INTEREST RATES AND PRICE MOVEMENTS

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The purpose of this paper is to critically survey some methods by which price expectations and real interest rates are obtained from regressing market interest rates on past price changes. Some further empirical findings are reported, supporting and expanding previous evidence. The use of modern econometric techniques is shown to improve our understanding of the relationship between interest rates and price changes and to clarify its theoretical underpinnings. The strength of this relationship in the after – 1960 period is confirmed for all nominal interest rates chosen and most price indices used.

The surge of inflation that took place in most countries after 1960 has rekindled our interest in the formation of price expectations. It is now recognized that expected prices should enter the investment and consumption function as well as many other important economic relationships. Hence the need for estimates of expected prices. There are several methods by which such estimates may be obtained. The approach employed in this paper is based on the relation between observed interest rates and changes in commodity prices. By regressing market interest rates on current and past price changes estimates of expected inflation can be obtained. These may be used then to get an estimate of the real interest rate. The relevant theoretical framework has been developed by Fisher (in [7]) and is given by the following relations:

$$RN_t = RR_t + \dot{P}_t^e + RR_t \cdot \dot{P}_t^e \qquad (1)$$

$$\dot{P}_t^e = \sum_{i=o}^{n} a_i \frac{\Delta P_{t-i}}{P_{t-i-1}} \qquad (2)$$

$$RR_t = \overline{RR} + U_t \qquad (3)$$

where
RN= nominal interest rate
RR= real interest rate
$\overline{RR}$= equilibrium real interest rate
P= price level
U= stochastic disturbance

The superscript (e) denotes an expected value and the dot the rate of change. Equation (I) states Fisher's hypothesis about the relation between nominal interest rates and price changes. Equation (2) specifies the formation of price expectations and eq. (3) helps to form the hypothesis usually tested.

The last term of eq. (I) is ignored in empirical tests since it is, in most cases, very small in magnitude. Substituting then (2) and (3) into (1) yields:

$$RN_t = \overline{RR} + \sum_{i=0}^{n} a_i \frac{\Delta P_{t-i}}{P_{t-i-1}} + U_t \tag{4}$$

It is assumed that there is no feedback effect from the interest rate to price changes and, furthermore, that $\overline{RR}$ is a constant interpreted as the long-run real rate of interest. Both assumptions have not gone unchallenged. The former is tantamount to taking the price change as exogenous, a procedure which can be considered at best an approximation. As for the latter, Mundell (in [10]) and Cagan [2] have challenged on theoretical grounds the Fisherian premise that the (expected) real rate is constant over time. Recent empirical work, for example, by Elliott and Carlson [5], has further undermined the validity of this assumption[1]. Accordingly, the performance of relation (4) ought to be appraised with some reservations.

Several methods have been employed for testing eq. (4). They can be grouped into three categories. The distinguishing feature of the first one is the type of distributed lag used to estimate the expected price change. For the second group, the distinguishing feature, is the adjustment of the historical data – e.g., "discounting" them by the contemporaneous unemployment rate –, undertaken on the premise that economic agents form expectations by considering both past price changes and general economic conditions[2]. In the third set, emphasis is given to the weights attached to the observed price changes; that is, the coefficients of the price terms themselves, (the a's), instead of the data, are adjusted there in an effort to improve the expectations mechanism (see [6]). Attention is given in this paper only to the methods of the first group[3]. On the basis of the lag scheme used to approximate the expected rate of

1. Additional papers on this subject can be found in the June 1977 issue of the American Economic Review (vol. 67, No. 3).

2. This approach is analyzed and implemented in [1].

3. For a discussion of the other methods see [16].

price changes one can discern four main methods or models belonging to this group:

1. the uncostrained lag
2. the geometric lag
3. the Pascal lag
4. the polynomial lag

In the following sections I give a brief critical evaluation of these methods and present some further empirical results based on the above techniques.

I. The Unconstrained Lag Method

The method of unconstrained lag suggests that we make use of the following regression model:

$$RN_t = \overline{RR} + a_1 \dot{P}_t + a_2 \dot{P}_{t-1} + \ldots + a_{n+1} \dot{P}_{t-n} + U_t \qquad (5)$$

where the weights are given by the a_i (i=1, . . . , n) coefficients and $\overline{RR}$ captures the equilibrium real interest rate. The advantage of equation (5) is, on the estimation side, that there is no **a priori** constraint on the shape of the distribution formed by the estimated coefficients and, on the analytiçal side, its simplicity. Its disadvantage is that it can not avoid autocorrelation and multicollinearity. Gibson (in [8]) ran this regression against U.S. data and obtained unsatisfactory results characterized by little explanatory power (i.e., low R^2) and statistically insignificant coefficients. Meiselman [9] introduced a slightly different model, in which the interest rate varies directly with the difference between expected and actual prices:

$$RN_t = \overline{RR} + a_0 (P_t^e - P_t) + U_t \qquad (6)$$

The performance of (6) as well as its logarithmic version were disappointing. Only when the sample period is confined to recent experience (1950's and, particularly, 1960's), is the explanatory power of (5) improved and the number of significant a_i's increased. Specifically, using montly data for the period 1952-69 Yohe and Karnosky were able to report for the first time short lags and rather high coefficients of determination, ranging from 0.59 to 0.63 [17][4]. In an effort to confirm the above mentioned findings and to test whether they hold true when the period is extended to include the years 1970-74, regression analysis of model (5) was performed using quarterly data for the period 1947 (1) - 1974 (IV). The dependent variable is Moody's average of corporate bond yield, taken from monthly issues of the Survey of Current Business; rates of change of the deflator for personal consumption expenditures are taken as the independent variable. Ordinary Least Squares (OLS) estima-

4. The previously reported lags were very long, a feature that rendered them implausible on several grounds; for a vigorous discussion of these grounds see Sargent [13].

tion was initially performed; it produced substantial evidence of autocorrelation of the residuals. To remove the undesirable consequences of autoregression, the Cochrane-Orcutt iterative technique (hereafter denoted by CORC) was employed [3]. Tables 1 and 2 present results which suggest that:

TABLE 1

Estimated coefficients of the relationship between interest rates and price changes when an unconstrained distributed lag is employed
Time period: 1951 (II) – 1974 (IV)

Estimator:	OLS		CORC	
Lag	a_i	t-stat.	a_i	t-stat.
0	0.342	3.8	0.029	2.0
1	0.044	0.5	0.047	3.7
2	0.072	0.8	0.065	5.4
3	−0.045	0.5	0.062	5.2
4	0.022	0.3	0.042	3.8
5	0.059	0.7	0.046	4.0
6	0.049	0.6	0.050	4.2
7	0.068	0.8	0.036	3.0
8	0.047	0.6	0.026	2.0
9	0.088	1.1	0.036	2.7
10	0.063	0.8	0.043	3.7
11	0.088	1.1	0.043	3.6
12	−0.009	0.1	0.022	1.9
13	0.033	0.4	0.009	0.9
14	0.019	0.2	0.013	1.2
15	0.045	0.6	0.015	1.4
16	0.000	0.0	0.013	1.2
17	−0.024	0.3	0.010	1.0
sum:	0.956		0.961	
Constant:	2.89 (t=9.4)		5.08(t=8.1)	
R^2:	.630		.933	
SEE:	1.23		0.17	
D.W.:	0.16		1.42	

SOURCE: Equation (5) in the text.

TABLE 2

Estimated coefficients of the relationship between interest rates and price changes when an unconstrained distributed lag is employed

	1952 (I) – 1960 (IV)			
Estimator	OLS		CORC	
Lag	a_i	t-stat.	a_i	t-stat.
0	0.193	1.3	0.003	0.1
1	0.185	1.4	0.033	0.7
2	0.057	0.4	0.045	1.1
3	–0.103	0.9	0.045	1.0
4	–0.075	1.2	0.029	0.9
5	–0.012	0.1	0.024	0.7
6	0.005	0.1	0.023	0.8
7	–0.063	1.3	0.011	0.5
8	–0.061	1.3	0.001	0.0
9	0.006	0.1	0.015	0.6
10	0.033	0.7	0.022	0.9
11	0.031	0.7	0.029	1.0
12	–0.036	0.8	0.015	0.6
13	–0.053	1.1	0.001	0.1
14	–0.050	1.0	0.003	0.1
15	–0.027	0.6	0.000	0.0
16	–0.048	1.1	0.002	0.1
17	–0.050	1.1	0.005	0.2
18	–0.084	1.9	0.003	0.2
19	–0.038	0.8	0.001	0.0
20	–0.014	0.3	0.004	0.3
21	–0.021	0.5		
22	–0.099	2.2		
23	–0.086	2.0		
24	–0.053	1.2		
Sum:	–0.462		0.314	

	OLS	CORC
Constant:	4.98 (t=6.1)	4.37 (t=4.7)
R^2:	.833	.967
SEE:	0.47	0.18
D.W.:	0.55	1.26

	1961 (I) – 1974 (IV)			
	OLS		CORC	
Lag	a_i	t-stat.	a_i	t-stat.
0	0.102	2.4	0.045	2.1
1	0.046	0.9	0.044	2.1
2	0.082	1.6	0.078	3.7
3	0.014	0.3	0.057	2.6
4	0.046	0.9	0.023	1.0
5	0.089	1.6	0.051	2.1
6	0.117	2.1	0.096	4.0
7	0.128	2.3	0.068	2.6
8	0.148	2.6	0.068	2.1
9	0.154	2.7	0.083	2.4
10	0.102	1.7	0.077	2.3
11	0.073	1.2	0.046	1.5
12	0.005	0.1	0.039	1.5
Sum:	1.106		0.775	
Constant:	3.43 (t= 33.6)		4.55 (7.7)	
R^2:	.964		.990	
SEE:	0.35		0.18	
D.W.:	0.43		1.30	

SOURCE: Equation no. (5) in the text.

(a) The relation between interest rates and price changes seems to be unstable; a break in the way price expectations are formed appears to have taken place and/or the extent to which nominal rates are affected by expected prices is different in the 1960's and 1970's from that prevailed in the 1950's[5]. This break appears to have occurred some time at the end of 1960 and the beginning of 1961. Some results (which can be made available on request) indicate that a similar but weaker break took place in 1965.

(b) There is a marked difference between the two subperiods as far as the statisti-

5. Estimates derived from the two subperiods seem to belong to different parameter spaces, a feature which is confirmed by a Chow test (as described in [4]) on coefficients obtained with the Cochrane-Orcutt technique. The implication is that the two subperiods are structurally different.

cal significance of the estimated coefficients is concerned.

Almost none of the coefficients (of both OLS and CORC type) obtained for the subperiod 1952-60 pass the test of statistical significance at the 0.05 level. The situation is reversed when the subperiod 1961-74 is examined (Table 2).

(c) The length of the distributed lag is considerably shorter in the latter period, suggesting that expectations are formed and revised faster after 1960.

(d) A stronger relationship between interest rates and price changes in the more recent period is implied by the fact that the sum of coefficients – reflecting the total impact of price changes on interest rates – is significantly higher in this period (see Table 2). The sensitivity of the 1950-60 regression to even small changes in the sample period is high, a feature which confirms the weakness and instability of the above relationship in the 1950's.

(e) According to Fisher's hypothesis the sum of the price term coefficients should be 1. His hypothesis is not rejected for the second subperiod. This also holds for the whole period, owing to the dominant influence of the recent years. However, it is clearly rejected for the 1950's.

(f) Autocorrelation in the residuals is present in the regressions estimated by OLS. Comparing these regressions with those estimated by the Cochrane-Orcutt method reveals that autocorrelation is responsible for holding down the explanatory power of the OLS equations and for rendering many coefficients estimated by OLS statistically insignificant, not to mention that OLS estimates frequently have a theoretically wrong sign.

g) The time pattern of the impact of past price changes on interest rates is not clear; it appears, in general, to be diminishing as time passes but is not "single peaked".

On the basis of these results one can conclude that the method of unconstrained lag, though it may be useful, is less than satisfactory mainly, because of long lags and poor estimates in the 1950's.

2. The Geometric Lag Method

Such a model is specified as follows:

$$RN_t = RR + \sum_{i=1}^{n} a_i \frac{\Delta P_{t-i}}{P_{t-i-1}} + U_t \qquad (7)$$

where $a_i = \gamma\lambda^i$, $i \geqslant 0$, $|\lambda| < 1$

The weithts are constrained to decline geometrically, thus giving rise to an **extrapolative** expectations model since past price changes are extrapolated into the future. The geometrically decaying lag is also known as a Koyck lag. The parameter λ indicates the rate at which the weight of past inflation rates on current expectations declines in time. It is worth noting that $\lambda=1$ means that the lagged terms never decay at all, $\lambda=0$ that only the current inflation rate has any effect on expectations and $\lambda>1$ that more distant rates of inflation have more bearing on current expectations than less distant lagged terms. The last case is, of course, inconsistent with the extrapolative expectations hypothesis.

Expression (7) contains an adaptive expectations mechanism; this can be shown from the following:

$$\dot{P}_t^e = \sum_{i=0}^{n} a_i \frac{\Delta P_{t-i}}{P_{t-i-1}} = \gamma \sum_{i=0}^{n} \lambda^i \frac{\Delta P_{t-i}}{P_{t-i-1}} \tag{8}$$

which defines expected prices on the basis of (7). Using the well-known Koyck transformation, we obtain

$$\dot{P}_t^e = \gamma \frac{\Delta P_t}{P_{t-1}} + \lambda \dot{P}_{t-1}^e \tag{9}$$

which in turn yields

$$\dot{P}_t^e - \dot{P}_{t-1}^e = \gamma \frac{\Delta P_t}{P_{t-1}} - (1-\lambda) \dot{P}_{t-1}^e \tag{10}$$

Expression (10) indicates that current expectations are adjusted in light of the error made in predicting inflation. Hence, equation (7) can also be interpreted as an adaptive expectations model.

Application of the Koyck transormation to (7) produces another version of the geometric lag:

$$RN_t = RR + \gamma \frac{\Delta P_t}{P_{t-1}} + \lambda RN_{t-1} \tag{11}$$

which has the advantage of avoiding multicollinearity present in the lagged price change terms of (7).

Several researchers have employed the geometric lag method. Sargent has estimated eq. (7) for both long-term and short-term interest rates and has obtained $\lambda= 0.91$, $R^2= 0.828$ and $\lambda= 0.99$, $R^2= 0.536$, respectively, using annual U.S. data for the period 1870-1940 [12]. Large values of λ imply very long distributed lags and, therefore, expectations which are very slowly formed. As for the R^2's, they may simply reflect the correlation between interest rates and the price level that characterizes the relevant data. Sargent has produced results which demonstrate that failure to make the real rate statistically independent of current and lagged rates of inflation forces the estimated a_i's to form a very long lag, even if the true a_i's form a short lag distribution [11,14]. Equally disappointing results were reported by Yohe and Karnosky, who (using monthly data for the period 1952-1969) obtained decay coefficients which were larger than one for both short and long-term interest rates[6].

TABLE 3

Estimates of the interest rate – price change relation obtained from the geometric lag model

Equat. Period: 1947 (III) – 1974 (IV)

1 OLS :
$$RN_t = \underset{(104.76)}{1.005}\, RN_{t-1} + \underset{(3.40)}{0.019}\, \dot{P}_t - \underset{(0.44)}{0.0002}$$
$R^2 = 0.9923$
D.W. = 1.3389

Period: 1947(III) – 1960 (IV)

2 OLS :
$$RN_t = \underset{(37.36)}{1.004}\, RN_{t-1} + \underset{(1.06)}{0.006}\, \dot{P}_t + \underset{(0.08)}{0.00008}$$
$R^2 = 0.9647$
D.W. = 1.4708

3 CORC :
$$RN_t = \underset{(26.66)}{0.983}\, RN_{t-1} + \underset{(0.39)}{0.002}\, \dot{P}_t + \underset{(0.67)}{0.0009}$$
$R^2 = 0.9667$
$\rho^{(a)} = 0.305$
D.W. = 1.8376
MEAN LAG$^{(b)}$ = 57.82

Period: 1961 (I) – 1974 (IV)

4 OLS :
$$RN_t = \underset{(42.68)}{0.954}\, RN_{t-1} + \underset{(4.33)}{0.050}\, \dot{P}_t + \underset{(1.73)}{0.002}$$
$R^2 = 0.9874$
D.W. = 1.3812
MEAN LAG$^{(b)}$ = 20.74

5 CORC :
$$RN_t = \underset{(31.82)}{0.947}\, RN_{t-1} + \underset{(3.38)}{0.048}\, \dot{P}_t + \underset{(1.56)}{0.003}$$
$R^2 = 0.9882$
$\rho^{(a)} = 0.312$
D.W. = 1.8139
MEAN LAG$^{(b)}$ = 17.87

Period: 1947 (III) – 1965 (IV)

6 OLS :
$$RN_t = \underset{(55.37)}{0.993}\, RN_{t-1} + \underset{(1.24)}{0.006}\, \dot{P}_t + \underset{(0.56)}{0.0004}$$
$R^2 = 0.9782$
D.W. = 1.4415
MEAN LAG$^{(b)}$ = 141.86

6. See [17]. Only when they divided the sample period into two parts, 1952-60 and 1961-69, were they able to get decay coefficients of less than unity.

Period: 1966 (I) – 1974 (IV)

7	OLS :	$RN_t = 0.914\ RN_{t-1} + 0.049\ \dot{P}_t + 0.005$		$R^2 = 0.9705$
		(26.64) (3.56) (2.31)		D.W. = 1.4550
				MEAN LAG [(b)] = 10.63

(a) Autocorrelation parameter, which is statistically significant at the 0.05 level in all equations.
(b) Computed from $\lambda/(1-\lambda)$.

Employing the same data as in the preceding section, I ran the geometric lag mode. in the version given by equation (11). The most important statistics are presented in Table 3, from which we can draw the following implications:

(a) OLS estimation applied to the entire post-war period as well as to the first subperiod produces decay coefficients that are slightly greater than one, for which it is difficult to find any theoretical justification (see equations 1 and 2)[7]; taken at face value, they suggest that the effect of lagged price changes does not decay. These results are in line with those reported by Yohe and Karnosky.

(b) There are some periods for which OLS estimation yields decay coefficients which are close to unity; these periods are 1947-65 and 1961-74 (eq. 4 and 6). Although they are acceptable, they still imply implausibly long lags. These features also hold true for the Cochrane-Orcutt estimation of the 1947-60 and 1961-74 periods (eq. 3 and 5).

(c) Plausible lags are obtained only for the 1966-74 period, for which the decay factor is 0.914 (eq. 7).

(d) Judging by the t-statistic and the estimated coefficient, there is a marked difference in the effect of the contemporaneous price change (the $\dot{P} = \Delta P/P$ variable) between the two subperiods. Concurrent rates of inflation exert a stronger influence in the later subperiod than in the earlier one.

(e) The goodness of fit appears to be quite satisfactory; this can be attributed mostly to the presence of a lagged dependent variable on the right-hand side of the equation. It should be noted that application of the Cochrane-Orcutt technique neither improves considerably the goodness of fit nor changes appreciably the estimated values of the parameters.

On the basis of the above findings one is led to the conclusion that the usefulness of this model is severely limited. The long lags, implied at least up to 1965, render it difficult to obtain estimates of expected price changes. Accordingly, the geometric lag method is seldom used to separate real interest rates from nominal ones.

7. The effect of the distributed lag is captured by the lagged dependent variable which is present on the right-hand side of equation (11). Therefore, few observations are lost and the test period could begin with 1947 (III).

3. The Pascal Lag Method

This model combines Fisher's basic equation with a Pascal-type of distributed lag and is given by the following expression:

$$RN_t = RR + \frac{(1-\mu)^r}{(1-\mu L)^r} \; \mathring{P}_t + U_t \tag{12}$$

where L is a lag operator defined by $L\mathring{P}_t = \mathring{P}_{t-1}$, r is the order of the Pascal distribution and μ a parameter with certain economic meaning (to be explained below). The weights (a_i's) are given by

$$a_i = \left(\frac{i+r-1}{i} \right) (1-\mu)^r \mu^i = \frac{(i+r-1)!}{i!\,(r-1)!} (1-\mu)^r \mu^i \tag{13}$$

for i= 0, 1, 2, ... and r>0

Expression (13) is a Pascal distribution showing the probability that the r^{th} success will occur on the $(r+i)^{th}$ trial. Alternatively, we may say that it shows the number of trials until the r^{th} success. The weights can be treated as probabilities a_i (μ, r) where:

1-μ= probability of a "success", constant from trial to trial, given that $0 \leqslant \mu \leqslant 1$,
r= a specified number of successes,
i= number of independent trials.

How can we interpret these weights in the context of the formation of inflationary expectations? The interpretation I will offer is based on Sinai [15]. Suppose that r indicates the proportion of investors who shift their focus from the actual rate of inflation to a new expected rate. Let r= 1, 2, 3, 4 be, for example, 25%, 50%, 75% and 100% of the investors. Assume, furthermore, that 1-μ is the probability that a single investor reaches the above threshold level of shift and take this probability to be constant from period to period. Then the Pascal distribution gives the probability that, say, 25% (i.e., r=1) of investors will reach their threshold level of expectation formation by the first period, second period, etc. The weights a_i (μ., r) of the distributed lag are, therefore, the probabilities describing the formation of expectations.

Furthermore, when a Pascal lag is of **specified order,** alternative distributions are summarized by a **single** parameter which is μ. Then (1-μ) indicates the speed of adjustment of expectations to new information. This means that, the lower μ is – or, equivalently, the higher is the probability, (1-μ), that an individual will reach the threshold of reaction to an actual change in inflation –, the less important are past (increasingly remote) inflation rates in determining current inflation expectations. Alternatively, we can put it in terms of a "permanent" theory of expectations:

> ... the more quickly investors perceive a change in inflation as being permanent, the more influence recent values of inflation have on inflation expectations ....[8]

My experimentation with Pascal lags produced the following results: the higher the order (r) of the distribution, the larger the mean and variance of the distributed lag weights. This was to be expected since the mean lag of a Pascal probability distribution is given by

$$\frac{r\mu}{(1-\mu)} \tag{14}$$

and its variance by

$$\frac{r\mu}{(1-\mu)^2} \tag{15}$$

A larger r means an increase in the proportion of investors who must reach a threshold level of shift before any major change in expectations occurs and a lengthening of the period in which larger numbers of investors do so[9]. For the entire period as well as for most subperiods, a second – order lag produced the best fit of equation (12). This may be attributed to two facts: first, that the shape of higher (than the second) order distributions is hardly different, and, second, that the peak response to a price change is constrained to come about with a considerable delay as r increases. As far as the parameter μ is concerned, its optimal value was found to depend upon the particular sample period chosen. An exhaustive search produced a range of optimal values which extended from $\mu = 0.75$ to $\mu = 0.86$ (depending on the sample period). As explained above, $(1-\mu)$ may be interpreted as the probability that, in a given period, an individual will react to a change in the actual rate. The higher is $(1-\mu)$, the faster is the reaction of an investor to such a change. Since this probability was estimated to be between 0.14 and 0.25, it suggests considerable delay and hesitation on the part of the public to let a change in actual inflation affect expected inflation. My findings did not support Yohe's and Karnosky's claim that results obtained with a second-order lag were not appreciably different from those obtained with the adaptive expectations model, which is in effect a Pascal lag model of first-order. In Table 4 I present statistics obtained from regressing nominal interest rates on expected prices generated by a Pascal distributed lag. Some comments on the relevant regressions are in order:

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8. Sinai [15], p. 28.

9. In other words, as r increases, it takes longer for the peak of the distribution (i.e., the peak response) to come about; accordingly, the effect of the lagged terms decays at a slower pace.

TABLE 4

Statistics obtained from regressing interest rates on expected prices when a pascal lag is employed

Period	Method	Price Variable	R^2	SEE	Constant	Estimated Coefficient of Expected Prices	D.W.
1951 (II) – 1974 (IV)	OLS	$\mu = 0.75$	.4778	.0133	.0289 (9.9) [a]	0.979 (9.2) [a]	0.03
»	»	$\mu = 0.78$	.4885	.0131	.0270 (8.8)	1.079 (9.4)	0.03
»	»	$\mu = 0.80$	.4765	0.133	.0260 (8.1)	1.133 (9.2)	0.02
»	»	$\mu = 0.86$	.4791	.0132	.0284 (9.5)	1.172 (9.3)	0.03
»	CORC	$\mu = 0.75$	.9917	.0017	.0637 (7.5)	0.452 (6.2)	1.41
»	»	$\mu = 0.78$	.9917	.0017	.0632 (6.7)	0.523 (6.1)	1.38
»	»	$\mu - 0.80$	.9916	.0017	.0631 (6.3)	0.569 (6.0)	1.34
»	»	$\mu = 0.86$	.9917	.0017	.0633 (7.3)	0.553 (6.2)	1.39
1951 (I) – 1960 (IV)	OLS	$\mu = 0.75$	.0425	.0062	.0408 (14.6)	– 0.158 (1.3)	0.06
»	»	$\mu = 0.78$	.0527	.0062	.0418 (13.2)	– 0.204 (1.4)	0.07
»	»	$\mu = 0.80$	.0725	.0061	.0430 (12.6)	– 0.264 (1.7)	0.07
»	»	$\mu = 0.86$	.0478	.0062	.0412 (14.2)	– 0.203 (1.4)	0.06
»	CORC	$\mu = 0.75$	.9567	.0013	.0490 (7.1)	0.126 (1.5)	1.33
»	»	$\mu = 0.78$	.9562	.0013	.0478 (7.1)	0.138 (1.4)	1.32
»	»	$\mu = 0.80$	.9560	.0013	.0489 (6.5)	0.153 (1.3)	1.32
»	»	$\mu = 0.86$	.9566	.0013	.0485 (7.2)	0.153 (1.5)	1.32

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»	»	$\mu = 0.78$	.4885	.0131	.0270 (8.8)	1.079 (9.4)	0.03
»	»	$\mu = 0.80$	.4765	0.133	.0260 (8.1)	1.133 (9.2)	0.02
»	»	$\mu = 0.86$	.4791	.0132	.0284 (9.5)	1.172 (9.3)	0.03
1961 (I) – 1974 (IV)	OLS	$\mu = 0.75$	.9173	.0047	.0362 (28.3)	1.033 (24.5)	0.22
»	»	$\mu = 0.78$	.9261	.0045	.0347 (27.6)	1.116 (26.0)	0.21
»	»	$\mu = 0.80$	.9188	.0047	.0339 (25.1)	1.171 (24.7)	0.18
»	»	$\mu = 0.86$	.9207	.0046	.0357 (28.2)	1.231 (25.0)	0.21
»	CORC	$\mu = 0.75$	.9880	.0018	.0555 (8.1)	0.576 (6.5)	1.56
»	»	$\mu = 0.78$	.9882	.0018	.0497 (8.8)	0.714 (7.3)	1.56
»	»	$\mu = 0.80$	.9882	.0018	.0479 (8.9)	0.781 (7.5)	1.49
»	»	$\mu = 0.86$	.9881	.0018	.0544 (8.0)	0.710 (6.6)	1.55

a : The figures in parentheses are t-statistics.
SOURCE: Equation (12) in the text.

(i) Introduction of the Cochrane-Orcutt adjustment leads to a substantial, and sometimes dramatic, improvement in the explanatory power of the equations. However, comparing CORC regressions with OLS ones reveals that this is accomplished at the expense of obtaining **higher** estimates of the real interest rate (which reach the unreasonable level of 6.3 percent for the entire 1951-74 period) and/or **lower** coefficients concerning the impact of expected prices on market interest rates. These coefficients are 30 to 50 percent lower than those generated by OLS.

(ii) If we ignore the sub-period 1951-60, we see that OLS estimation produces expected price coefficients which are close to unity (or not significantly different from unity). On the contrary, the auto-correlation adjustment generates coefficients which are persistently below unity, implying less than full adjustment of nominal interest rates to expected prices.

(iii) The adjustment to expected prices is larger in the 1961-74 subperiod than in the 1951-60 one. In the early period, the relationship between interest rates and expected prices is weak or even negative.

(iv) Although the best value of μ seems to depend on the sample period used, it is fair to say that a value of 0.78 for μ is bound to generate quite satisfactory results.

On the basis of the above remarks, one can conclude that the Pascal method is superiod to all models examined thus far in several respects (e.g., it produces short lags with good statistical fit and meaningful coefficients). Nevertheless, there remain some aspects in which this model is somewhat unsatisfactory. That is to say, the improvement of the CORC equations vis-a-vis the OLS counterparts is gained at the cost of possibly distorting the estimates of the real interest rate and the expected price. Accordingly, there is still room for improvement in the statistical estimation of the relationship between interest rates and expected prices. The search for further improvement leads us to the polynomial lag method.

4. The Polynomial Lag Method

The introduction of the polynomial distributed lag (PDL hereafter) into the nominal interest rate equation gives rise to a new model of expected inflation, which can be written as follows:

$$RN_t = RR + \sum_{i=0}^{h} a_i \dot{P}_{t-i} + U_t \qquad (16)$$

$$\text{where } a_i = \sum_{j=0}^{K} g_j i^j, \; i = 0, 1, 2, \ldots, n$$

$$a_i = 0 \qquad \text{for } i = n+1, n+2, \ldots$$

The weights follow a finite polynomial of order n and degree K and are made to lie along a K^{th} degree polynomial curve. The major advantage of this lag is its ability to reduce multicollinearity among the price change terms, thus helping us to identify the exact parameters associated with each independent variable. Nevertheless, the polynomial (or Almon) lag does not eliminate inconsistency, which may be present if the residuals (U_t's) are autocorrelated. Hence the need to combine the Almon lag with the Cochrane – Orcutt correction.

Empirical evidence presented by Yoge and Karnosky [17] suggests that introduction of PDL may improve the performance of Fisher's equation (i.e., eq. 4). Since my findings were mostly in line with theirs, I will not comment on their study except for divergent findings. Tables 5 and 6 present results from which one can draw the following implications:

(a) The explanatory power of the PDL equation fitted to the whole period, 1951-74, is quite satisfactory. An examination, however, of the subperiods reveals that this is due to the superior fit of the subperiod 1961 (I) – 1974 (IV) (see Table 6). The difference between the regressions for the two subperiods is not limited to their statistical fit but is even more profound when one looks at the estimated coefficients. Most coefficients are negative and statistically insignificant when the regression is run with 1950-60 data; on the contrary, estimation of the second subperiod turns out positive and statistically significant coefficients.

(b) PDL as well as PDL-CORC (i.e., PDL combined with Cochrane-Orcutt iteration) produces lags which are shorter than those implied by most other estimating techniques. The mean lags implied by the equations of Table 6 are[10]:

Period	Estimating Technique	Mean Lag (in quarters)
1951-1960	PDL	8.4
»	PDL-CORC	6.7
1961-1974	PDL	6.3
»	PDL-CORC	6.4

These figures provide some support to the view that faster formation of price expectations and/or more powerful price effects on interest rates characterized the 1961-74 period.

(c) The data reject the Fisherian hypothesis for the 1951-60 period but they lend support to it for the 1961-74 period. Further tests for the 1966 (I) – 1973 (III) subperiod reveal that it conforms particularly well to a Fisherian world since the sum of coefficients is very close to unity; this implies that a given increase in the expected rate of inflation will be fully reflected in the nominal interest rate. It is worth noting

10. The mean lag is defined as $\sum_{i=0}^{h} ia_i \Big/ \sum_{i=0}^{h} a_i$.

TABLE 5

Estimated coefficients of the relationship between interest rates and price changes when a polynomial distributed lag is employed

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1951 (II) – 1974 (IV)

Lag	PDL a_i	PDL t-stat.	PDL-CORC a_i	PDL-CORC t-stat.
0	.093	4.6	.032	3.0
1	.090	6.1	.040	5.1
2	.087	8.0	.046	6.8
3	.083	9.0	.049	7.1
4	.080	8.2	.051	6.8
5	.076	6.9	.051	6.5
6	.071	5.8	.050	6.1
7	.067	5.1	.048	5.7
8	.062	4.6	.044	5.2
9	.057	4.3	.040	4.7
10	.052	4.0	.036	4.1
11	.047	3.7	.031	3.5
12	.041	3.2	.026	3.0
13	.036	2.4	.021	2.6
14	.030	1.6	.017	2.2
15	.024	1.0	.013	1.9
16			.010	1.6
17			.009	1.2
Sum:	.997		.614	

	PDL	PDL-CORC
Constant:	2.81 (t= 6.3)	5.28 (t= 8.0)
Mean Lag:	5.9 quarters	6.7 quarters
R^2 :	0.558	0.992
SEE :	1.23	0.17
D.W. :	0.03	1.48

SOURCE: Equation (16) in the text.

TABLE 6

Estimated coefficients of the relationship between interest rates and price changes when a polynomial distributed lag is employed

1951 (I) – 1960 (IV)

	PDL		PDL-CORC	
Lag	a_i	t-stat.	a_i	t-stat.
0	−.054	1.05	.010	0.72
1	−.049	1.19	.018	1.21
2	−.015	0.60	.028	1.89
3	.007	0.28	.032	2.38
4	.009	0.36	.030	2.57
5	−.005	0.23	.023	2.23
6	−.022	1.01	.015	1.64
7	−.033	1.54	.008	0.95
8	−.035	1.70	.006	0.71
9	−.027	1.20	.010	1.20
10	−.015	0.68	.019	2.27
11	−.005	0.29	.027	2.89
12	−.002	0.12	.010	1.27
13	−.009	0.41		
14	−.024	1.16		
15	−.042	1.19		
16	−.061	1.53		
Sum:	−0.384			0.237

	PDL	PDL-CORC
Constant:	4.56	5.49
Mean Lag:	8.4 quarters	5.5
R^2:	0.208	0.967
SEE:	0.627	0.126
D.W.	0.14	1.45

1961 (I) – 1974 (IV)

	PDL		PDL-CORC	
Lag	a_i	t-stat.	a_i	t-stat.
0	.091	2.44	.047	2.30
1	.070	1.75	.052	2.59
2	.049	1.69	.068	3.83
3	.040	1.43	.062	3.64
4	.050	2.00	.049	2.94
5	.076	2.81	.049	2.73
6	.109	4.37	.065	3.54
7	.138	4.71	.086	3.82
8	.151	5.60	.094	3.68
9	.142	4.69	.086	3.00
10	.112	3.76	.072	2.52
11	.067	1.38	.067	2.28
12	.011	0.24	.053	2.11
Sum:	1.107		0.852	

	PDL	PDL-CORC
Constant:	3.44	4.25
Mean lag:	6.3	6.4
R^2:	0.964	0.989
SEE:	0.334	0.183
D.W.:	0.43	1.42

SOURCE: Equation (16) in the text

that during 1973 (IV) – 1974 (IV) market interest rates, though exceptionally high, have admittedly failed to fully incorporate price expectations; since including these five quarters in the estimation period considerably suppressed the sum of coefficients, they were left out.

(d) Utilizing the Cochrane-Orcutt technique reveals the presence of serious autoregression in the residuals. It is easily understood then why combining a polynomial lag, which almost eliminates multicollinearity, with the Cochrane – Orcutt iteration, which drastically reduces autocorrelation, results in an impressive goodness of fit.

(e) The real interest rate shows a tendency to fall during the 1960's and early

1970's. This tendency appears to be halted or reversed beginning with 1973 (II) as a period by period run of the regression makes clear[11].

The record of empirical studies as well as my tests show that a polynomial distributed lag scheme, reinforced by a Cochrane-Orcutt iteration, can produce statistical results that are superior to those obtained by any other distributed lag model. Compared with the Pascal lag, the PDL stands better as far as the estimates of the real rate and the price coefficients are concerned. The former is lower in the PDL equations for both the entire period and the 1961-74 subperiod. As for the total impact of the lagged price terms, it is larger in the PDL-CORC equations for all sample periods. Therefore, the PDL method seems to be preferable in estimating the interest rate – expected price relationship. If the presence of other distributed lags calls for the use of an alternative econometric technique, one's best choice is the Pascal lag. However, before we draw any conclusion, we have to answer an additional question.

5. Does it Make a Difference Which Interest Rate and Which Price Variable We Use?

One may pose the question whether the pattern of the empirical relations presented in this paper holds invariably for all (or most) interest rates given the price index or for all price variables given the interest rate. That is to say, we have to examine how general and representative the reported results are. For this purpose, several market interest rates were regressed on various price indices. The former included, among others, the average yield on new issues of **high grade** corporate bonds (frequently used in recent research and published in the **Business Conditions Digest),** the average rate of industrial bonds, the average yield on bonds issued by public utilities and the commercial paper rate. The price variables included, besides the deflator for personal consumption expenditures (PCE), the consumer price index (CPI), the wholesale price index (WPI), the deflator for the gross national product (GNP), the deflator for the business gross product (BGP) and the price index for fixed investment (FIXINV). Table 7 presents, selectively, some results obtained by means of the polynomial distributed lag (combined with a Cochrane-Orcutt adjustment) for the periods 1951 (I) – 1960 (IV), 1961 (I) – 1974 (IV) and 1961 (I) – 1973 (IV). The last period was included for the reasons explained in section 4 (C). The findigs reported in Table 7 confirm, with few exceptions, the remarks made in the preceding section. Deviations from the common pattern are observed only when the wholesale price index is employed. This happens irrespectively of the particular interest rate used and may be attributed to the lack of any systematic synchronization between changes in the WPI and changes in market interest rates. As a result, regressions of nominal

11. A decrease in real interest rates during the 1960's is not supported by all studies; see, for example, Andersen and Carlson [1].

TABLE 7

The relationship between interest rates and expected price changes

EQ.	INTEREST RATE	PRICE	PERIOD	CONSTANT	SUM OF COEFFI-CIENTS	R^2	SEE	D.W.
1.	AVERAGE BOND RATE	CPI	1951-60	4.49	0.21	.964	.0013	1.46
2.	» » »	CPI	1961-74	3.71	0.85	.992	.0016	1.66
3.	» » »	CPI	1961-73	3.48	0.93	.992	.0015	1.58
4.	» » »	WPI	1951-60	7.32	0.16	.964	.0013	1.57
5.	» » »	WPI	1961-74	6.69	0.27	.988	.0019	1.29
6.	» » »	WPI	1961-73	6.77	0.29	.985	.0020	1.25
7.	» » »	GNP DEFLATOR	1951-60	1.87	0.25	.971	.0012	1.52
8.	» » »	GNP DEFLATOR	1961-74	3.42	0.97	.989	.0018	1.37
9.	» » »	GNP DEFLATOR	1961-73	3.32	1.00	.989	.0017	1.26
10.	» » »	BGP	1951-60	6.57	0.23	.965	.0013	1.59
11.	» » »	BGP	1961-74	3.89	0.96	.989	.0019	1.34
12.	» » »	BGP	1961-73	3.56	1.09	.990	.0017	1.20
13.	» » »	FIXINV	1951-60	4.66	0.23	.968	.0013	1.44
14.	» » »	FIXINV	1961-74	3.83	0.86	.987	.0020	1.37
15.	» » »	FIXINV	1961-73	3.74	0.87	.986	.0019	1.27
16.	HIGH GRADE RATE	PCE	1951-60	3.87	0.32	.900	.0025	1.57
17.	» » »	PCE	1961-74	4.60	0.62	.964	.0034	1.92
18.	» » »	PCE	1961-73	3.46	1.04	.967	.0030	1.79

interest rates on WPI changes produce coefficients of lagged terms which are frequently negative and statistically insignificant and which are summed up to considerably less than unity.

6. Conclusions

A survey of the available empirical studies and some further tests presented in this paper reveal that there exists a systematic relation between contemporaneous and past price changes on the one hand and nominal interest rates on the other. The former may be taken as a proxy for **expected** price changes. This relationship appears to be rather weak during the 1960's; the estimated coefficients frequently have the wrong sign and/or are statistically insignificant. However, after 1961 (I) and more profoundly after 1966 (I), these negative aspects disappear. Since that time price movements and changes in market interest rates are highly correlated. During the latter part of the sample period, the sum of the price term coefficients is either very close to unity or not significantly different from unity, implying that Fisher's hypothesis is not rejected. The overall superior statistical fit obtained for the latter subperiod is, furthermore, reflected by a significant reduction of autoregression. Specifically, the coefficient of autocorrelation is 20 to 40 percent lower in the post-1960 period[12].

When one comes to the mechanics of expectations formation, the choice of the distributed lag is the crucial one. Which particular scheme of distributed lag is employed seems to be the single most important factor determining both **how fast** expectations are formed and **how strong** their impact is on observed interest rates. The former has to do with the length of the lag, while the latter indicates the extent to which market interest rates incorporate price expectations. If one is to rank the various distributed lags on the basis of their **overall** performance, the polynomial lag ranks first, followed closely by the Pascal lag and remotely by the geometric and unconstrained ones.

Finally, it was found that most statistical parameters of the interest rate-price change relationship were not conditional upon the particular interest rate or price variable used. Accordingly, distinct series of market interest rates were shown to be highly correlated with various price indices, with the notable exception of the WPI. Such a strong correlation has important implications for both economic policy and theoretical analysis.

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12. The only exception takes place when nominal interest rates are regressed on the WPI; then the autocorrelation parameter of the latter subperiod is, sometimes, lower and, at other times, higher than that of the earlier subperiod.

19.	» » »	CPI	1951-60	3.87	0.28	.891	.0026	1.50
20.	» » »	CPI	1961-74	3.84	0.73	.970	.0031	1.93
21.	» » »	CPI	1961-73	3.34	0.90	.975	.0027	1.81
22.	» » »	WPI	1951-60	4.36	0.25	.894	.0026	1.65
23.	» » »	WPI	1961-74	6.50	0.13	.969	.0032	1.89
24.	» » »	WPI	1961-73	6.41	0.18	.968	.0030	1.86
25.	» » »	GNP DEFLATOR	1951-60	4.12	0.37	.914	.0024	1.62
26.	» » »	GNP DEFLATOR	1961-74	3.44	0.90	.965	.0034	1.81
27.	» » »	GNP DEFLATOR	1961-73	3.31	0.94	.969	.0029	1.76
28.	» » »	BGP	1951-60	4.26	0.34	.907	.0024	1.66
29.	» » »	BGP	1961-74	3.94	0.86	.962	.0035	1.72
30.	» » »	BGP	1961-73	3.48	1.03	.968	.0030	1.64
31.	» » »	FIXINV	1951-60	3.88	0.31	.906	.0025	1.66
32.	» » »	FIXINV	1961-74	3.77	0.84	.963	.0035	1.83
33.	» » »	FIXINV	1961-73	3.55	0.87	.967	.0030	1.74
34.	INDUSTRIAL BOND RATE	PCE	1951-60	5.19	0.20	.968	.0013	1.31
35.	» » »	PCE	1961-74	4.82	0.65	.989	.0017	1.38
36.	» » »	PCE	1961-73	3.61	1.01	.988	.0017	1.27
37.	» » »	BGP	1951-60	5.49	0.21	.971	.0012	1.38
38.	» » »	BGP	1961-74	5.65	0.50	.989	.0018	1.40
39.	» » »	BGP	1961-73	3.60	1.00	.989	.0016	1.22
40.	» » »	FIXINV	1951-60	4.42	0.22	.969	.0012	1.32
41.	» » »	FIXINV	1961-74	5.99	0.48	.987	.0017	1.25
42.	» » »	FIXINV	1961-73	3.77	0.81	.987	.0018	1.14

SOURCE: Equation (16) in the text.

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