac{1}{2-n}\,[1-m\,ln\,\frac{T}{T_0}], \tag{12.b}$$

which represent the demand for cash, and the interest and transactions elasticities, respectively [6].

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6. At this point the reasons for expanding (5) to read as in (9) should be clear. Raising $\underline{e}$ to the power - m (lnT/T) makes (5) shift upward or downward according as $T \lessgtr T_0$. This feature of the specification captures the essence of the assertion that the relative size of the investor does influence transfer costs. Had we, however, stopped here, the resulting elasticities would

This solution is characterized by three very desirable properties. First, it permits the transactions elasticity to vary from $-\infty$ to $+\infty$ according as $T \lesseqgtr T_0$. Second, the elasticities, no longer identical in size, may be identified separately. Third, and most important, by making the transactions elasticity a function of T, the solution is able to account for the possibility that firms with different assets or volume of sales and households of different wealth may have different propensities for holding cash. In this respect (12. b) suggests that larger firms and wealthier households, their size being measured by T, can be expected to hold less cash per dollar of T than smaller firms and less wealthy households. However, two objectionable features still remain : a) the interest elasticity can vary only between $-1/2$ and -1, while, as mentioned earlier, recent studies report interest elasticities closer to zero, and b) the predicted cash - holding behavior of firms or individuals continues to be invariant with respect to changes in the level of the interest rate, since neither ni nor nT depend on i. This rigidity can be ameliorated by relaxing the postulate (i.e., assumption 1, part (a), on page 3) that the opportunity costs, as represented by the interest rate, are independent of the size of the cash managing unit.

Specifically, to keep the analysis simple [7], we propose to set

$$(13) \qquad i = B\left(\frac{T}{T_0}\right)^{qR} \qquad \text{for} \qquad 0 \leqslant qR \leqslant 1,$$

where B and q are positive constants and R stands for the return on an asset that is offered in perfectly divisible denominations, so that even the smallest investors can obtain it and i is now the opportunity cost as the relative size of an investor (T/T_0) tends to infinity. Deferring discussion of the empirical plausibility of this assumption for the Appendix, if (13) is substituted into (10), the cost minimization operations yield solution (14) :

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depend solely on the absolute rather than the relative size of the investor. Thus, in order to avoid this drawback we have raised e to the power $\frac{m}{2}(ln T/T)^2$. No doubt, one may be able to come up with alternative specifications having more general implications. Yet, equation (9) besides being easily testable, is very useful because it helps us discuss the issues while keeping thr analysis simple.

7. In the course of our inquiry we also experimented with several analytical forms from the class of learning curves, such as

$$i = B + (R - B)\, e^{-q \frac{T}{T_0}}$$

Nevertheless, such specifications make the signs of certain crucial derivatives ambiguous and as such limit the analyticaltsimplicity of the solution.

$$(14) \qquad C = \left[2(1-n)\,DT \left(\frac{T}{T_0} \right)^{-qR} e^{\frac{-m}{2} \left(ln \frac{T}{T_0} \right)^2} \right]^{\frac{1}{2-n}}$$

with $D = AB^{-1}$

Finally, the interest and transactions elasticities from (14) are given by

$$(15.a) \qquad n_R = -\frac{q}{2-n} R\,ln \left(\frac{T}{T_0} \right) \quad \text{and}$$

$$(15.b) \qquad n_T = \frac{1}{2-n} \left[1 - m\,ln \frac{T}{T_0} - qR \right]$$

From (15.a) and (15.b) it appears that solution (14) generalizes the model in a way consistent with the tenets of economic theory. For instance, as Hansen (7) has suggested and Tobin (21) has subsequently accepted, the following derivative:

$$(16) \qquad \frac{\partial n_R}{\partial R} = -\frac{q}{2-n}\, ln \frac{T}{T_0} < 0 .$$

This predicts that, if the transfer and opportunity costs are as specified above, cash holders can be expected to become more sparing in their use of cash as the interest rate climbs to higher levels. A similar prediction is suggested by the following relationship

$$(17) \qquad \frac{\partial n_T}{\partial R} = -\frac{q}{2-n} < 0 .$$

Since the transactions elasticity varies inversely with the interest rate, *ceteris paribus*, cash holders can be expected to use less cash per unit of T at higher interest rates than they do at lower interest rates, and vice versa.

The cash - holding behavior that is implied by the model as the size of economic agents changes is also revealing. Differentiating (15.a) and (15.b) with respect to T there result

$$(18) \qquad \frac{\partial n_R}{\partial T} = -\frac{q}{2-n} \left(\frac{R}{T} \right) < 0 \text{ , and}$$

$$(19) \qquad \frac{\partial n_T}{\partial T} = -\frac{m}{2-n} \left(\frac{1}{T} \right) < 0 .$$

Expression (18) signifies that firms and households are more sensitive to changes in R at higher levels of T. If not for any other reason, this implication is as might be expected, since the ability to pay for and utilize information concerning investment opportunities increases with T. Expression (19), on the other hand, indicates that the returns to the scale of cash management vary directly with the size of the operation. Consequently, if there are economies of scale ($n_T < 1$), these will tend to increase (decrease) according as the volume of transactions increases (decreases).

This generalization of the deterministic inventory approach to the demand for money, besides being novel and going a long way towards meeting the familiar criticisms as summarized in [16], has also been undertaken for another reason. That is, it has been advanced in order to serve as a means by which to demonstrate that the common practice of ignoring the possibility of non - linearities may have been partly responsible for the existing conflicts in the empirical evidence that has been examined up to this point. To this task we now turn.

IV. IMPLICATIONS OF THE SUGGESTED NON - LINEARITIES FOR MONEY DEMAND

A closer look at formulae (15.a) and (15.b) reveals that the interest and transactions elasticities of money demand are completely variable, depending on both the interest rate R and the volume of transactions T. to the extent, therefore, that the postulated non - linearities are present, the elasticities that have been obtained in such previous investigations as [11], [25], [18] and [6], to cite a few, entail biases of unknown dimensions. This can be seen by contrasting model (20)

$$(20) \qquad lnC = a_{11} + a_{12}\, lnT + a_{13}\, lnR$$

which underlies most, if not all, estimated demand functions, with model (21)

$$(21) \qquad lnC = a_{21} + a_{22}\, lnT + a_{23}\, R + a_{24}\, (lnT)^2,$$

$$\text{for all } a_{ij} \text{ parameters but } a_{23} = \frac{q}{2-n}\,(lnT_0 - lnT),$$

which is suggested by our analysis. Obviously, even in the absence of definitional problems and sampling errors in the variables, a_{13} would diverge systematically from a_{23}. This is also true with repect to a_{12}, which may differ significantly from a_{22} due to the presence of the term $(lnT)^2$ in (21).

Besides the issue of bias, however, what is probably more important is that functions along the lines suggested by (20) are misspecified and, by implication, unable to pin down the characteristics of money demand sought. Observe, for example, that, so long as cash - management costs are non - linear, a relationship like (20) will be unstable, since a_{13} will shift every time the volume of transactions changes. Poole's [17] finding that it is impossible to obtain a firm estimate of the income elasticity may, in fact, be due largely to non - linearities that have been ignored. And the same may be true for Goldfeld's [6] results relating to the interest elasticities.

Taking explicit account of the transfer costs makes it obvious that model (20) may be unstable for still another reason. As formulae (15.a) and (15.b) reveal, the interest and transactions elasticities are directly related, through parameter

$\underline{n}$, to the cost structure of the securities industry. Whenever the costs of exchanging money balances for interest - bearing assets change say, for institutional reasons, parameter $\underline{n}$ will change in value and so will the corresponding elasticities. In a complete specification of model (21) one may then need to account for influences due to changes in the costs of the securities industry. This may be accomplished by adding one more equation to explain $\underline{n}$. In the absence of such an equation, simultaneous equations bias may affect estimates of the demand for money.

Aside from the light shed on the possibility of bias and instability in conventional money demand functions, the incorporation of non - linear costs extends substantially the predictive ability of the transactions model. Referring to the calculations presented in Table I below (for an explanation of the way these calculations were performed see the footnotes to this table) observe that, in contrast to the uniform prediction of scale economies in Baumol's [1] model, the solution obtained here predicts both economies (i.e., $n_T < 1$) and diseconomies (i.e., $n_T > 1$) of scale. For example, under the assumed parameter values and an interest rate of 5%, two households or firms with cash flows of \$ 5,000 and \$ 30,000 are projected to have $n_T = 1.38$ and $n_T = 79$, respectively. It turns out from these results that the prediction of economies of scale is not inherent to the inventory - theoretic approach as many people may have thought. The prediction of economies of scale by the original model has rather resulted from the customary assumption of linear transfer and opportunity costs.

As hinted to in the previous section and reiterated by our arithmetic calculations below the model also predicts diminishing diseconomies of scale up to a certain volume of transactions and increasing economies of scale thereafter. Contrasted to the existing empirical evidence this prediction supports the findings of Madala and Vogel [8] and contradicts those obtained by Meltzer [11], who has reported diminishing economies to scale for Business firms in the manufacturing sector of this country.

With respect to the interest elasticity of money demand, three characteristics of our solution call particularly for comment. First, observe that even if all cash management units in an economy had transactions volumes T equal to T_0, where $n_R = 0$, money demand would still be affected by changes in the interest rate. As the calculations in Table I show, some interest rate effects would be channeled to money demand through changes in the transactions elasticity e_T. This implication suggests that Friedman's [5] empirical finding of almost zero interest elasticity of the total demand for money is a necessary but may not be a sufficient condition for stability of the velocity of money, on which he and other proponents of the Quantity Theory have based much of their case. Second, notice that the interest rate elasticities for all $T < T_0$ are predicted to be positive. Although on first thought this prediction might appear to be questionable, in the context of our solution it is not hard to explain. From the definition of T_0 in section III above it follows that for all $T \leqq T_0$ the transfer costs are larger than the opportunity costs so that it pays to keep all of one's money balances in cash. A change in the interest rate then may induce people to adjust their demand for cash along their downward sloping demand curves but, so long as the opportunity costs continue to remain below transfer costs and all money balances have to be retained in cash, the demand

Table I

Hypothetical Predictions of nR and nT Obtained from the Revised Model[1].

Volume of Transactions T	Assumed Parameter Values[2] : n = .95, m = .11, q=3, T_0=20x10[3]							
	e_R				e_T			
	R = .03	R = .05	R = .07	R = .09	R=.03	R=.05	R=.07	R=.09
5×10^3	.051	.086	.120	.155	1.440	1.383	1.326	1.269
10×10^3	.026	.043	.060	.077	.898	.841	.784	.726
15×10^3	.011	.018	.025	.032	.879	.822	.764	.707
20×10^3	.000	.000	.000	.000	.867	.809	.752	.695
25×10^3	—.008	—.013	—.019	—.024	.856	.799	.742	.685
30×10^3	—.015	—.025	—.035	—.045	.848	.791	.734	.677
50×10^3	—.034	—.057	—.079	—.102	.824	.768	.711	.653
10×10^4	—.059	—.100	—.140	—.180	.793	.736	.679	.622
10×10^5	—.145	—.242	—.340	—.437	.688	.631	.574	.517
10×10^6	—.231	—.385	—.540	—.694	.583	.526	.470	.412
10×10^7	—.317	—.528	—.740	—.951	.479	.422	.365	.308
10×10^8	—.402	—.671	—.940	—1.208	.374	.317	.260	.203

Notes

1. The interest and transactions elasticity figures shown in this table were calculated using formulae (15.a) and (15.b).
2. In the calculations these parameter values have been used in conjunction with common logarithms.

curves must shift in order to provide for the necessary compensating effects. Thus, the positive interest elasticities of economic agents having volumes of transactions less than T_0 might be interpreted as measuring shifts in the individual demand curves for cash rather than movements along it. Thirdly, observe that the predicted values for n_R at each and every T increase at a decreasing rate as R rises. At the level T = $ 25,000, for example, the interest elasticity increases by 62.5 % as R moves from 0.03 to 0.05 and then the increase declines to 46 % as R jumps from 0.05 to 0.07. Since this feature may be taken to account for the progressive difficulty of reducing the usage of cash down to its technically minimum levels in response to economic stimuli, the flexibility of the revised transactions model can hardly be overemphasized.

Finally, Table I reveals that, even though the motives for holding cash by modest households and major corporations are held by the model to be the same, estimates of money demand functions for the household and corporate sectors, or any other disaggregation scheme for that matter, will be different, if the scale of the average cash management unit from the one sector to the other varies significantly. According to the numerical calculations presented, if the average household has an annual volume of transactions equal to \$ 20,000, application of the model to the household sector will tend to show zero interest elasticity of money demand and limited economies of scale (i.e., $n = .80$ at $R = .05$). An application of the model to the business sector on the other hand, where the average firm might have an annual volume of transactions equal to, say \$10,000,000, will probably reveal a significant interest elasticity of money demand (i.e., $n_R = -.38$ at $R = .05$) and substantial economies of scale (i.e., $n_T = .52$ at $R = .05$). It is clear, therefore, that the transactions model with non-linear costs provides a framework within which, aside from discrepancies due to definitions in the construction of variables and computational procedures, the observed differences in empirical estimates of interest and income elasticities of money demand among various sectors are found to be consistent with the assumed premises of rational behavior.

V. CONCLUSIONS

This paper has used the familiar Baumol - Tobin framework to investigate the implications of non - linear transfer and opportunity costs for the demand for money. The most significant results obtained are the following.

1. To the extent that portfolio - adjustment costs are non - linear, standard estimates of money - demand functions, which assume implicitly lumpy and/or proportional costs, are misspecified and thus involve biases and instabilities of unknown dimensions. In this respect, the proper model to estimate is not (20) but some variant of (21), depending on the particular assumptions one would care to make concerning the form of the pertinent non - linearities considered above.

2. The assumption of lumpy and/or proportional costs is highly questionable. According to the modest evidence reviewed in the Appendix, non - linear specifications of these cost relationships, such as (9) and (13), are closer to the facts than those using the conventional linearity premise. Specifically, the latter assumption may have been partly responsible for the difficulties that empirical research has faced in tracking down the interest and transactions elasticities of money demand.

3. The view that Baumol's model of the transactions demand for money lacks predictive, and therefore analytical substance, is unjustified. Our analysis has shown clearly that this critical view is directly related to the usual assumptions

of linear transfer and opportunity costs. Once the linearity premise is relaxed and replaced by non - linear specifications the model becomes very robust. In particular, the model becomes capable of dealing both with economies and diseconomies of scale as well as explaining the observed intra - sectoral differences in the elasticities that have been reported by most empirical investigations.

An obvious line of further inquiry would be to combine stochastic cash flows with non - linear costs. It stands to reason that, to the extent that this can be done, additional insights may be gained. However, this was not attempted here because the simpler analysis was sufficient to demonstrate our results.

APPENDIX

This appendix undertakes to present some elementary pieces of evidence that appear to lend support to the non - linearities hypothesis suggested in the text. As far as the nature of transfer costs is concerned, it is our contention that equation (9), or some other similar specification is nearer to the facts than the assumptions of «lumpy» and/or «proportional» transfer cost used in the literature. This contention may be based on the behavior of price spreads [8] in the money market and the current structure of brokerage fees in the capital market [9]. No doubt, besides the middleman's payment, a transaction often also incurs linearly behaving costs such as postage, bank service charges and other resources expended in making and effecting cash management decisions. These costs, however, are small relative to the total trading expenses and substantially fixed, so that they may be subsumed into the shift parameter of the proposed transfer cost function without loss of generality.

In the money market, where government securities and other highly liquid debt instruments are traded, transactions are made by firms serving as either large - scale dealers (i.e., buying and selling securities for their own accounts) or as commission brokers for customers. In the dealer's capacity these firms draw their income from the trading spreads between bid and asked prices and any capital appreciation that occurs while holding the securities. Strictly speaking, then, there are no explicit transfer costs charged in such transactions. The charge is simply included in the prices at which the transactions are settled. The spread may, however, serve as a very good proxy for transfer costs and its behavior should point to the likely nature of the dealer's transaction charges.

According to the account of Nadler and Engberg (13) :

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8. «Spread» is the difference between the bid and asked prices quoted by the dealers in the money market.

9. If the money market were perfectly competitive we would expect the spreads to be such that would just cover the dealers' costs of doing business.To the extent, therefore, that this market is imperfect the spreads will include elements other than pure costs.

> ...the daily quoted spreads by government securities dealers are outside quotations, while inside spreads, applicable to interdealer or large - lot transactions normally are considerably narrower. The inside market spread is *frequently* narrowed to 1/64 of a point and sometimes to 1/128 of a point, or about $ 78 on a one million dollar transaction. Occasionally, a dealer may sell at his bid price or buy at the asked price in order to acquire a large customer....

Obviously, to the extent that this quotation describes the normal pricing practices of dealers, transfer costs must be non - linear. For, if a $100,000 order is executed with a spread of 1/32 of a point, while an order of $1,000,000 involves a spread of 1/128, some non - linearities are present. Moreover, the possibility that transfer costs may vary with the size of the order and the business volume of a particular investor is also accepted by another eminent student in this field. As Malkiel (9) has put it,

> The cost per bond of executing a very large order is undoubtedly much less than for a small order. Thus, trading costs are not likely to be identical for all investors...

Thus, while brokerage charges *per se* are not explicitly specified in transactions taking place in the dealer's market, experienced investigators do believe that transfer costs in this section of the brokerage industry are non - linear.

When, however, orders are executed on a commission basis brokers impose specific charges that originate in the schedules of rates approved by the Securities and Exchange Commission. (23) The schedule that is currently in effect for trades in stocks certainly indicates that the structure of non - member commission rates is meant to be non - linear [10]. Table II in the next page summarizes the commissions that apply to 100 - share transactions in securities at various prices. Although this table relates only to a 100 - share transaction the structure of commissions follows the same pattern of change from 5 to 1,000 shares. That is, commissions increase at a decreasing rate up to certain price level and then remain fixed. In our table this occurs at $50. Since the average transaction involves less than 300 shares and an average price of $32.29 per share, it falls in the range of variable commissions [11]. Thus transactions of above average size are charged proportionately lower fees than are transactions of below average size.

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10. Non - linearities are also present in the suggested schedule of minimum non - member commission rates (19) for corporate bonds.

11. The study carried out by National Research Associates, Inc. (14), for example, found an average of 229 shares per order in the first half of 1969. On the other hand, the average price per share reported in the text is given in (22) and refers to the year 1972.

Table II
Non - Member Commission Rates

Price of Stock Per Share	Commission on a Round Lot of 100 Shares	% Increase in the Commission Charged on a Round Lot of 100 Shares as the Price per Share Increases
$ 10	$ 25.00	
$ 20	$ 38.00	52.0
$ 30	$ 49.00	28.9
$ 40	$ 58.00	18.3
$ 50	$ 65.00	12.0
$ 60	$ 65.00	0.0

The assertion that the transfer costs involved in the trading of stocks may be nonlinearly dependent on the size of the funds transferred is also supported, although indirectly, by the available evidence on the cost behavior of brokerage industry. For example, a study conducted by the New York Stock Exchange (15) finds that as the volume of a firm's transactions increases, total expenses increase less than proportionately [12]. Similar results have been reported by Demsetz (2) who suggested that, statistical analysis strongly indicates that the cost of exchanging a security declines as trading activity in that security increases. Later he also testified (23) to the existence of moderate economies of scale in the brokerage industry on the basis of the data shown in Table III.

Table III
Average Total Cost Per Transaction

Size of Firm by Number of Brokerage Transactions			Average Cost Per Transaction
up	to	40,000	$ 46
40,000	to	100,000	43
100,000	to	200,000	40
200,000	to	500,000	28
500,000	to	1,000,000	34
1,000,000	to	1,800,000	28
over		1,800,000	28

Source : H. Demsetz, testimony before the Securities and Exchange Commission, «Commission Rate Structure of Registered National Securities Exchanges, 1969.

12. Customers will enjoy, of course, whatever economies of scale exist to the extent that they are passed on to them through competition among brokerage firms.

Finally, as an additional indication that economies of scale may be present in this area we note that a volume discount was introduced in 1968.

The evidence discussed so far favors the narrow specification, equation (5), rather than the generalized form of equation (9). Obviously, the latter equation suggests that for any size of the cash lot C big customers are charged lower commissions than are smaller customers. In other words, the transfer costs do not depend solely on the size of the transferred funds, but they also depend on some measure of relative importance of the particular customers. Unfortunately, besides several instances of newspaper reports alleging that brokers do discriminate between small and big customers, no hard empirical evidence exists in this respect.

On the other hand, the specification of opportunity costs as in (13) may be an approximation superior to the linear specification that has been employed generally. This assertion is based on the observation that financial markets are so segmented by the denominations of the various credit instruments that small investors do not have access to some of them. While, for example, all firms and households can acquire interest - bearing assets in the form of time deposits in the banks, only the larger firms and households can buy certain money - market instruments that come in denominations of $10,000, $ 50,000 or even $ 100,000. Accordingly, since larger investors have a wider choice of investment opportunities than smaller investors, it is likely that the former may obtain higher returns than the latter.

Moreover, the substantial gap that exists between the interest paid by a bank on a regular passbook account and the rates offered by the same bank on large Certificates of Deposit suggests that the ability to trade in large denominations does result in a higher rate of return. Moreover, Evans (4) has found that, whether transactions costs are accounted for or not, the return to a portfolio adjusted for risk increases, though at a decreasing rate as its size expands. Thus, although the empirical evidence in support of this conjecture is not overpowering all indications point to the prevalence of the practices it describes.

In conclusion, this appendix has provided a number of reasons and some empirical evidence that appear to lend support to the view that transfer and opportunity costs are non - linear. Though the evidence discussed is certainly scanty, we believe it offers some presumption favoring the non - linearity premise over the linear treatment of transfer and opportunity costs.

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