CONSUMER DEMAND SYSTEMS : AN APPLICATION OF THE INDIRECT ADDILOG EXPENDITURE SYSTEM *

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1. The Theoretical Model

In this paper we shall present a complete set of demand equations which is different from those developed in [5], [6] and [7], in a fundamental respect : it permits marginal budget shares to vary with income.

A set of Engel curves, say $q_i\,(y)$ $(i = 1,\ldots,n)$, satisfies the «adding-up» condition if $\Sigma_{i=1}^{n} q_i\,(y) = y$. C.E.V. Leser (1942) pointed out that it is possible to transform any set of Engel curves which fail to satisfy the condition into a set which do, by using the transformation :

$$(1) \qquad g_i\,(y) = \frac{y q_i(y)}{\Sigma_{j=1}^{n} q_j\,(y)} \qquad (i = 1, \ldots, n).$$

This is obvious since $\Sigma_{i=1}^{n} g_i\,(y) = y$.

Because of the difficulty in estimating equation (1) Leser suggested taking pairs of commodities :

$$(2) \qquad \frac{q_i}{q_j} = \frac{g_i\,(y)}{g_j\,(y)} = \frac{q_i\,(y)}{q_j\,(y)} \qquad (i, j = 1,\ldots,n).$$

Now equation (2) is easier to estimate than equation (1). In particular, «the case of the double-logarithmic function is relatively simple because the ratio of two double-log functions is itself double - logarithmic»-Russell (1965, p. 17). That is, if

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$$(3) \qquad g_i(y) = \frac{y\,(\alpha_i\, y^{\beta_i})}{\Sigma_{j=1}^{n} \alpha_j\, y^{\beta_i})} \qquad (i = 1, \ldots, n).$$

then

$$(4) \qquad \frac{q_i}{q_j} = \frac{\alpha_i}{\alpha_j} \cdot y\,(\beta_i - \beta_j) \qquad (i, j = 1, \ldots, n);$$

which in logarithmic terms becomes

$$(5) \qquad \ln q_i - \ln q_j = (\ln\alpha_i - \ln\alpha_j) + (\beta_i - \beta_j)\ln y \qquad (i, j = 1, \ldots, n).$$

As we see we can estimate (5) very easily.

If we now generalize by introducing prices, equation (3) above becomes

$$(6) \qquad g_i(y) = \frac{\alpha_i y^{\beta_i + 1}\, p_1^{\gamma i1} \ldots p_n^{\gamma in}}{\Sigma_{j=1}^{n} \alpha_j y\, \beta_j\, p_1^{\gamma j1} \ldots p_n^{\gamma jn}} \qquad (i = 1, \ldots, n).$$

Houthakker (1960b), using the relation

$$(7) \qquad q_i = -\lambda^{-1}\,(\partial u^* / \partial p_i) \qquad (i = 1, \ldots, n),$$

which is known as Roy's identity, has found that, if certain relations between the coefficients of this function are satisfied, (6) can be integrated into the following indirect utility function :

$$(8) \qquad u^* = \Sigma_{i=1}^{n} \alpha_i\,(y/p_i)^{\beta_i}$$

where a_i and β_i are parameters with $\alpha_i < 0$, $-1 < \beta_i < 0$ $(i = 1, \ldots, n)$, $-\Sigma_{i=1}^{n} \alpha_i = 1$.

Empirical work has in fact been confined to this indirect utility function, which has been christened by Houthakker (1960a,p. 252) the Indirect -Addilog utility function. The demand functions obtained from this utility function have already been used by Somermeyer and Wit (1956), Somermeyer (1961), Russell (1965), Parks (1965), 1969), Yoshihara (1969), and Gamaletsos (1973).

The demand functions generated by (8) are of the form :

$$(9) \qquad q_i = \frac{\alpha_i\, \beta_i\, (y/p_i)^{1+\beta_i}}{\Sigma_{j=1}^{n} \alpha_j\, \beta_j (y/p_j)\, \beta_j} \qquad (i = 1, \ldots, n).$$

In terms of expenditures, the form is

(10) $$e_i = \alpha_i\beta_i\, y\, (y/p_i)^{\beta_i} \left(\Sigma_{j=1}^{n}\alpha_j\, \beta_j\, (y/p_j)^{\beta_j}\right)^{-1} \qquad (i = 1,\ldots,n),$$

It is convenient to reparametrize these equations into

(11) $$e_i\,(t) = \gamma_i\, y\,(t)\,(y(t)/\,p_i\,(t))^{\beta_i} \left(\Sigma_{j=1}^{n}\gamma_j\,(y(t)/p_j(t)^{\beta_j}\right)^{-1}$$

$$(i = 1\ \ldots,n\ ;\ t = 1\ldots,T)$$

where $\gamma_i = \alpha_i\beta_i$, and $\Sigma_{i=1}^{n}\gamma_i = 1$. The features of this indirect addilog model are given as follows. Differentiating (11) with respect to y we have

(12) $$\mu_i\,(t) = \partial e_i\,(t)/\partial y\,(t)$$

$$= \frac{\gamma_i\,(y(t)/p_i\,(t)^{\beta_i}}{\Sigma_{i=1}^{n}\gamma_j\,(y(t)/\,p_j(t))^{\beta_j}}$$

$$\cdot\left[(1+\beta_i) - \frac{\Sigma_{i=1}^{n}\beta_j\,\gamma_j(y(t)/p_j(t))^{\beta_j}}{\Sigma_{i=1}^{n}\gamma_j\,(y(t)/p_j(t))^{\beta_j}}\right]$$

$$(i = ,\ldots,n\ ;\ t = 1\ldots,T).$$

Equation (12) gives the income slopes of the expenditure equations (11), which depend on income and prices. These income slopes are the marginal budget shares for the Indirect Addilog Expenditure System-henceforth IAES model. The average budget shares for this model are

(13) $$w_i(t) = \gamma_i(y(t)/p_i(t))^{\beta_i} \left(\Sigma_{i=1}^{n}\gamma_j(y(t)/p_j(t))^{\beta_j}\right)^{-1}$$

$$(i=1,\ldots,n\ ;\ t = 1,\ldots,T).$$

The income elasticities, obtained as $\mu_i\,(t)\,/\,w_i(t)$ are

(14) $$\eta_i(t) = (1+\beta_i) - \Sigma_{j=1}^{n}\beta_j w_j\,(t) \qquad (i = 1,\ldots,n\ ;\ t = 1,\ldots,T).$$

From equation (14) we observe that differences between income elasticities, $\eta_i(t) - \eta_j\,(t) = \beta_i - \beta_j$, are constant over income and prices.

The uncompensated (Cournot) own-price elasticities are

(15) $$\eta_{ii}\,(t) = -(1+\beta_i) + \beta_i w_i(t) \qquad (i = 1,\ldots,n\ ;\ t = 1,\ldots,T).$$

The uncompensated (Cournot) cross-price elasticities are

(16) $$\eta_{ij}\,(t) = \beta_j\, w_j\,(t) \qquad (i \neq j)$$

$$(i, j = 1,\ldots,n\ ;\ t = 1,\ldots,T).$$

In this model, since it has an indirectly additive utility basis, cross-price elasticities do not depend on the good whose quantity is responding-i.e., η_{ij} is the same for all $i \neq j$ (1).

The compensated (Slutsky) own-and cross-price elasticities are given by

(17) $$\eta^*_{ij}(t) = \eta_{ii}(t) + w_i(t)\eta_i(t) \qquad (i = 1, \ldots, n \; ; \; t = 1, \ldots, T),$$

and

(18 $$\eta^*_{ij}(t) = \eta_{ij}(t) + w_j(t)\,\eta_i(t) \qquad (i \neq j)$$

$$(i, j = 1, \ldots, n \; ; \; t = 1, \ldots, T).$$

Because the β_i's must lie between -1 and 0, the income elasticities lie between 0 and 2, and the uncompensated own-price elasticities lie between 0 and -1.

The income elasticity of the marginal utility of income for the IAES model is given by

(19) $$\eta_{\lambda y}(t) = -1 + \Sigma_{j=1}^{n}\beta_j\,\gamma_j\,(y(t)/p_j(t))^{\beta_j}\left(\Sigma_{j=1}^{n}\gamma_j\,(y_j(t)/p_j(t))^{\beta_j}\right)$$

$$= -1 + \Sigma_{j=1}^{n}\beta_j\,w_j(t) \qquad (t = 1, \ldots, T),$$

which lies between —1 and —2. From (19) we obtain the «income flexibility» for the IAES model, which is given by

(20) $$\varphi(t) = \eta_{\lambda y}^{-1}(t)$$

$$= (-1 + \Sigma_{j=1}^{n}\beta_j\,w_j(t))^{-1} \qquad (t = 1, \ldots, T),$$

which lies between —0.5 and —1.

2. Stochastic Specification and Estimation of the Model

An attempt to empirically implement a complete system of demand functions by means of aggregate time series must also face up to the problem of stochastic specification.

An obvious problem under this heading concerns identification and simultaneity in multi-equation models. In the present context, the question arises as to whether observed relations between quantities, on the one hand, and prices and income, on the other, can be interpreted as demand functions. It appears possible that this problem can be approached by an interpretation of the demand functions

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(1) See Houthakker (1960a, pp. 244-256).

as conditional expectation functions, thus automatically ensuring the «classical» disturbance properties and justifying the use of least squares regression in estimation. However, further analysis of this problem is required and may necessitate some refinements.

A less obvious problem is concerned with the linkages among the demand equations themselves. If income is measured-as it will be in this paper-as the sum of the expenditures on each of the n goods-then there is an inherent correlation between the disturbances in the n demand functions. (If more than the expected amount is spent on one good, less than the expected amount must be spent on some other, since total expenditures are taken as given). There is a related problem concerned with heteroskedasticity, error variation presumably being larger for some items in the budget (durables, very likely) than it is for others. It appears that recent work by Barten (1969), Parks (1971), Pollak-Wales (1969) Berndt and Savin (1975), Powell (1973), LLuch and Williams (1975), LLuch and Powell (1975), approach to this set of problems.

Earlier work on estimation of the indirect addilog system-e.g. Houthakker (1960a)-has exploited the simplification which results from considering expenditure ratios. For the expenditure equations (11), for each pair i, j of commodities, these ratios are

$$(21) \qquad \frac{e_i(t)}{e_j(t)} = \frac{\gamma_i (y(t)/p_i(t))^{\beta_i}}{\gamma_j (y(t)/p_j(t))^{\beta_j}} \qquad (i \neq j)$$

$$(i, j = 1, \ldots, n \; ; \; t = 1, \ldots, T).$$

If we take logarithms in (21) then we have

$$(22) \qquad \ln (e_i(t)/e_j(t)) = \ln (\gamma_i/\gamma_j) + \beta_i \ln (y(t)/p_i(t)) - \beta_j \ln (y(t)/p_j(t)) \qquad (i \neq j),$$

$$(i, j = 1, \ldots, n \; ; \; t = 1, \ldots, T).$$

Now equations (22) are easy to estimate, but we must keep in mind that for each different pair of commodities we would get different estimates for γ_i's and β_i's. Indeed, using all the pairs of equations, we would get n—1 different estimates of the β_i's.

Parks (1969) handles this by estimating jointly the (n-1) equations of the form (22) above for $i = 1$ and $j = 2, \ldots, n$. Parks assumes a multiplicative disturbance in expenditure equations to get an additive disturbance in the equations expressed as differences of logarithms. That is, he gives the expenditure equations in stochastic form as follows

$$(23)\qquad e_i(t) = \frac{\gamma_i\,(y(t)/p_i(t))^{\beta_i}\, e^{\varepsilon_i(t)}}{\Sigma_j\, \gamma_j\,(y(t)/p_j(t))^{\beta_j}}$$

$$(i = 1,\dots,n\ ;\ t = 1,\dots,T),$$

with the assumptions

$$E(\varepsilon_i(t)) = 0,\qquad E(\varepsilon_i(t)\,\varepsilon_j(s)) = \begin{cases} \pi_{ij}(t) & \text{for } t = s \\ \pi_{ij}(t,s) & \text{for } t \neq s \end{cases}$$

$$(i, j = 1,\dots,n\ ;\ t,s = 1,\dots,T).$$

He then estimates the differences of logarithms of (n-1) pairs of commodities, namely, the

$$\ln(e_1(t)/e_j(t)) = \gamma_{1j} + \beta_1 \ln(y(t)/p_1(t))$$

$$-\beta_j \ln(y(t)/p_j(t)) + u_j(t)$$

$$(j = 2,\dots,n\ ;\ t = 1,\dots,T),$$

where $\gamma_{1j} = \ln(\gamma_1/\gamma_j)$ and $u_j(t) = \varepsilon_1(t) - \varepsilon_j(t)$.

For our purposes, however, it seems more natural to maintain an additive disturbance specification. We use an additive disturbance for interpreting each of the n expenditure equations as the conditional expectation of an $e_i(t)$ given $y(t)$, $p_1(t),\dots, p_n(t)$. To do so, we let $\varepsilon_i(t)$ = disturbance in i^{th} expenditure equation at time t and specify that $E\varepsilon_i(t) = 0$ independently of $y(t)$, $p_1(t),\dots, p_n(t)$, and

$$(24)\qquad E\varepsilon(t)\,\varepsilon'(s) = \begin{cases} \Sigma & \text{for } t = s \\ 0 & \text{for } t \neq s \end{cases}\qquad (t, s = 1,\dots,T).$$

That is we permit heteroskedasticity and contemporaneous correlation among the disturbances but assume independence over time. Now in stochastic form the IAES model (11) becomes

$$(25)\qquad e_i(t) = \gamma_i\, y(t)\,(y(t)/p_i(t))^{\beta_i} \left(\Sigma_{j=1}^{n} \gamma_j\,(y(t)/p_j(t))^{\beta_j}\right)^{-1} + \varepsilon_i(t)$$

$$(i = 1,\dots,n\ ;\ t = 1,\dots,T).$$

To fit such a system of n equations, a scalar criterion is required. We choose our estimates of γ's and β's so as to minimize the residual sum of squares across all observations across all equations. If we write $\hat{\gamma},\dots,\hat{\gamma}_n$ and $b_1,\dots, b_n$ for the estimates of $\gamma_1,\dots,\gamma_n$ and $\beta_1,\dots,\beta_n$ then our equations (25) above become

(26) $e_i(t) = \hat{\gamma}_i \, y(t) \, (y(t)/p_i(t))^{b_i} \, (\Sigma_{j=1}^{n} \hat{\gamma}_j \, (y(t)/p_j(t))^{b_i})^{-1} + v_i(t)$

$(i = 1,\ldots,n \; ; \; t = 1,\ldots,T),$

where $v_i(t)$ = residual in i^{th} expenditure equation at time t. We choose the $\hat{\gamma}$'s and b's so as to minimize $\Sigma_{t=1}^{T} \Sigma_{i=1}^{n} (v_i(t))^2$.

For fitting, we use the Gauss-Newton (1967) computer program, a straightforward non-linear regrrssion one. The program obtains a least squares fit, say $y = f(x_1,\ldots,x_m; \vartheta_1,\ldots,\vartheta_p) + v$ of a user specified function f to data values $x_1,\ldots,x_m,y$ by means of stepwise Gauss-Newton iterations on the parameters $\vartheta_1,\ldots,\vartheta_p$, allowing minimum and maximum constraints on these parameters. In fitting our model, we did not impose any of those minimum and maximum constraints. Within each iteration parameters are selected for modification in a stepwise manner. The parameter selected at a given step is the one which, differentially at least, makes the greatest reductions in the error sum of squares. Beginning with an initial set of parameter values $\vartheta = (\vartheta_1,\ldots,\vartheta_p)$ the program minimizes the error mean square

$$s^2 = \frac{1}{T-p} \, \Sigma_{t=1}^{T} \left[y_t - f(x_{t1},\ldots,x_{tm}; \vartheta_1,\ldots,\vartheta_p) \right]^2 .$$

The function f and its partial derivatives with respect to its parameters are needed. The convergence criterion for the error mean square is. 001 % which we considered as satisfactory. Generally we get fast convergence but have no guarantee that the minimum is the global one[1].

The computer program is a single-equation one. For this reason we must convert our system of n equations into one equation. We do so by making use of the «constructed variables» :

(1) As a matter of fact, in some cases when we started from different initial values of the parameters, we converged to different minima. So in this case it is better to talk about «local» minima and not just minimum. When we obtained different minima we adopted the smallest one.

$$e = \begin{bmatrix} e_1(1) \\ \vdots \\ e_1(T) \\ e_2(1) \\ \vdots \\ e_2(T) \\ \vdots \\ e_n(1) \\ \vdots \\ e_n(T) \end{bmatrix}, y = \begin{bmatrix} y(1) \\ \vdots \\ y(T) \\ y(1) \\ \vdots \\ y(T) \\ \vdots \\ y(1) \\ \vdots \\ y(T) \end{bmatrix}, p_1 = \begin{bmatrix} p_1(1) \\ \vdots \\ p_1(T) \\ p_1(1) \\ \vdots \\ p_1(T) \\ \vdots \\ p_1(1) \\ \vdots \\ p_1(T) \end{bmatrix}, \ldots, p_n = \begin{bmatrix} p_n(1) \\ \vdots \\ p_n(T) \\ p_n(1) \\ \vdots \\ p_n(T) \\ \vdots \\ p_n(1) \\ \vdots \\ p_n(T) \end{bmatrix}, z_1 = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ -1 \\ \vdots \\ -1 \end{bmatrix}$$

$$z_2 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \\ -1 \\ \vdots \\ -1 \end{bmatrix}, \ldots, z_{(n-1)} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \\ -1 \\ \vdots \\ -1 \end{bmatrix}, z_n = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, v = \begin{bmatrix} v_1(1) \\ \vdots \\ v_1(T) \\ v_2(1) \\ \vdots \\ v_2(T) \\ \vdots \\ v_n(1) \\ \vdots \\ v_n(T) \end{bmatrix}$$

$$,\omega_1 = \begin{bmatrix} 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \ldots, \omega_n = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}.$$

Each of these vectors is of dimension $nT \times 1$, where T is the number of observations and n is the number of commodities for each country. Individual elements of the vectors will be designated by the index k ($k = 1,\ldots,nT$).

In terms of the constructed variables, the system (26) is expressed as

$$\text{(27)} \qquad e(k) = \left\{ \omega_n(k) + \Sigma_{i=1}^{n-1} z_i(k)\hat{\gamma}_i \right\} \left\{ \Sigma_{i=1}^{n} \omega_i(k)\, y(k)\, (y(k)/p_i(k))^{b_i} \right\}$$

$$\cdot \left\{ \Sigma_{j=1}^{n-1} \hat{\gamma}_j \, (y(k)/p_j(k))^{b_j} + (1 - \Sigma_{j=1}^{n-1} \hat{\gamma}_j) \right.$$

$$\left. (y(k)/p_n(k))^{b_n} \right\}^{-1} + v(k) \qquad (k = 1,\ldots,nT).$$

Equation (27) uses the restriction $\Sigma_{i=1}^{n} \hat{\gamma}_i = 1$, so we estimate $(2n - 1)$ free parameters: $(n-1)$ γ_i's and n β_i's, while our independent variables are 3n in number: one y, n p's, n ω's, and $(n-1)$ z's.

The partial derivatives of equation (27) with respect to the parameters are of the form :

$$D_1(k) = \partial e(k)/\partial \hat{\gamma}_1$$

$$= X_2(k) X_3^{-1}(k) \{ z_1(k) - X_1(k) X_3^{-1}(k) \{ (y(k)/p_1(k))^{b_1} - (y(k)/p_n(k))^{b_n} \} \}$$

$$D_2(k) = \partial e(k) / \partial \hat{\gamma}_2$$

$$= X_2(k) X_3^{-1}(k) \{ z_2(k) - X_1(k) X_3^{-1}(k) \{ (y(k)/p_2(k))^{b_2} - (y(k)/p_n(k))^{b_n} \} \}$$

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$$D_{n-1}(k) = \partial e(k) / \partial \hat{\gamma}_{n-1}$$

$$= X_2(k) X_3^{-1}(k) \{ z_{n-1}(k) - X_1(k) X_3^{-1}(k) \{ (y(k)/p_{n-1}(k))^{b_{n-1}} - (y(k)/p_n(k))^{b_n} \} \}$$

$$D_n(k) = \partial e(k) / \partial b_1$$

$$= X_1(k) X_3^{-1}(k) [\ln (y(k)/p_1(k))] (y(k)/p_1(k))^{b_1} \cdot [\omega_1(k) y(k) - \hat{\gamma}_1 X_2(k) X_3^{-1}(k)]$$

$$D_{n+1}(k) = \partial e(k) / \partial b_2$$

$$= X_1(k) X_3^{-1}(k) [\ln (y(k)/p_2(k))] (y(k)/p_2(k))^{b_2} \cdot [\omega_2(k) y(k) - \hat{\gamma}_2 X_2(k) X_3^{-1}(k)]$$

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$$D_{2n-1}(k) = \partial e(k) / \partial b_n$$

$$= X_1(k) X_3^{-1}(k) [\ln (y(k)/p_n(k))] (y(k)/p_n(k))^{b_n} \cdot [\omega_n(k) - (1 - \Sigma_{j=1}^{n-1} \hat{\gamma}_j) X_2(k) X_3^{-1}(k)]$$

where

$$X_1(k) = \omega_n(k) + \Sigma_{j=1}^{n-1} \hat{\gamma}_j z_j(k),$$

$$X_2(k) = y(k)\,\omega_1(k)\,(y(k)\,p_1(k))^{b_1} + y(k)\,\omega_2(k)\,(y(k)/p_2(k))^{b_2} + \ldots$$

$$+\, y(k)\,\omega_n(k)\,(y(k)/p_n(k))^{b_n}$$

$$= y(k)\,\Sigma_{j=}^{n}\,\omega_j(k)\,(y(k)/p_j(k))^{b_j}$$

and

$$X_3(k) = \hat{\gamma}_1 (y(k)/p_1(k))^{b_1} + \hat{\gamma}_2 (y(k)/p_2(k))^{b_2} + \ldots$$

$$+\, \hat{\gamma}_{n-1} (y(k)/p_{n-1}(k))^{b_{n-1}} + (1 - \Sigma_{j=1}^{n-1} \hat{\gamma}_j)$$

$$\cdot\, (y(k)/p_n(k))^{b_n}\,.$$

Equation (27) is the one we estimate using the simple least squares criterion.

3. Data Base

The data which we use are based on the O.E.C.D. volumes (1964, 1967) of national accounts statistics for the years 1950-65. Goldberger and Gamaletsos (1970) drew on the same source, but only for the years 1950-61.

The five components of total consumer expenditures constitute our five commodities. These are Food, Clothing, Rent, Durables and Other. Further characterization of these categories is to be found in OECD (1958). The countries which we finally use are; Belgium, Denmark, France, Greece, Ireland, Italy, Netherlands, Norway, Sweden, United Kingdom and U.S.A.

4. Estimates

Estimates of the parameters of the IAES model are given in Table 1. In examining this table we see that 8 out of 55 $\hat{\gamma}$'s and 10 out of 55 b's are not significantly different from zero-on the convention used here, namely a t-ratio less than two in absolute value.

All $\hat{\gamma}$'s are positive as they should be according to the theoretical model. Of the b_i's 11 out of 55 are positive and 12 out of 55 are less than -1, contrary to the theoretical model. But 4 of those 11 positive b_i s are not significantly different from zero. Most of the positive b_i's appear for Durables.

The finding that estimates of the β_i parameters fall outside the theoretical model, does not appear for the first time in this paper. Houthakker (1960b) estimated the parameters of this model using equation (22). Of his 260 estimated β_i's, 55 are positive and 113 are less than -1. Parks (1969) estimated the parameters of this model using equetion (23) with three different assumptions about the disturbances. Assuming no correlation among the disturbances half of his estimated β_i's had the wrong sign or were less than -1. He also obtained the same results even when he took account of contemporaneous correlation of the disturbances. But when serial correlation was also treated, all of the estimated β_i's were found to lie between 0 and -1.

It seems after all this that a possible appreach would be to impose sign constraints in the fitting process.

In Table 2 we tabulate the marginal budget shares evaluated at the sample mean point $(\bar{y}, \bar{p}_1, \ldots, \bar{p}_n)$. In Table 3 we present the mean income and own-price elasticities, and in Table 4 we tabulate the terminal income and ownprice elasticities of the IAES model.

In examining Table 3 we observe that one out of 55 income elasticities is negative. This happened because the corresponding b_i (for France for commodity «Rent») is less than -2, and this together with the positive b_i for «Durables» makes the income elasticity for «Rent» negative [1]. All the remaining mean income elasticities are between 0 and 2 with the exception of 4 which are greater than 2. These are associated with the corresponding b_i's being positive. In examining the uncompensated mean own-price clasticities we observe that 8 out of 55 are positive. Also in the case of the compensated mean own-price elasticities 9 out of 55 are positive. This is because the corresponding b_i's are less than —1. Furthermore 10 out of the 55 uncompensated mean own-price elasticities and 7 out of the 55 compensated ones are less than —1, contrary to the theoretical model.

In examining Table 4 we observe that 4 out of 55 terminal income elasticities are greater than 2 and 1 is negative, for the same reasons as in the case of the mean income elasticities. With regard to the terminal own-price elasticities 16 out of 110 are positive, while 19 out of 110 are less than -1, for the same reasons as in the case of the mean own-price elasticities.

In Table 5 we present the uncompensated mean own-and cross-price elasticities. With the exception of those elasticities which are positive because the corresponding b_i's are less than -1, all the remaining uncompensated cross-price elasticities lie between 0 and -1.

Finally Table 6 presents the estimated «income flexibility» evaluated at the first, mean, and last year of our time series for each country. It lies between-1 and —.5 (with the exception of Denmark for the first year) according to the theoretical model. There is no substantial variation of the estimates over the examined period of sixteen years.

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(1) As we know $n_i(t) = (1 + \beta_i) - \Sigma_{j=1}^{n} \beta_j w_j(t)$ and because in this case $b_i < -2$ the result is to give a negative sign.

5. Fitting Criteria

There is general agreement that the best criterion for choice among alternative models would be predictive power, that is the best model is that one which predicts the best. But when there is no predictive evidence available at the time the researcher has to make a choice, there are other criteria upon which he has to rely in order to solve his problem of choice. The most common criterion is to select that model which fits the best within the sample period-that is, that one which has the highest R^2. Another criterion involves examining whether parameter estimates have theoretical support. These two criteria are not examined independently, of course, and a researcher has to depend on both. A third criterion in choosing among alternative models is simplicity.

Having these in mind we test the IAES model on the basis of its goodness of fit, by using an R^2 statistic, which measures the predictive ability over the sample period with respect to the expenditures on each commodity. Table 7 tabulates this R^2-type statistic, for each country and commodity, for the IAES model. This R^2-type statistic was computed using the formula

$$R_i^2 = 1 - \frac{\Sigma_{t=1}^{T} (v_i(t))^2}{\Sigma_{t=1}^{T} (e_i(t) - \bar{e}_i)^2} \qquad (i = 1,\ldots,n),$$

where $\bar{e}_i = \Sigma_{t=1}^{T} e_i(t)/T$ denotes sample mean expenditures on commodity i. Clearly the IAES model can account for most of the variation over time in expenditures.

Another criterion for evaluation of the models is to consider their predictive ability in term of the budget shares over the sample period. Variation of average qudget shares is, after all, of prime interest in economic planning.

The procedure which has been used is the following : Let

$$\hat{e}_i(t) = e_i(t) - v_i(t) \qquad (i = 1,\ldots,n \;\; ; \;\; t = 1,\ldots,T)$$

be the calculated values in our fitted model. The calculated average budget shares

$$\hat{w}_i(t) = \frac{\hat{e}\,t)}{y(t)} \qquad (i = 1, \ldots, n \;\; ; \;\; t = 1, \ldots, T),$$

and the resulting errors in «predicting» the average budget shares are given by:

$$\hat{u}_i(t) = w_i(t) - \hat{w}_i(t) \qquad (i = 1, \ldots, n \;\; ; \;\; (t = 1, \ldots, T)$$

Since at each observation we have $\Sigma_{i=1}^{n} v_i(t) = 0$, this guarantees that $\Sigma_{i=1}^{n} \hat{u}(t) = 0$.

Table 8 reports the «proportion of variation explained», an R^2 - type statistic, as a measure of goodness of fit, recording

$$\hat{R}^2_i = 1 - \frac{TS^2_i}{\Sigma_{t=1}^{T} (w_i(t) - \bar{w}_i)^2} \qquad (i = 1, \ldots, n),$$

where

$$S^2_i = \Sigma_{t=1}^{T} (\hat{u}_i(t))^2 / T$$

is the mean square error.

These «proportions of variation explained» can be positive or negative. Comparing the columns in Table 7 with those of Table 8 we observe that even when the proportion of expenditure variation explained is close to unity, much of the variation in average budget shares may remain unaccounted for. Table 9 reports mean square errors for the average budget shares for the IAES model.

Another measure which we can use to describe the predictive ability of the IAES model for the average budget shares is the *average information inaccuracy* statistic. Theil (1967) and Theil and Mnookin (1966) have applied the techniques of information theory to the evaluation of the average budget share predictions. Since $\Sigma_{i=1}^{n} w_i(t) = 1$ and $\Sigma_{i=1}^{n} \hat{w}_i(t) = 1$, and $0 < \hat{w}_i(t) < 1$, for $i = 1, \ldots, n$, this means that we can regard each of n value shares (predicted as well as observed) as a complete set of probabilities. «The forecasts are the «prior» probabilities; at some point of time a message comes in that states what the value shares actually are and that thus changes the prior probabilities $\hat{w}_i(t)$ into «posterior» probabilities $w_i(t)$. The information content of such a message is defined in information theory as

$$\hat{I}(t) = \Sigma_{i=1}^{n} w_i(t) \ln \frac{w_i(t)}{\hat{w}_i(t)},$$

which is always positive unless $w_i(t) = \hat{w}_i(t)$ for each i (perfect forecasts), in which case $\hat{I}(t) = 0$. The larger the differences between $w_i(t)$ and $\hat{w}_i(t)$, the worse the forecasts are and the larger the information content of the message on the realization is. Therefore, $\hat{I}(t)$ is called the *information inaccuracy*, of the forecasts

$\hat{w}_1(t),\ldots,\hat{w}_n(t)$ with respect to the corresponding realizations $w_1(t),\ldots,w_n(t)$» — Theil and Mnookin (1966, pp. 37-38).

Table 10 reports the average information inaccuracy

$$\bar{I} = \Sigma_{t=1}^{T} \hat{I}(t) / T$$

for the IAES model. That measure reasserts the predictive ability of the IAES model. A comparison of this model with the linear expenditire system (LES) and the generalized linear expenditure system (GLES), using the above fitting oriteria, gives us as a result that the GLES model, generally speaking, has a better predictive ability than the IAES and LES models[1].

6. Conclusions

In this paper we have explored consumer expenditure patterns within the framework of the classical demand theory. Our estimation of the Indirect Addilog Expenditure system gave an indication that empirical demand models could justify this theory. Working at a highly aggregative level with respect to commodities and to observational units, we fount that expenditures respond to movements in prices as well as in income.

The results of this paper tell us that the IAES model could be used to account for variation overtime in expenditures within each country in terms of variation in income and prices. However, in this model the estimated marginal budget shares appear to vary across countries. In view of the small standard errors for these parameters, their differences should be considered as signiicant. These results tell us that the cross-country variation in expendituresf cannot possibly be exptained by these models. However, more formal testing of the hypothesis of cross-country constancy of parameters is needed. For more conclusive statements about cross-country variability, an extension of thi study to a wide range of countries would be required.

(1) See Gamaletsos (1973, pp. 16-19).

TABLE 1

PARAMETER ESTIMATES

	g_i					β_i				
	F	C	R	D	O	F	C	R	D	O
Belgium	.292	.016	.662	.000	.030	—.730	—.235	—1.163	.340	.000
Denmark	.486	.114	.077	.017	.306	—1.569	—1.219	—1.138	.105	—.942
France	.342	.085	.280	.020	.273	—.980	—.648	—2.245	.137	—.639
Greece	.745	.106	.001	.002	.146	—1.207	—.811	.364	—.384	—.608
Ireland	.524	.159	.012	.000	.305	—.638	—.635	—.253	1.000	—.507
Italy	.334	.087	.047	.153	.379	—.713	—.741	—1.029	.344	—.537
Netherlands	.365	.152	.112	.052	.319	—.820	—.542	—1.307	.396	—.519
Norway	.487	.138	.137	.012	.226	—1.092	—.757	—1.190	.445	—.529
Sweden	.659	.123	.042	.020	.156	—1.338	—.768	—.288	.279	—.227
U. Kingdom	.135	.117	.131	.218	.399	—1.503	—.790	—.570	—.058	—.886
U.S.A.	.268	.112	.106	.112	.402	—.773	—.722	.102	—.288	—.232

Underlined coefficients are less, in abselute value, than twice their their standard errors.

F = Food, C = Clothing, R = Rent, D = Durables, O = Other.

TABLE 2

MEAN MARGINAL BUDGET SHARFS

	F	C	R	D	O
Belgium	.168	.108	.021	.172	.531
Denmark	.104	.075	.065	.303	.453
France	.262	.146	—.026	.156	.462
Greece	.292	.152	.156	.077	.323
Ireland	.297	.093	.072	.145	.393
Italy	.368	.098	.053	.087	.394
Netherlands	.242	.172	.020	.207	.359
Norway	.186	.145	.036	.201	.432
Sweden	.067	.105	.118	.199	.511
U. Kingdom	.130	.130	.131	.158	.451
U.S.A.	.126	.060	.202	.125	.487

$\bar{\mu}_i = \bar{w}_i \, \bar{\eta}_j$ income slopes evaluated at the point $(\bar{y}, \bar{p}_1, \ldots \bar{p}_n)$.

TABLE 3

MEAN ELASTICITIES

	η_i Income elasticity					η_{ii} Own-price elasticity (Uncompensated))					η^*_{ii} Own-price elasticity (Compensated)				
	F	C	R	D	O	F	C	R	D	O	F	C	R	D	O
Belgium	.60	1.10	.17	1.67	1.34	—.48	—.79	.02	—1.30	—1.00	—.31	—.68	.04	—1.13	—.47
Denmark	.42	.77	.85	2.09	1.05	.18	.10	.05	—1.09	—.47	.28	.28	.12	—.79	—.02
France	.80	1.13	46	1.92	1.14	—.34	—.44	1.12	—1.13	—.62	—.08	—.29	1.09	—.97	—.16
Greece	.63	1.02	2.20	1.45	1.23	—.35	—.31	—1.34	—.64	—.55	—.06	—.16	—1.18	—.56	—.23
Ireland	.83	.83	1.21	2.46	.96	—.59	—.44	—.76	—1.94	—.70	—.29	—.35	—.69	—1.79	—.31
Italy	.92	.89	.61	1.98	1.10	—.57	—.34	—.06	—1.33	—.66	—.20	—.24	—.01	—1.24	—.27
Netherlands	.76	1.04	.27	1.97	1.06	—.44	—.55	.21	—1.35	—.66	—.20	—.38	.23	—1.14	—.30
Norway	.60	.94	.51	2.14	1.17	—.25	—.36	.11	—1.40	—.67	—.06	—.21	.15	—1.20	—.24
Sweden	.23	.80	1.28	1.85	1.35	—.05	—.33	—.74	—1.25	—.86	.02	—.22	—.62	—1.05	—.35
U. Kingdom	.45	1.16	1.38	1.90	1.07	.07	—.30	—.48	—.95	—.49	.20	—.17	—.35	—.79	—.04
U.S.A.	.58	.63	1.46	1.07	1.12	—.39	—.35	—1.09	—.75	—.87	—.26	—.29	—.89	—.62	—.38

F = Food, C = Clothing, R = Rent, D = Durables, O = Other

TABLE 4

TERMINAL ELASTICITIES

	η_i Income Elasticity					η_{ii} Own-price Elasticity (Uncompensated)					η^*_{ii} Own-price Elasticity (Compensated)				
	F	C	R	D	O	F	C	R	D	O	F	C	R	D	O
Belgium	.56	1.06	.13	1.63	1.29	—.46	—.79	.05	—1.30	—1.00	—.31	—.68	.06	—1.11	—.46
Denmark	.35	.70	.78	2.03	.98	.23	.13	.04	—1.09	—.47	.31	.18	.11	—.72	—.04
France	.80	1.14	—.46	1.92	1.14	—.31	—.43	1.08	—1.12	—.62	—.07	—.29	1.05	—.94	—.15
Greece	.59	.99	2.16	1.41	1.19	—.32	—.31	—1.33	—.64	—.56	—.06	—.17	—1.14	—.56	—.30
Ireland	.77	.77	1.15	2.40	.90	—.57	—.43	—.76	—1.90	—.70	—.31	—.35	—.69	—1.67	—.33
Italy	.90	.88	.59	1.96	1.08	—.56	—.33	—.06	—1.32	—.66	—.21	—.24	—.01	—1.20	—.27
Netherlands	.72	1.00	.23	1.94	1.02	—.42	—.54	.21	—1.34	—.66	—.21	—.39	.23	—1.07	—.31
Norway	.57	.90	.47	2.10	1.13	—.23	—.35	.10	—1.39	—.67	—.06	—.22	.13	—1.14	—.25
Sweden	.19	.76	1.24	1.80	1.30	—.02	—.32	—.74	—1.24	—.86	.03	—.23	—.62	—1.01	—.34
U. Kingdom	.42	1.13	1.35	1.86	1.04	.12	—.29	—.49	—.95	—.50	.23	—.17	—.34	—.77	—.05
U.S.A.	.57	.62	1.44	1.05	1.11	—.38	—.34	—1.09	—.75	—.87	—.27	—.29	—.87	—.63	—.37

F = Food, C = Clothing, R = Rent, D = Durables, O = Other.

TABLE 5

MEAN OWN-AND CROSS-PRICE ELASTICITIES

(ALL ENTRIES SHOULD BE MULTIPLIED BY 10^{-2})

	F	C	R	D	O
Belgium					
F	—48	—02	—14	04	00
C	—20	—79	—14	04	00
R	—20	—02	02	04	00
D	—20	—02	—14	—130	00
O	—20	—02	—14	04	—100
Denmark					
F	18	—12	—09	02	—41
C	—39	10	—09	02	—41
R	—39	—12	05	02	—41
D	—39	—12	—09	—109	—41
O	—39	—12	—09	02	—47
France					
F	—34	—08	—13	01	—26
C	—32	—44	—13	01	—26
R	—32	—08	112	01	—26
D	—32	—08	—13	—113	—26
O	—32	—08	—13	01	—62
Greece					
F	—35	—12	03	—02	—16
C	—56	—31	03	—02	—16
R	—56	—12	—134	—02	—16
D	—56	—12	03	—64	—16
O	—56	—12	03	—02	—55
Ireland					
F	—59	—07	—02	06	—21
C	—23	—44	—02	06	—21
R	—23	—07	—76	06	—21
D	—23	—07	—02	—194	—21
O	—23	—07	—02	06	—70
Italy					
F	—57	—08	—09	01	—19
C	—29	—34	—09	01	—19
R	—29	—08	—06	01	—19
D	—29	—08	—09	—133	—19
O	—29	—08	—09	01	—66

TABLE 5 (continued)

	F	C	R	D	O
Netherlands					
F	—44	—09	—09	04	—18
C	—26	—55	—09	04	—18
R	—26	—09	21	04	—18
D	—26	—09	—09	—135	—18
O	—26	—09	—09	04	—66
Norway					
F	—25	—12	—09	04	—20
C	—34	—36	—09	04	—20
R	—34	—12	11	04	—20
D	—34	—12	—09	—140	—20
O	—34	—12	—09	04	—67
Sweden					
F	—05	—10	—03	03	—09
C	—39	—33	—03	03	—09
R	—39	—10	—74	03	—09
D	—39	—10	—03	—125	—09
O	—39	—10	—03	03	—86
U. Kingdom					
F	07	—09	—05	—01	—37
C	—43	—30	—05	—01	—37
R	—43	—09	—48	—01	—37
D	—43	—09	—05	—95	—37
O	—43	—09	—05	—01	—49
U.S.A.					
F	—39	—07	01	—03	—10
C	—17	—35	01	—03	—10
R	—17	07—	01	—03	—10
D	—17	—07	—109	—75	—10
O	—17	—07	01	—03	—87

F = Food, C = Clothing, R = Rent, D = Durables O = Other

TABLE 6

INCOME FLEXIBILITY

	$\varphi(1)$	$\overline{\varphi}$	$\varphi(T)$
Belgium	—.72	—.75	—.78
Denmark	—.49	—.50	—.52
France	—.56	—.56	—.56
Greece	—.53	—.54	—.56
Ireland	—.68	—.68	—.71
Italy	—.61	—.61	—.62
Netherlands	—.63	—.64	—.65
Norway	—.58	—.59	—.60
Sweden	—.63	—.64	—.66
U. Kingdom	—.51	—.51	—.52
U.S.A.	—.73	—.74	—.75

$\varphi(t) = (-1 + \Sigma_j \, b_j \, w_j \, (t))^{-1}$

$\overline{\varphi} = (-1 + \Sigma_j \, b_j \, \overline{w}_j \,)^{-1}$

TABLE 7

GOODNESS OF FIT FOR EXPENDITURES

	F	C	R	D	O
Belgium	.990	.985	.979	.995	.987
Denmark	.988	.826	.996	.970	.999
France	.999	.996	.984	.987	.998
Greece	.997	.983	.973	.990	.993
Ireland	.957	.900	.968	.982	.990
Italy	.999	.979	.999	.968	.996
Netherlands	.997	.979	.997	.998	.998
Norway	.996	.989	.995	.974	.994
Sweden	.997	.981	.994	.992	.998
U. Kingdom	.998	.985	.998	.966	.997
U.S.A.	.935	.948	.987	.812	.996

Figures give proportions of variation explained.

F = Food, C = Clothing, R = Rent, D = Durables O = Other.

TABLE 8

GOODNESS OF FIT FOR AVERAGE BUDGET SHARES

	F	C	R	D	O
Belgium	.947	.186	.977	.954	.906
Denmark	.939	.843	.916	.794	.629
France	.985	.894	.870	.864	.712
Greece	.893	.427	.721	.604	.632
Ireland	.377	.619	.286	.896	.048
Italy	.894	.820	.969	.860	.286
Netherlands	.920	.779	.961	.973	.205
Norway	.862	.927	.966	.846	.304
Sweden	.936	.899	.822	.912	.790
U. Kingdom	.976	.788	.962	.698	.773
U.S.A.	.586	.852	.796	—.064	.745

Figures give «proportion of variation explained» but see text.
E = Food, C = Clothing, R = Rent, D = Durables, O = Other.

TABLE 9

BADNESS OF FIT FOR AVERAGE BUDGET SHARES

(Entries should be multiplied by 10^{-7})

	F	C	R	D	O
Belgium	120	062	030	038	244
Denmark	387	511	042	1799	214
France	137	100	264	292	405
Greece	783	647	872	080	1060
Ireland	1408	274	076	204	614
Italy	122	258	017	151	770
Netherlands	274	511	019	103	227
Norway	284	113	038	409	513
Sweden	272	226	065	156	596
U. Kingdom	80	076	027	359	299
U.S.A.	664	058	143	805	333

Figures give mean square error.
F = Food, C = Clothing, R = Rent, D = Durables, O = Other.

TABLE 10

AVERAGE INFORMATION INACCURACY :

(All entries should be multiplied by 10^{-6})

Belgium	110
Denmark	1080
France	580
Greece	1430
Ireland	640
Italy	410
Netherlands	290
Norway	410
Sweden	310
U. Kingdom	330
U.S.A.	610

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