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ALGEBRAIC SPECIFICATION OF THE ELASTICITY OF SUBSTITUTION FUNCTION AND THE EMPIRICAL BASIS OF THE h — CES PRODUCTION FUNCTION

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I. INTRODUCTION

Since the pathbreaking article by Arrow, Chenery, Minhas and Solow, hereafter referred to as ACMS (2)[1], a considerable professional interest has been concentrated on the empirical estimation of the elasticity of substitution between factors in production. The reason for this is that the value of the elasticity of substitution «σ» is a crucial parameter in the modern theories of growth, production, distribution and international trade. Moreover, the degree of substitutability between inputs in production is a decisive factor in choosing among different algebraic specifications of the empirical production function. For instance, if the value of «σ» is zero, the data under investigation are interpretted by *Leontief's model of fixed proportions,* while if the value of «σ» equals to unity the appropriate model to be used is the well known *Cobb - Douglas function.* A fixed value of «σ» between zero and infinity implies that the investigator should use a *constant elasticity of substitution function.* Finally, if the value of the elasticity of substitution is not constant the appropriate model to be used is the *variable elasticity of substitution production function.*

It follows that if the choice of the production function is made without previous empirical information concerning the value of the elasticity of substitution, the researcher automatically imposes a priori restrictions on the conclusions that will be drawn from the empirical investigation. It turns out that the estimation of the degree of substitution between factors in production is the first task of the empirical investigator in the field of production functions.

Part of this paper is based on my Ph.D Thesis (4).

1. Numbers in parentheses refer to the corresponding numbers of reference, as they are listed at the end of the paper.

The analysis of the present paper is limitted to the case where the elasticity of substitution is constant over all points of the production function. The purpose of the paper is twofold : *firstly* to derive a relationship for the empirical estimation of the elasticity of substitution that is subject to less restrictive assumptions than those used by ACMS and *secondly* to show that the derived relationship is the empirical basis of the non - constant returns to scale CES production function.

II. SPECIFICATION OF THE ELASTICITY OF SUBSTITUTION FUNCTION

1. *Preliminaries*

Arrow, Chenery, Minhas and Solow showed that under certain very restrictive assumptions the elasticity of substitution can be estimated easily from data on output, labour input and real wages. More specifically, they regress the logarithm of output per labour unit on the logarithm of the real wage rate, e.i.:

$$\log \frac{Y}{L} = \alpha + \sigma \log \frac{W}{P} + \lambda (1 - \sigma)\, t + u \qquad (1)$$

where $\frac{Y}{L}$ = value added per head at constant prices

$\frac{W}{P}$ = average wage rate deflated by the output price

σ = elasticity of substitution

λ = neutral technological change

t = time

α = constant

u = the disturbance term, which is assumed to obey all desirable properties

The main advantage of model (1) is that one does not need data on capital - which are usually the most difficult to obtain and the most unreliable - to estimate the elasticity of substitution. To derive the above very attractive formula ACMS used the following restrictive assumptions :

(i) there are constant returns to scale

(ii) there is perfect competition in product and factor markets

(iii) the objective of the production unit is to maximize profit

(iv) technological progress is Hicks - neutral

(v) output per head is independent of capital per head

Kmenta (5) pointed out that if we drop the assumption of constant returns to scale, then we cannot any more use the approach of ACMS to estimate the elasticity of substitution. In order to estimate the constant elasticity of substitution function for the case of non - constant returns to scale, hereafter referred to as h - CES production function, by simple least squares techniques, Kmenta uses a Taylor series expansion and by disregarding higher than second order terms he derived the following approximation for the h - CES function :

$$\log Y = \alpha + h\delta \log K + h(1-\delta)\log L - \frac{h\rho}{2}(1-\delta)\{\log K - \log L\}^2 + u \quad (2)$$

where : h = returns to scale parameter

δ = distribution parameter

ρ = substitution parameter

The substitution parameter 'ρ' is related with the elasticity of substitution by the formula :

$$\sigma = \frac{1}{1+\rho}$$

If $\rho = 0$, one can easily see that model (2) is reduced to the well known Cobb - Douglas production function.

Using model (2) one can estimate by linear estimation techniques all parameters of the h - CES production function. There are some deficiencies with this method, however. Firstly, equation (2) is an approximation of the h - CES function and not the function itself. However, this is not the only approximation of the h - CES function; Cobb - Douglas model is another approximation of the same function. Of course (2) is a better approximation than the Cobb - Douglas, but still is an approximation. Secondly, one should assume that errors resulting from the neglect of higher than second order terms are not serious and that they are uncorrelated with the disturbance term in the production function, otherwise one will face problems in estimating the function. Thirdly, for the estimation of model (2) one needs data on capital which are not easily avai lable.

Dhrymes (3) in order to relax the assumption of constant returns to scale postulated the following relation among real wages (W), output (Q) and labour input (L) :

$$W = AQ^{\beta} L^{\gamma} \quad (3)$$

Moreover, he showed that the elasticity of substitution 'σ' and the returns to scale 'h' parameters are given by the relations :

$$\sigma = -\frac{1}{\gamma} \text{ and } h = \frac{1+\gamma}{1-\beta}$$

Obviously, one can estimate 'σ' and 'h' from (3) by simple least squares techniques without using data on capital. However, Dhrymes did not derive model

(3) from the production function,as ACMS derived their relation for the estimation of 'σ'. He simply postulated it.

In the following section we shall derived a model for the estimation of the elasticity of substitution which does not require data on capital and the restrictive assumptions of constant returns to scale and perfect competition. In deriving the model we shall use as a starting point the concept of an unspecified production function, as ACMS did in their case.

2. *Derivation of the elasticity of substitution and returns to scale function*

Let the production function be :

$$Y = G(L, K) \tag{4}$$

where Y, L and K stand for output, labour and capital respectively.We assume that (4) is homogeneous of degree h and possecess the following properties :

$$\frac{\partial Y}{\partial L} > 0 \text{ and } \frac{\partial Y}{\partial K} > 0$$

$$\frac{\partial^2 Y}{\partial L^2} < 0 \text{ and } \frac{\partial^2 Y}{\partial K^2} < 0$$

$$\frac{\partial^2 Y}{\partial L\, \partial K} = \frac{\partial^2 Y}{\partial K\, \partial L} > 0$$

Under the assumption of homogeneity, function (4) can be written :

$$Y = L^h F\left(\frac{K}{L}\right) \tag{5}$$

If we let : $y = \frac{Y}{L^h}$ and $x = \frac{K}{L}$

equation (5) becomes :

$$y = F(x) \tag{6}$$

The marginal product of labour will be :

$$\frac{\partial Y}{\partial L} = L^{h-1}\{h\,F(x) - xF'(x)\} \tag{7}$$

We now assume that the production unit under consideration is a profit maximizer, but the markets in which it operates are not perfect. Then in equilibrium we shall have :

$$P \frac{\partial Y}{\partial L} = \frac{\eta_1 W}{\eta_0} \tag{8}$$

where W is the price of L, $\eta_0 = 1 - \frac{1}{e_0}$ and $\eta_1 = 1 + \frac{1}{e_1}$. Mereover e_0 is the price elasticity of demand for the output and e_1 is the elasticity of labour supply. Furthermore, we have :

$$\lambda_0 = \frac{\eta_0 P}{P} = \frac{\text{marginal revenue}}{\text{price of the product}} \tag{9}$$

$$\lambda_1 = \frac{1}{\eta_1} = \frac{W}{\eta_1 W} = \frac{\text{price of L}}{\text{marginal revenue of L}} \tag{10}$$

λ_0 can be interpreted as an index of the degree of product market imperfiction, since in equilibrium the term $1 - \eta_0 = + \frac{1}{e_0}$ registers the percentage by which marginal cost falls below price. Also λ_1 measures the degree of factor exploitation. If $\lambda_1 = 1$ labour is reciving its marginal revenue, while if $\lambda_1 < 1$ labour is exploited.

If $\frac{\partial Y}{\partial L}$ in (7) is expressed in value terms, then upon substitution and rearrangement we get :

$$\frac{1}{L^{h-1}} \cdot \frac{W}{\lambda_0 \lambda_1} = hF(x) - xF'(x) = z \tag{11}$$

Differentiating (11) with respect to y and taking into account (6) we obtain :

$$\frac{dz}{dy} = h - y' \frac{dx}{dy} - xy'' \frac{dx}{dy}$$

$$= \frac{hF'(x) - F'(x) - xF''(x)}{F'(x)}$$

Multiplying through by $\frac{y}{z}$ and rearranging we finally get :

$$\frac{z}{y} \cdot \frac{dy}{dz} = \frac{F'(x)\{h\,F(x) - xF'(x)\}}{(h-1)\,F(x)\,F'(x) - xF(x)\,F''(x)} \tag{12}$$

Furthermore, from (11) we have :

$$F(x) = \frac{z + xF'(x)}{h} \tag{13}$$

Let us now for convenience call the right hand side of (12) ρ. Then upon substitution from (13) we get :

$$\rho = \frac{hF'(x)\{hF(x) - xF'(x)\}}{(h-1)\,xF'^2(x) - hxF(x)\,F''(x) + (h-1)\,F'(x)\,z} \tag{14}$$

From (14) we obtain :

$$\frac{1}{\rho} = \frac{(h-1)\,xF'^2(x) - hxF(x)\,F''(x)}{hF'(x)\{hF(x) - xF'(x)\}} + \frac{(h-1)}{h} \tag{15}$$

The first term in the right hand side of (15) is equal to $\frac{1}{h\sigma}$, where σ is the elasticity of substitution [1]. Hence :

$$\rho = \frac{h\sigma}{1 + \sigma h - \sigma} \tag{16}$$

Equation (12) upon substitution and rearrangement becomes :

$$\frac{dy}{y} = \frac{h\sigma}{1 + \sigma h - \sigma} \cdot \frac{dz}{y} \tag{17}$$

Integrating (17) we get :

$$y = A^* z^{\frac{h\sigma}{1 + h\sigma - \sigma}} \tag{18}$$

where A^* is a constant of integration. Substituting for y and z, we have :

$$\frac{Y}{L^h} = A^* \left\{\frac{W}{\lambda_0 \lambda_1 L^{h-1}}\right\}^{\frac{h\sigma}{1 + h\sigma - \sigma}}$$

$$\text{or } Y = A^* \left(\frac{1}{\lambda_0 \lambda_1}\right)^{\frac{h\sigma}{1 + h\sigma - \sigma}} W^{\frac{h\sigma}{1 + h\sigma - \sigma}} L^{h - \frac{(h-1)\sigma h}{1 + h\sigma - \sigma}}$$

$$= A\, W^{\frac{h\sigma}{1 + h\sigma - \sigma}} L^{\frac{h}{1 + h\sigma - \sigma}}$$

$$= A\, W^{\beta} L^{\gamma} \tag{19}$$

where $A = A^* \left(\frac{1}{\lambda_0 \lambda_1}\right)^{\frac{h\sigma}{1 + h\sigma - \sigma}}$, $\beta = \frac{h\sigma}{1 + h\sigma - \sigma}$ and $\gamma = \frac{h}{1 + h\sigma - \sigma}$

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1) For the proof of this see appendix A.

Solving the last two relations in terms of β and γ we get:

$$\sigma = \frac{\beta}{\gamma} \text{ and } h = \frac{\gamma - \beta}{1 - \beta} \tag{20}$$

for $\beta \neq 1$

Equation (19) is the marginal productivity relation when the production function is homogeneous of degree h. It is obvious from 19 and 20 that we can estimate the elasticity of substitution "σ" and the homogeneity parameter h if we have data on output, the wage rate and the labour input.

In what follows we shall prove that equation (19) is the empirical basis for the estimation of the h-CES production function.

III. EMPIRICAL BASIS OF THE h-CES PRODUCTION FUNCTION

Taking lognrithms of (19) we obtain :

$$\log Y = \log A + \beta \log W + \gamma \log L \tag{21}$$

Moreover from (20) we have :

$$\gamma = h - h\beta + \beta \tag{22}$$

Substituting into (21) we get :

$$\log Y = \log A + \beta \log W + h \log L - h\beta \log L + \beta \log L$$

or

$$\log Y - h \log L = \log A + \beta \log W + \beta (\log L - h \log L)$$

Hence :

$$\log \frac{Y}{L^h} = \log A + \beta \{ \log W - \log L^{h-1} \}$$

$$= \log A + \beta \log \frac{W}{L^{h-1}} \tag{23}$$

$$\text{But } \frac{Y}{L^h} = y \text{ and } \frac{W}{L^{h-1}} = \lambda_0 \lambda_1 \{ hF(x) - xF'(x) \}$$

Substituting into (23) we get :

$$\log y = \log A + \beta \log \{ hF(x) - xF'(x) \} \tag{24}$$

(for the moment we put $\lambda_0 \lambda_1 = 1$)

from which we have :

$$\{ hF(x) - xF'(x) \}^{\beta} = \frac{y}{A} \tag{25}$$

Equation (25) can be written as :

$$hF(x) - xF'(x) = \left(\frac{y}{A}\right)^{\frac{1}{\beta}} = y^{\frac{1}{\beta}} A^{-\frac{1}{\beta}}$$

$$\frac{dx}{x} = \frac{dy}{hy - A^{-\frac{1}{\beta}} y^{\frac{1}{\beta}}} \tag{26}$$

Integrating both sides of (26) and simplifying we obtain[1]:

$$x = A^* [hA^{\frac{1}{\beta}} y^{1-\frac{1}{\beta}} - 1]^{\frac{\beta}{h(\beta-1)}} \tag{27}$$

Raising both sides of (27) to $\frac{1-\beta}{\beta}$ and simplifying we get :

$$x^{-\rho} = A' h A^{\frac{1}{\beta}} y^{1-\frac{1}{\beta}} - A' \tag{27a}$$

$$\text{where } \rho = \frac{h(1-\beta)}{\beta} \text{ and } A' = A^{*-\rho} \tag{28}$$

re-arranging terms we obtain :

$$y^{-\frac{\rho}{h}} = h^{-1} A^{-\frac{1}{\beta}} + \frac{1}{A'} h^{-1} A^{-\frac{1}{\beta}} x^{-\rho}$$

$$= h^{-1} A^{-\frac{1}{\beta}} \left(1 + \frac{1}{A'} x^{-\rho}\right)$$

$$= \alpha \left(1 + \frac{1}{A'} x^{-\rho}\right)$$

Hence :

$$y = \left(\alpha + \frac{\alpha}{A'} x^{-\rho}\right)^{-\frac{h}{\rho}}$$

$$= (\alpha + B x^{-\rho})^{-\frac{h}{\rho}} \tag{29}$$

But $y = \frac{Y}{L^h}$ and $x = \frac{K}{L}$. Substituting into (29) we obtain :

$$Y = L^h \left\{ B\left(\frac{K}{L}\right)^{-\rho} + \alpha \right\}^{-\frac{h}{\rho}}$$

$$= L^h \{ B K^{-\rho} L^{\rho} + \alpha \}^{-\frac{h}{\rho}}$$

$$= \{ B K^{-\rho} + \alpha L^{-\rho} \}^{-\frac{h}{\rho}} \tag{30}$$

1. The integral in the right hand side of (27) is estimated in the appendix B.

setting $\alpha + B = A_1^{-\frac{\rho}{h}}$ and $BA_1^{\frac{\rho}{h}} = \delta$ and substituting into (30) we get :

$$Y = \left\{ \frac{\delta}{A_1^{\frac{\rho}{h}}} K^{-\rho} + \frac{(1-\delta)}{A_1^{\frac{\rho}{h}}} L^{-\rho} \right\}^{-\frac{h}{\rho}}$$

$$= \{ A_1^{-\frac{\rho}{h}} [\delta K^{-\rho} + (1-\delta) L^{-\rho}] \}^{-\frac{h}{\rho}}$$

$$= A_1 \{ \delta K^{-\rho} + (1-\delta) L^{-\rho} \}^{-\frac{h}{\rho}} \qquad (31)$$

(31) is the standard form for the h-CES production function. Thus, we have proved that equation (19) is the empirical basis for the homogeneous of degree h-CES production function.

There are the following very interesting relationships between the parameters of (19) and (31) :

$$h = \frac{\gamma - \beta}{1 - \beta}, \quad \rho = \frac{1 - \sigma}{\sigma}, \quad \sigma = \frac{\beta}{\gamma}$$

Moreover, the constan term in (30) is given by :

$$\alpha = h^{-1} A^{-\frac{1}{\beta}}$$

where A is the constant term in (19). It turns out that we can estimate the most important parameters of (31) from equation (19).

In order to establish the relationship between (19) and (30) we disregarded the term $\lambda_0 \lambda_1$ from (24). Nothing will change if we introduce into the analysis this term as an index of markets imperfections. In such a case however the constant term A_1 and the distribution parameter δ in (31) will not only be determined by the technological conditions of production, but they shall reflect the institutional structure of the merket as well.

APPENDIX A

Let the production be given by :

$$Y = F(L, K) \qquad (A.1)$$

where Y stands for output and L and K denote the labour and capital input respectively. (A. 1) is assumed to be twice-differentiable and homogeneous of degree h. Under these assumptions we have :

$$Y = F(L, K) = L^h F\left(1, \frac{K}{L}\right) = L^h f(x) \qquad (A.2)$$

where $x = \dfrac{K}{L}$

Now define $y = \dfrac{Y}{L} = L^{h-1} f(x)$ (A. 3)

Then the elasticity of substitution is given by :

$$\sigma = \frac{\left(\frac{y}{x}\right)' \Big/ \left(\frac{y}{x}\right)}{y''/y'} = \frac{\frac{y'x-y}{x^2} \cdot \frac{x}{y}}{y''/y'} = \frac{y'(xy'-y)}{xyy''} \qquad (A.4)$$

We have to determine y' and y'' in terms of $f(x)$. From (A.3) we obtain :

$$\frac{dy}{dx} = y' = (h-1) L^{h-2} L' f(x) + L^{h-1} f'(x) \qquad (A.5)$$

Differentiating (A.2) for given Y we get:

$$h L^{h-1} L' f(x) + L^h f'(x) = 0$$

hence :

$$L' = -\frac{L^h f'(x)}{hL^{h-1} f(x)} = -\frac{Lf'(x)}{hf(x)} \qquad (A.6)$$

Substituting (A.6) in (A.5) one obtains :

$$y' = (h-1) L^{h-2}\left\{-\frac{Lf'(x)}{hf(x)}\right\} f(x) + L^{h-1} f'(x)$$

$$= -\frac{h-1}{h} L^{h-1} f'(x) + L^{h-1} f'(x) = \frac{1}{h} L^{h-1} f'(x) \qquad (A.7)$$

from (A.6) and (A.7) we have :

$$y'' = \frac{1}{h}\{(h-1) L^{h-2} L' f'(x) + L^{h-1} f''(x)\}$$

$$= \frac{1}{h}\left\{(h-1) L^{h-2}\left(-\frac{Lf'(x)}{hf(x)}\right) f'(x) + L^{h-1} f''(x)\right\}$$

$$= \frac{L^{h-1}}{h}\left\{f''(x) - \frac{h-1}{hf(x)} f'^2(x)\right\}$$

$$= \frac{L^{h-1}}{h^2 f(x)}\{hf(x) f''(x) - (h-1) f'^2(x)\} \qquad (A.8)$$

Substituting (A. 7) and (A. 8) in (A. 4) we get :

$$\sigma = \frac{L^{h-1} f'(x)}{hxL^{h-1} f(x)} \cdot \frac{x \frac{1}{h} L^{h-1} f'(x) - L^{h-1} f(x)}{\frac{L^{h-1}}{h^2 f(x)} \{ hf(x)f''(x) - (h-1)f'^2(x) \}}$$

$$= \frac{f'(x) \{ xf'(x) - hf(x) \}}{x \{ hf(x) f''(x) - (h-1) f'^2(x) \}}$$

$$= \frac{f'(x) \{ hf(x) - xf'(x) \}}{(h-1)\, xf'^2(x) - hxf(x) f''(x)} \tag{A. 9}$$

(A. 9) gives the elasticity of substitution when the production function is homogeneous of degree h. If the production function is homogeneous of degrec one, then put $h = 1$ in (A. 9) to obtain :

$$\sigma = -\frac{f'(x) \{ f(x) - xf'(x) \}}{xf(x) f''(x)} \tag{A. 10}$$

APPENDIX B

In this appendix we shall estimate the integral :

$$J = \int \frac{dy}{hy - A^{-1/\beta} y^{1/\beta}} \tag{B. 1}$$

For convenience we put $A^{-1/\beta} = b$ and $1/\beta = m$, then (B. 1) becomes :

$$J = \int \frac{dy}{hy - by^m} = \int \frac{dy}{y(h - by^{m-1})} \tag{B. 2}$$

$$dy = e^u \, du = y \, du$$

so

$$\frac{dy}{y} = du \tag{B. 3}$$

Substituting (B. 3) into (B. 2) we obtain :

$$J = \int \frac{du}{h - be^{(m-1)u}} \tag{B. 4}$$

put $h - be^{(m-1)u} = z$

then

$$dz = -b(m-1) e^{(m-1)u} du$$
$$= (1-m)\, be^{(m-1)u} du$$
$$= (1-m)(h-z) du$$

Hence

$$du = \frac{dz}{(1-m)(h-z)} \tag{B. 5}$$

Substituting (B. 5) into (B. 4) we get:

$$J = \int \frac{dz}{(1-m)(h-z)\ z} = \frac{1}{1-m} \int \frac{dz}{z\,(h-z)} \tag{B. 6}$$

To estimate (B. 6) we write:

$$\frac{1}{z(h-z)} = \frac{B}{z} + \frac{\Gamma}{h-z}$$

$$Bh - Bz + \Gamma z = 1$$

$$Bh = 1, \quad B = \frac{1}{h}$$

$$\Gamma - B = 0, \quad \Gamma = B$$

Now (B. 6) becomes:

$$J = \frac{1}{1-m} \left\{ \int \frac{dz}{hz} + \int \frac{dz}{h(h-z)} \right\}$$

$$= \frac{1}{(1-m)h} \left\{ \int \frac{dz}{z} + \int \frac{dz}{h-z} \right\}$$

$$= \frac{1}{(1-m)\,h} \left\{ \int \frac{dz}{z} - \int \frac{d\,(h-z)}{h-z} \right\}$$

$$= \frac{1}{(1-m)\,h} \{ \log|z| - \log|h-z| \} + C$$

$$= \frac{1}{(1-m)\,h} \log \left| \frac{z}{h-z} \right| + C \tag{B. 7}$$

$$= \log \left| \frac{z}{h-z} \right|^{1/h\,(1-m)} + C \tag{B. 8}$$

but

$$z = h - be^{(m-1)u} = h - by^{m-1}$$

Substituting into (B.7) we get:

$$J = \log \left| \frac{h - by^{m-1}}{by^{m-1}} \right|^{1/h\,(1-m)} + C$$

Now by (26) we have:

$$x = A^* \left(\frac{h - by^{m-1}}{by^{m-1}} \right)^{1/h\,(1-m)}$$

$$= A^* \left(\frac{h}{b} \; y^{1-m} - 1 \right)^{1/h\,(1-m)}$$

$$= A^* (hA^{1/\beta} \; y^{1-1/\beta} - 1)^{\beta/h\,(\beta-1)} \qquad \text{(B. 9)}$$

Equation (B. 9) is the same as equation (27) in the text.

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