HYBRID OPTIMIZATION TECHNIQUE FOR THE SOLID WASTE COLLECTION AND DISPOSAL PROBLEM

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Introduction

The United States is an economically rich country whose production of consumer goods is high relative to the size of its population. It is therefore faced increasingly with the problem of what to do with these consumer goods when their utility, as judged by the consumer, has been exhausted.

All "used" products (solid waste) are disposed of by one of two methods: recycling[1] or collection and disposal by any of several different ways. Since the collection of solid waste involves routing refuse via various transport modes from the point of generation to the point of disposal, the object of this study is to develop an algorithm which may be used to define and schedule near-optimal collection plans for refuse systems facing high seasonal variation in the quantity of solid waste generated.

Section 1

The general distrubution problem was formulated by F.L. Hitchcock in 1941[2]. Independently, in 1949, T. C. Koopmans[3] also sutdied it, thus the "Hitchcock-Koopmans Transportation Problem". George Dantzig[4], in a 1951 application of the

1. Recycling is probably the necessary long-run method. See arguments in chapter one of unpublished paper. "Hybrid Optimization Technique for the Solid Waste Collection and Disposable Problem" by Angelos A Tsaklanganos and Michael Clayton presented at the Joint National Meeting of the Institute of Management Services (TMS), The Operations Research Society of America (ORSA) and the Systems Engineering Group of the American Institute of Industrial Engineers (ALLE) Atlantic City, New Jersey, November 8-10, 1972.

2. Hitchcock, F.L., "Distribution of a Product from Several Sources to Numerous Localities", *Journal of Mathematical Physics (Studies in Applied Mathematics)*, Vol. 20, 1941, pp. 224-230.

3. Koopmans, T.C., "Optimum Utilization of the Transportation System", *Econometrica*, supplement to Vol. 17, 1949, pp. 136-145.

4. Dantzig, George B., "Application of the Simplex Method to a Transportation Problem, *"Activity Analysis of Production and Allocation*, Chapters XXIII, Cowels Commission Monograph No. 13, ed. T. C. Koopmans, New York, 1951.

simplex method, proposed a solution technique which was widely utilized until Ford and Fulkerson[5] published their paper in 1956, basing their algorithm on the combinatorial procedure developed by H. Kuhn called the "Hungarian Method"[6]. No further work appears to have been done in this area until the late 1960's when F. Tillman considered the multiterminal distribution problem with probabilistic demands[7]. J. Quon, *et. al.*,[8] approached the specific problem of distribution in a solid waste collection system in 1965, using a simulation technique to determine the significant parameters and to differentiate between those of primary and those of secondary importance. Since 1965, two other simulations in the area of solid waste collection have been published, the first by Truitt, *et. al.*,[9] in 1969, and the second by R. M. Bodner, *et. al.*,[10] in 1970.

The general distribution problem as formulated by Hitchcock may be stated as follows[11]. Denote the cost of shipping one ton of a produce from the i^{th} factory to the j^{th} city by a_{ij}, and the number of tons shipped by x_{ij}. The total cost of distribution, y, for m factories and n cities is, therefore:

$$y = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} x_{ij} \tag{1}$$

The problem is to minimize y. By the nature of the problem, x_{ij} is greater than or equal to zero for all i and j:

$$X_{ij} > 0 \quad i = 1, 2, \ldots m \qquad j = 1, 2, \ldots n \tag{2}$$

Hitchcock autlined a solution procedure, which is similar to the simplex method.

Koopmans' technique was based upon marginal costs whereby the movement of vehicles from point to point in the distribution network was defined in terms of marginal costs, and the objective became to minimize the sum of these marginal costs. Citing Koopmans' example[12], consider two points, A and B. Assume A to need three train loads of products a day, while B requires six train loads. The net empty movement of train cars is therefore three cars a day from B to A. The marginal cost from

5. Ford, L. R., Jr., and Fulkerson, D.R., "Solving the Transportation Problem.", *Management Science*, Vol. 3, 1956, pp. 24-32.

6. Kuhn, H.W., "The Hungarian Method for the Assignment Problem", *Naval Research Logistics Quarterly*, Vol. 2, 1955, pp. 83-97.

7. Tillman, F.A., "The Multiterminal Delivery Problem with Probabilistic Demands", Transportation Science, Vol. 3, No. 3, August, 1969, pp. 192-204.

8. Quon, J., Charnes, A., and Werson, S., "Simulation and Analysis of a Refuse Collection System", *Journal of the Sanitary Engineering Division*, ASCE, Vol. 91, No. SA5, 1965.

9. Truitt, M.M., Liebman, J.C., and Kruse, C.W., "Simulation Model of Urban Refuse Collection", *Journal of the Sanitary Engineering Division*, ASCE, No. SA2, April, 1969.

10. Bodner, R.M., Cassell, E.A., and Andros, P.J., "Optimal Routing of Refuse Collection Vehicles, "*Journal of the Sanitary Engineering Division*, ASCE, No. SA4, August, 1970.

11. Hitchcok, *op. cit.*

12. Koopmans, *op. cit.*

A to B for one additional carload a day equals the sum of the loading, A to B travel, unloading, and B to A return travel costs. The marginal cost from B to A for one additional carload a day equals the sum of the loading and unloading costs only. The optimal routing is achieved by arranging routes so that the sum of the marginal costs is minimized.

Significant assumptions made by both Hitchcock and Koopmans, which have importance in the solid waste problem, concern demand, supply, and costs. Demand for a commodity at each demand point was assumed to be known and constant, although in practice, it is a variable, distributed about some mean value. The constant demand assumption for the general distribution problem is an approximation made by letting the means of the demands of each demand point represent the known, fixed demand. The accuracy of the resulting solution is dependent upon the distribution around the mean of the actual values of the demand. If the distribution shows little significant variation from the mean, then the assumption of known, fixed demand yields plausible results. Yet, on the other hand, if the disbritution is spread significantly about the mean, then assuming the value of the mean as a fixed demand may yield inaccurate results, particularly if the distribution is other than a normal distribution. (For the urban solid waste problem, it has been found that the quantity of refuse generated at each demand point is variable and distributed geometrically)[13]. This solution's unacceptability lies in the costs of inadequate and excessive capacities. For example, if a vehicle fails to complete a scheduled refuse collection route because of insufficient capacity, then the cost of such failure is reflected in the extra trip to the disposal site necessary to complete the assigned route, the overtime wages which may accrue, and the influence upon other routes assigned to the same vehicle. The amount of time spent traveling to and from the disposal site represents a significant portion of the collection costs, and so the fewer disposal trip made, the lower the collections costs will be. A consideration of a technique for handling the problem of variable demand will be discussed later. What is of importance at this point is recognition that the demand is a solid waste problem is not constant and known, and that therefore the solution techniques of Hitchcock and Koopmans need to be applied with caution.

Several studies have discovered that the quantity and composition of solid waste has cyclical variations over time[14]. Quon found that variations in the amount of time necessary to service each demand point had a relatively important effect on the overall efficiency of collection[15]. (The time necessary to service a demand point depends upon the amount of refuse at that demand point, so variations in the quantity of re-

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13. Unpublished letter from William S. Galler, Associate Professor of Civil Engineering, North Carolina State University at Raleigh, March 16, 1971.

14. Hickman, H.L. Jr., "Studies of Characteristics of Municipal and Residential Solid Waste", *Engineering Foundation Research Conference, Proceedings*, Conterence Reprint Number F-3, July, 1967.

15. Quon, *op. cit.*, Efficiency has units *man* — min / ton.

fuse generated will have a significant effect upon the overall collection efficiency). Recent studies, notably those of Quon[16] and Galler[17] make use of this fact. If a forecast of seasonal variations of refuse generation could be made with reasonable confidence, then route assignments could be adjusted by applying Hitchcock's or Koopmans' techniques to several sets of demands over an entire cycle. Hance the Hitchcock-Koopmans solutions, although not suitable for the solid waste collection problem as it stands, could be utilized if the proper modifications were made, and if such modifications provided models with constant demand.

The second significant assumption is that each distrubution point has a known, fixed supply of the commodity in question. For the ensuing discussion, let the demand points correspond to the points of refuse generation and the supply points to the disposal sites. (The particular labels used are merely a convention). It is true that each disposal site has a known maximum capacity, and in this sense a fixed supply. Unless each disposal site is operating at maximum capacity, however, the "supply" at each site is unknown, bounded by zero at the lower extreme and its capacity at the upper. It would be unwise to have only enough disposal capacity to enable all disposal sites to operate at full capacity at all times, since any increase in the amount of solid waste generated would overload such a system. Therefore, a reasonable assumption to make would be that disposal sites operate sufficiently below capacity to be able to handle seasonal fluctuations, hence causing the value of the supply to be unknown, in contrast to the known supply assumption.

The third assumption, central to Hitchcock's study, was that the unit costs of transporting the commodity between any two points of the system were known and constant. Most studies done on the distribution problem made this same assumption: that costs were linearly dependent upon the number of number of miles driven. On e exception was Koopmans, who recognized that the marginal cost of shipping one additional unit from point A to point B within a system was dependent upon the net flow of commodities between these points as well as being linearly related to the distance between them. Koopmans' marginal cost concept requires flow in both directions, from A to B, and from B to A. In the solid waste problem, if B is the disposal site and A is the demand point there would be no flow from B to A, for no one would wish refuse to be delivered to his home. Since flow does not exist in both directions, Koopmans' marginal cost concept is inappropriate here.

Another significant factor affecting the accuracy of the linear dependence assumption is that of traffic congestion. It is suggested that congestion may be a significant parameter of refuse collection costs. Collecting refuse in New York City between, say, 4 p.m. and 6 p.m. would obviously be more time consuming per unit refuse than between 2 a.m. and 4 a.m. Accessibility of the refuse containers may be greatly re-

16. *Ibid.*

17. Galler, W.S., "Study and Investigation of Solid Waste Disposal, City of Raleigh, North Carolina", *Engineering Foundation Research Conference, Proceedings*, Conference Reprint Number a-7, July 1967.

duced due to vehicles parked along the roadway or people moving along the sidewalks, and movement of the collection vehicles may be greatly inhibited, especially movement to and from the disposal site. The collection vehicles themselves may also be a cause of congestion, thus becoming a diseconomy to the community. Since the congestion parameter primarily affects the length of time required for collection, and thus labor costs, the most significant costs in refuse collection[18 19 20], are directly related to time factors which cause any increase in the time required for collection simultaneously cause increases in total collection costs. Therefore, when congestion becomes a significant parameter in the solid waste problem, costs can no longer be assumed to be linearly dependent upon the miles driven, but must reflect the non-uniform effect of congestion[21].

Since the early publications of Hitchcock and Koopmans, some variations on their techniques have emerged, seeking to decrease the computation time necessary for large problems[22]. The same basic assumptions are made, however, so the improvement gained lies in the fact that the same results can be obtained much faster. Tillman was the first to alter one of the basic assumptions when he suggested dealing with probabilistic demand[23]. In order to incorporate the concept of probabilistic demand, he used a twophase procedure – the route development phase has come to be known as the "savings" approach (that is, the decision to join two points on a route based upon the "savings" of distance rendered by such joining). For example, consider the case shown in Figure (1), where T_1 is a distribution point, and P_1 and P_2 are demand points. The distance from the j^{th} terminal to the ith demand point is given by d_i^j while intra-demand point distances (e.g., between i and k) are denoted by d_{ik}. Assuming that initially a vehicle from the terminal is assigned to each demand point, the total distance traveled, D, is given by:

$$D = a\ (d_1^1 + d_2^1) \tag{3}$$

18. Unpublished memorandum from South District Maintenance Supervisor to Bill Hap, Maintenance Supervisor, Yellowstone National Park, December 9, 1970.

19. Quon, *op. cit.*

20. Paul, I.L., and Wilson, D.G., "Reviewing the Prospects for Automating Urban Solid Waste Collections", *ASME* paper 69-WA/PID-21.

21. This is true in light of the fact that congestion is not uniform throughout the model, and therefore cannot exert a linear influence. For a more complete discussion of the errors due to linearity assumptions, see: Baumol, W.J., and Bushnell, R.C., "Error Produced by Linearization in Mathematical Programming", *Econometrica*, Vol. 35, No. 3-4, July - October, 1967.

22. As examples, see: Minty, G.J., "A Varieant on the Shortest-Route Problem", *JORSA*, Vol. 6, No 6, 1958.
Gleyzal, A., "An Algorithm for Solving the Transportation Problem", *Journal of Research of the National Bureau of Standards*, Vol. 54, No. 4, April, 1955, Research Paper 2583, pp. 213-216.
Russell, Edward, J., "Extension of Dantzig's Algorithm to Finding an Initial Near-Optimal Basis for the Transportation Problem", *Operations Research*, Vol. 17, No. 1, Jan. - Feb. 1969, pp. 187-191.

23. Tillman, *op. cit.*

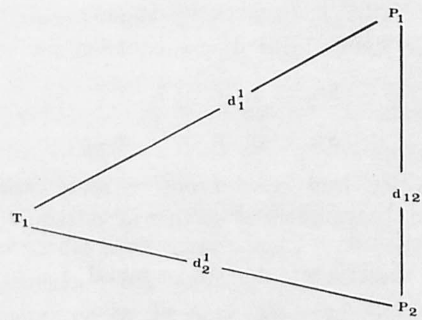

FIGURE I: Illustration of distances between a distribution point T_1 and two demand points P_1, P_2, and of intra-demand distance.

The "savings" gained by joining points P_1 and P_2 to form the new route T_1 P_1 P_2 T_1 are given by:

$$S_{12} = d_1^1 + d_2^1 - d_{12} \tag{4}$$

Caution should be exercised, however, for when two demand points close to one terminal and far from another are joined together, the greatest savings would be obtained by assigning them to the remoter terminal. Figure (II) illustrates this case.

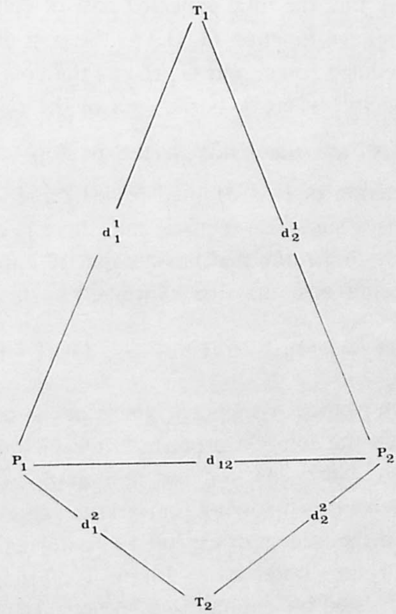

FIGURE II: The case of two demand points P_1, P_2, close to one terminal T_2 and far from another T_1, joined together.

The "savings" of route $T_1P_1P_2T_1$ is given by equation 5a) and the "savings" of route $T_2P_1P_2T_2$ by equation 5b).

$$S^1_{12} = d^1_1 + d^1_2 - d_{12} \tag{5a}$$

$$S^2_{12} = d^2_1 + d^2_2 - d_{12} \tag{5b}$$

Here, the greater savings lead to an erroneous route assignment. Therefore, the savings equations 3) and 4) are modified so that all distances are relative to the closer terminal.

$$\dot{d}^i_j = \min d^s_j - (d^i_j - \min d^s_j) \tag{6a}$$

$$s^i_{jk} = \ddot{d}^i_j + \ddot{d}^i_k - d_{jk} \tag{6b}$$

Equation 6a, 6b) gives the modifications where j is the terminal index and i and k are demand point indices. The notation min d^s_j means to choose the distance from j to the closest terminal s. The objective then becomes to arrange the routes so that the sum of the savings of the routes assigned to each terminal is maximized. Points considered for joining are those joined to a terminal and not already on the same route. The joining of the points must not violate the restrictions of the system, such as causing a route assignment to be greater than the capacity of the largest available vehicle.

The vehicle assignment phase of Tillman's technique utilizes the concept of probabilistic demand to assign vehicles of various capacities to the routes determined in Phase 1, in such a way that the total expected cost of collection for the given routes is minimized. He defines the function G_1 (L) as the cost of filling a truck prior to the completion of the scheduled route, and G_2 (L) as the cost of completing a scheduled route with excess capacity, where L is the sum of the random variables x_1, x_2, ... x_n, $L = \sum_1^n x_i$. The x_i are the loads at the n stops on a scheduled route. The probability density function of L is denoted by h(L) and the minimum expected cost for route by R[24]. If the assumption is made that there is enough flexibility in the system to assign a vehicle to a route that has a capacity approximately equal to R, i.e., C = R, then the expected cost may be expressed as in equation 7).

$$E\,(\text{cost}) = \int_0^R G_1\,(L)\,h\,(L)\,dL + \int_R^\infty G_2\,(L)\,h\,(L)\,dL \tag{7}$$

The objective of the vehicle assignment phase of the procedure is to minimize the expected cost. To begin the solution procedure, the G functions and the distribution of x_i are defined. An expression for the minimum cost may thus be found by evaluating equation 7), and then solving for various values of R and n. The values of R which correspond to the minimum expected cost for each n are the R values used to assign vehicles to routes obtained in Phase 1 of the procedure.

The procedure suggested by Tillman is a "practical method that provides "good"

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24. The value of R determined for each route is the load assigned to the vehicle for that route.

answers to a rather difficult problem"[25]. His two primary assumptions seem applicable to the solid waste problem. The first – that enough flexibility exists so that a vehicle of capacity C can be assigned to a route of load R thus making C approximately equal R – is self satisfied by the restrictions of the system itself, for the length of the routes is bounded by the capacity of the larges available vehicle. Thus, the routes are combined, generating further "savings" until the capacity restriction comes into play. The other assumption – that all demand points have the same mean – may also be applicable to the solid waste collection problem if a modification of the system under study is made. While the type of distribution at each demand point in a refuse collection system may be the same, the means of the distribution may not be equal except in a micro-analysis[26]. If the demand points have different means, the points with the larger means may be divided into several points at the same location ($d_{i,j} = 0$) so that the means do become equal. The size of the problem may increase considerably if the range of the means is great, thus becoming a limiting factor for the implementation of Tillman's technique. As with the Hitchcock and Koopmans procedures, the Tillman technique does not take large seasonal fluctuations of the quantity of the refuse generated into consideration. The distributions could be made in such a way that these fluctuations are accounted for, but handling the seasonal fluctuations as outlined previously woyld clearly result in a more accurate solution. None of the techniques discussed above provide insight into the effect of alterations upon the system under study.

With the advent of high-speed computers and the development of simulation techniques, the effects of varying parameters could be gauged. The simulation techniques to not provide optimal solutions per se, although by proper definition, an optimal solution could be found. If the optimal solution were defined to be the "best" solution of 100 random trials, for example, clearly an optimal solution could be obtained, since although 100 trials may not be total enumeration, the probability of at least a near - optimal solution being among them is relatively high[27]. Quon noted that the value of simulation lay not in its optimizing capabilities, but in its ability to provide information about the system at low cost. "The use of the information developed from a verified simulation program as the basis for an over – all economic analysis of refuse collection and disposal systems promises a method of optimization that is cognizant of stochastic variation, is capable of singling out the most significant parameters, and can delineate the range of values for each parameter consistent with the optimal solution"[28].

As more accuracy is demanded of a solution technique, so the amount of known

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25. Tillman, *op. cit.*

26. Bodner, *op. cit.*, the DOWAR program is an example of a micro-analysis.

27. Len, S., "Computer Solutions of the Traveling Salesman Problem", *Bell System Technical Journal*, Vol. 44, No. 10, p. 2245, 1965.

28. Quon, *op. cit.*, p. 23.

information necessary to solve the problem is increased. Simulation of a system requires extensive knowledge of that system, thus creating large data collection costs. As with all techniques used in operations research, the marginal cost of any extra information must be weighted against the marginal utility of that information. This should be kept in mind when choosing a technique for application to a specific problem.

Section 2 outlined the development of a hybrid optimization technique designed for application to a particular aspect of the solid waste problem, the high seasonal variation of the quantity of refuse generated at each demand point.

Section 2

The Algorithm developed in this Section applies to solid waste collection problems with a high variability of quantities of refuse generated at each demand point over time. Such a situation occurs, for example, in recreational areas where there is a seasonal variation in the number of visitors. (The amount of refuse generated at a summer resort during February would be significantly less than the amount generated during the summer). Indeed, within the season itself, there may be rapid change in the amounts of refuse generated – Figure (III) shows the quantity of refuse generated in Yellowstone National Park over the period of one year[1]. The rapid and significant change during the summer months is the type of variability for which this algorithm is designed.

Techniques discussed heretofore have been applicable solely to a static situation where either the quantities of refuse were assumed to be constant, or the means of the related distributions were considered to be constant. The case of high variability, however, requires not only defining the optimal routing, but also the scheduling of several optimal routing plans, each with a different number of vehicles, reflecting the rapid changes in the quantity of refuse generated within the system over the season. That several plans are necessary may be seen by examining Figure III – the number of vehicles required to service Yellowstone Park during February is obviously different from the number required in July.

The algorithm is based on the relationship between the feasible shortrun average cost (FSAC) curve and the ideal short-run average cost (ISAC) curve, where the FSAC curve contains those points physically attainable by the collection firm in the short-run. General economic theory suggests that a firm is capable of operating at the minimum point of its FSAC curve by adjusting its mix of labor, capital, and output over time, but in the solid waste problem, the output corresponds to the re-

1. Unpublished Report, "Average Monthly Refuse Collection-Main Areas", National Park Service, Yellowstone National Park, 1969.

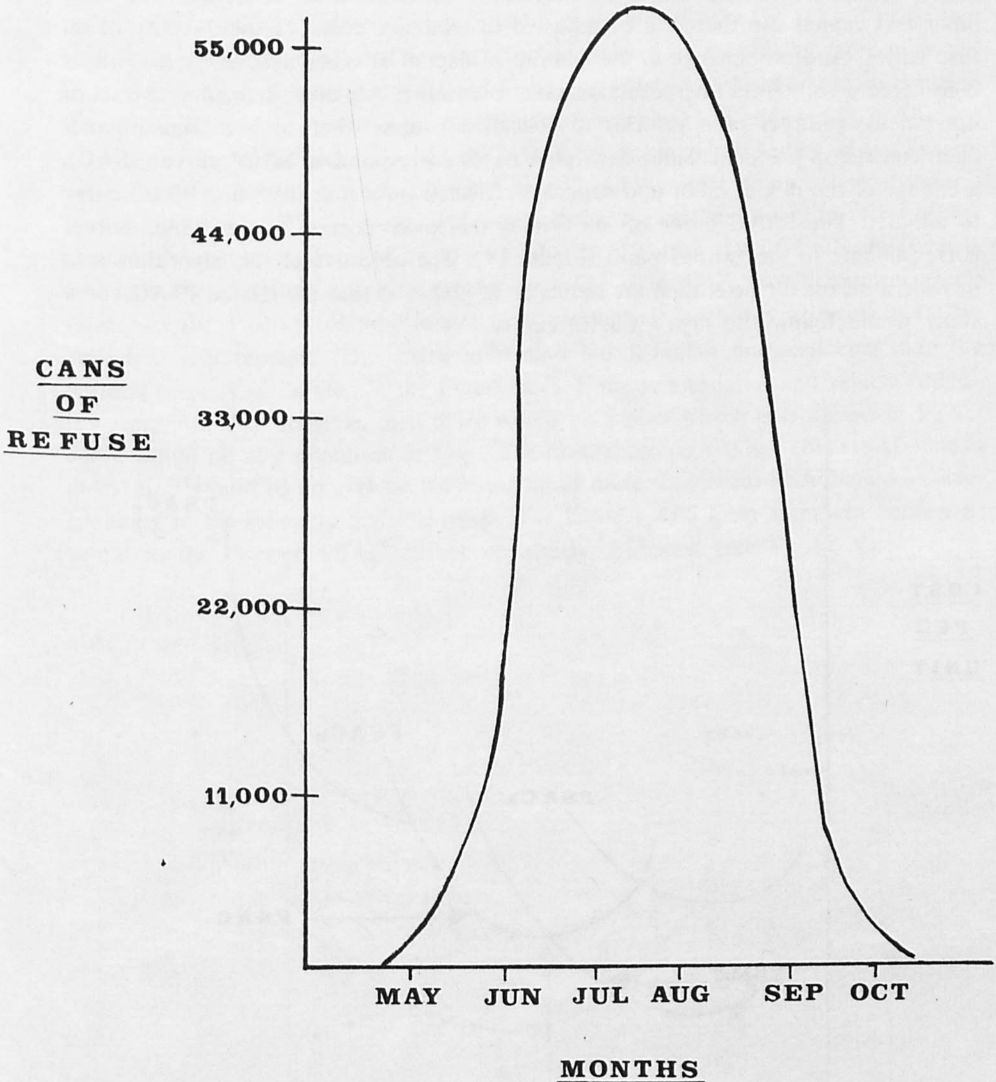

FIGURE III: Variations of refuse generated at Yellowston National Park (1969)

fuse generated in the system and is, therefore, exogenous to the collection firm. Only labor and capital can therefore be adjusted to minimize costs, as the quantity of refuse varies. And any change in the amount of capital or labor utilized by the firm is considered a shift from one collection plan to another. A plan is defined as the set of optimal assignments of n vehicles to collection routes, where n is a fixed number characteristic of the plan. Since each plan has a corresponding SAC* curve (PSAC), a change of the mix of labor and capital is reflected by a shift from one PSAC curve to another. The FSAC curve of the firm is the lower portion of the PSAC curves corresponding to the various plans (Figure IV). The objective of this algorithm is to provide a means for determing the sequence of plans, so that the related PSAC curve shifts would follow the firm's FSAC curve.

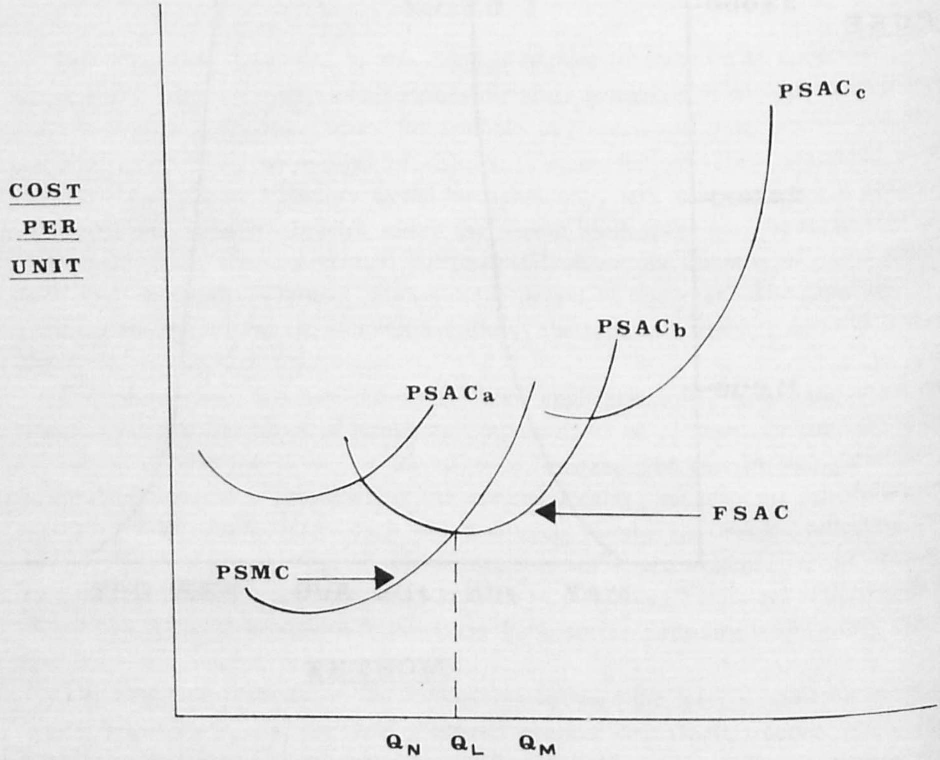

FIGURE IV.

* Short-run average cost

The technique is to locate the intersections of the PSAC curves. To find the points of intersection, the solid waste system is simulated over the season while two successive plans are evaluated for the optimal routing of vehicles and the expected cost of operation. A plan is considered to be operating in tis least-cost configuration if two conditions are met: (1) all vehicles (with workers) are engaged in productive activity for the entire workday; (2) the refuse collection requirements of the system are satisfied within the workday. Such a plan would therefore be operating at the minimum point of its PSAC curve, and would define the optimal quantity of refuse for that plan's operation, denoted by Q_L for plan b in Figure (IV). Clearly, for plan b, any qyantity of refuse which is greater than Q_L, say Q_M, would require vehicle utilization for a period of time longer than the normal work day in order to satisfy collection requirements. This extra utilization has a higher marginal cost than the optimal point, 1, as is seen in the Figure (IV). If the mix of labor and vehicle capacities were infinitely divisible, then there would be a plan whose corresponding PSAC curve would be at a minimum at Q_M. The ideal shortrun average cost (ISAC) curve of the firm would be an evelope enclosing all of these theoretical PSAC curves corresponding to the infinitely divisible plans. The ISAC would form a smooth curve tangental to the discrete PSAC curves previously discussed (see Figure V).

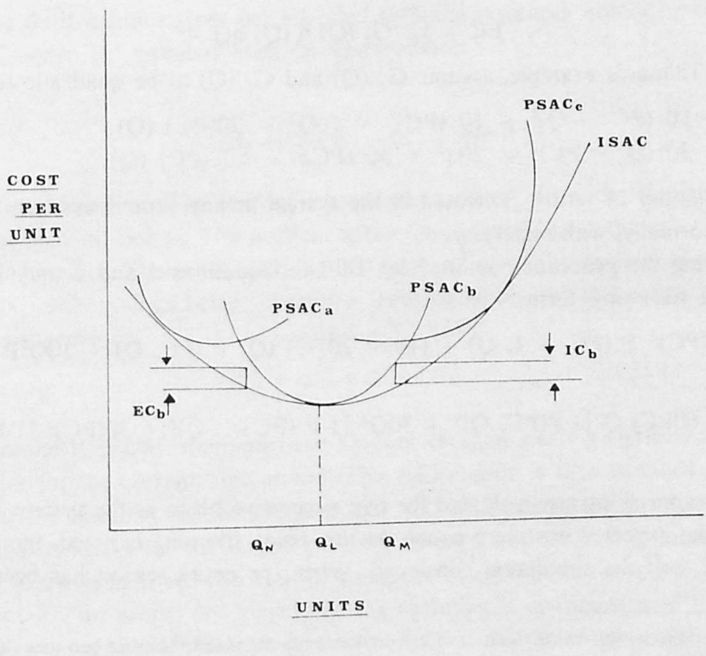

FIGURE V.

The difference between the expected cost of plan b at Q_M and the ISAC for Q_M is termed the expected cost of insufficient capacity of plan b, IC_b. This cost is attributed to insufficient capacity for the plan capacity, and the PC of plan b at this point would be less than the PC of the ideal plan. Similarly, for a quantity of refuse less than Q_L, say O_N, there would be an expected cost of excessive capacity of plan b, EC_b. The intersections of any two successive plans correspond to the situation where the IC of one plan equals the EC of the next. This alogrithm takes advantage of this fact to determine the time at which to change from one plan to the other[2].

For a given plan, the expected cost of insufficient capacity is denoted by G_1 (Q) and the expected cost of excess capacity by G_2 (Q). This is an extension of Tillman's technique for assigning vehicles to routes, to a method of assigning plans to time periods. The probability density function of Q is denoted by h(Q). Whereas Tillman evaluated the sum of the expected costs for each route, this algorithm calls only for the computation of the IC for one plan and the EC of the other, thus the G_1 (Q), and the G_2 (Q), and the h (Q) are related to the plan capacity, PC, rather than the route capacity. The expected costs may be expressed by:

$$IC = \int_0^{PC} G_1 (Q)\, hQ)dQ \tag{1}$$

$$EC = \int_0^{PC} G_2 (Q)\, h (Q)\, dQ \tag{2}$$

Using Tillman's example, assume G_1 (Q) and G_2 (Q) to be quadratic functions[3]:

$$G_1 (Q) = 10 (PC - Q)^2 = 10 (PC)^2 + 10Q^2 - 20(PC) (Q) \tag{3}$$
$$G_2 (Q) = 30 (Q - PC)^2 = 30q^2 + 30 (PC)^2 - 60 (PC) (Q) \tag{4}$$

The quantity of refuse generated in the system at any time is assumed to be distributed normally, with mean μ_Q.

Following the procedure outlined by Tillman, equations 1 and 2 may be expended to the following form:

$$IC = 10 (PC)^2 P (PC + 1, Q) + (10 - 20PC) (Q) P (PC, Q) + 10Q^2P (PC -1, Q) \tag{5}$$

$$EC = (30\text{-}60PC) Q\, |1\text{-}P(PC, Q)| + 30Q^2\, |1\text{-}P (PC-1, Q)| + 30(PC)^2\, |1\text{-}P (PC+1, Q)|. \tag{6}$$

These expressions are evaluated for two successive plans as the system is simulated, until the expected costs are equal. At this point, the time is noted, the next plan determined, and the simulation continued. When the entire season has been simula-

2. It should be noted that if the cost of conversion from one plan to another becomes significant relative to the expected costs, the shift point will be defined by the relationship, IC = EC + CC, where CC is the cost of conversion.

3. Tillman, F.A., "The Multiterminal Delivery Problem with Probabilistic Demands, "*Transportation Science*, Vol. 3 NO. 3, August, 1969.

ted, a schedule of collection plans will have generated, as well as the optimal routing for each plan.

The algorithm proceeds, as follows. Information about the solid waste system is input, including locations of disposal sites, locations of demand points, refuse generation functions for each demand point, capacities of the vehicles, and mileage among points of the system. Each demand point (point of refuse generation) is assigned to the nearest terminal, and the quantities of refuse at each demand point simulated for the first time period. The optimal collection routes are then determined using the "savings" approach. The IC for this plan is computed and compared with the EC of the next plan. If the IC is significantly less than the EC, the simulation proceeds to the next time period. If the IC does not significantly differ from the EC, the succeeding plan is adopted, the time period noted, and the simulation continued. When the IC is greater than the EC, it is an indication that the intersection point has been passed. Therefore, if the IC is found to be significantly greater than the EC, the time flow is reduced and reversed to return to the point of intersection. Figure (VI) presents the flow of the algorithm, which is iterated over the entire season.

Subroutine DIST computes the terminal matrix, for each terminal. The first column is reserved for the quantities of refuse at each demand point computed by REFUSE. The second column gives the distance from the terminal to each demand point. The third column gives the adjusted terminal distances according to equation (6a) of Chapter II, repeated here for convenience:

$$\mathring{d}^{i}_{j} = \min_{S} d^{s}_{j} - (d^{i}_{j} - \min_{S} d^{s}_{j})$$

The rest of the terminal matrix is a square array of the order r, where r is the number of demand points. The portion below the diagonal gives the mileages among the demand points. The portion above the diagonal gives the savings in joining any two points, such savings being computed from equation 6b) of Section 1, repeated here for convenience:

$$s^{i}_{jk} = \mathring{d}^{-i}_{j} + \mathring{d}^{-i}_{k} - d_{jk}$$

Subroutine REFUSE computes the amount of solid waste generated at each demand point for the current time period. This information is then installed as the first column of the terminal matrices. Subroutine COMBO scans the savings portion of the terminal matrices for the largest possible savings, and when it is located, the restrictions of the system are checked for violation, assuming the two points are joined. If none occurs, the points are joined and the scanning is continued until the next largest savings is located. If the restrictions are violated this time, the points are not joined, and are never again considered for joining. This method of developing the collection routes is the "savings" approach discussed in Section 1. Subroutine COSTS computes the IC and the EC for the current plan and the next plan respectively,

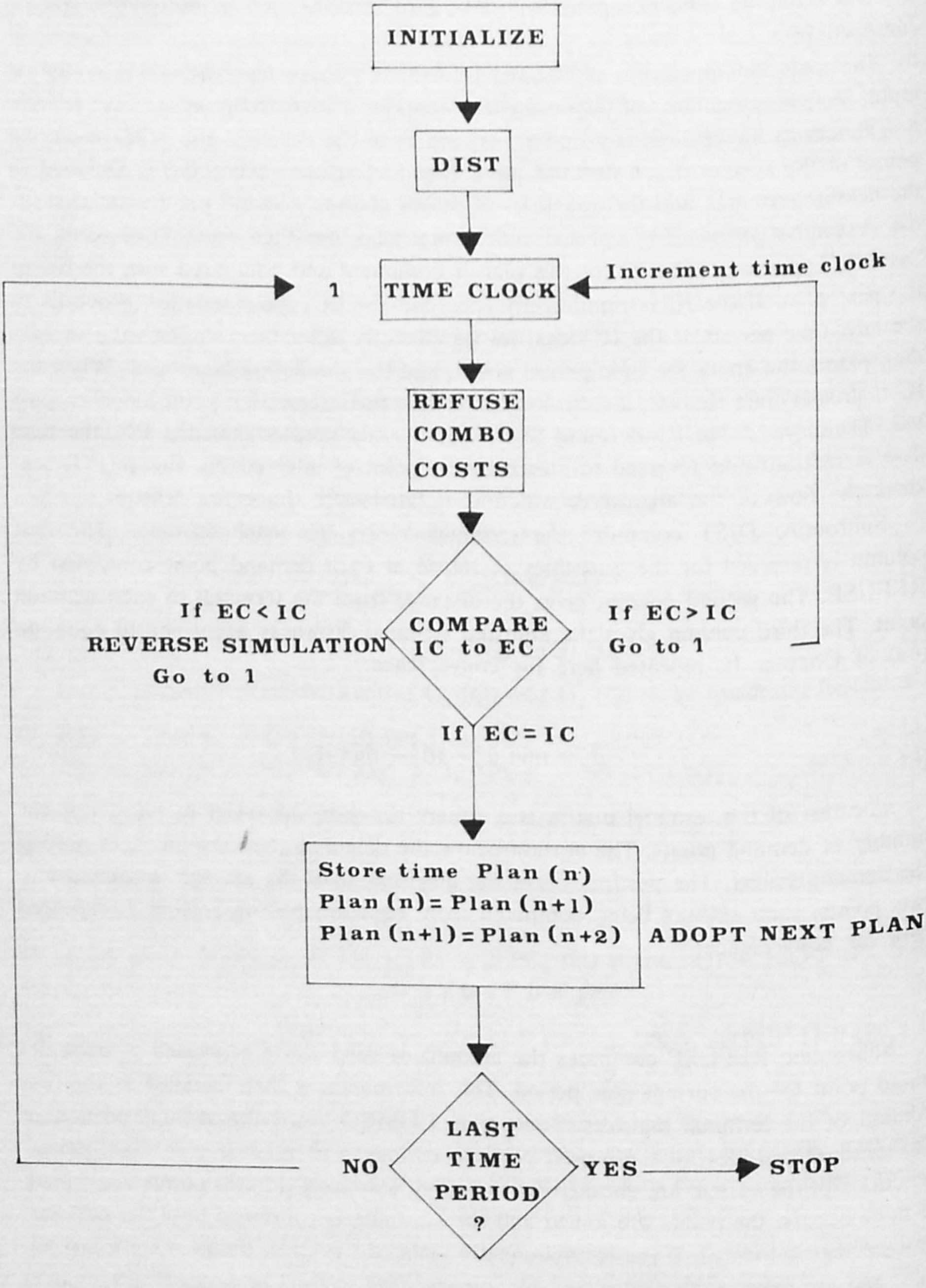
INITIALIZE
DIST
1
TIME CLOCK
Increment time clock
REFUSE
COMBO
COSTS
If EC<IC
REVERSE SIMULATION
Go to 1
COMPARE
IC to EC
If EC>IC
Go to 1
If EC=IC
Store time Plan (n)
Plan (n) = Plan (n+1)
Plan (n+1) = Plan (n+2)
ADOPT NEXT PLAN
NO
LAST
TIME
PERIOD
?
YES
STOP

FIGURE VI

using equations 5 and 6 developed in this chapter.

The primary drawback of this algorithm is the large amount of data required for its implementation. The refuse generation functions themselves need careful analysis in order to derive significant equations. The accuracy of this algorithm depends upon the accuracy of the input data, and this being the case, the less reliable the refuse generation functions become, the less rekiable the results of the algorithm will be. The use of this algorithm, as any operations research method, must therefore be evaluated in terms of the data collection cost, that is, the marginal cost of additional data must be compared to the marginal utility of extra information that would be derived from the data. Simulation, accompanied by its large data collection cost, was chosen or use in this algorithm because, as a practical technique, it has proven to be an effective, inexpensive method of analyzing compex systems. Theorists are often criticized for making numerous simplifying assumptions, so many that the technique finally derived cannot be applied to a practical problem. The number of simplifying assumptions that produce errors in the results of an applied theoretical technique are reduced by simulations, thereby improving the reliability of the solution obtained. By blending the relevant aspects of both theoretical and practical methods, this hybrid optimization technique attempts to bridge the gap between theory and practice with a viable technique which yields significant results for a given problem.

APPENDIX 1

A Numerical Example

Assume a situation of two terminals and seven demand points as illustrated in Figure (VII)

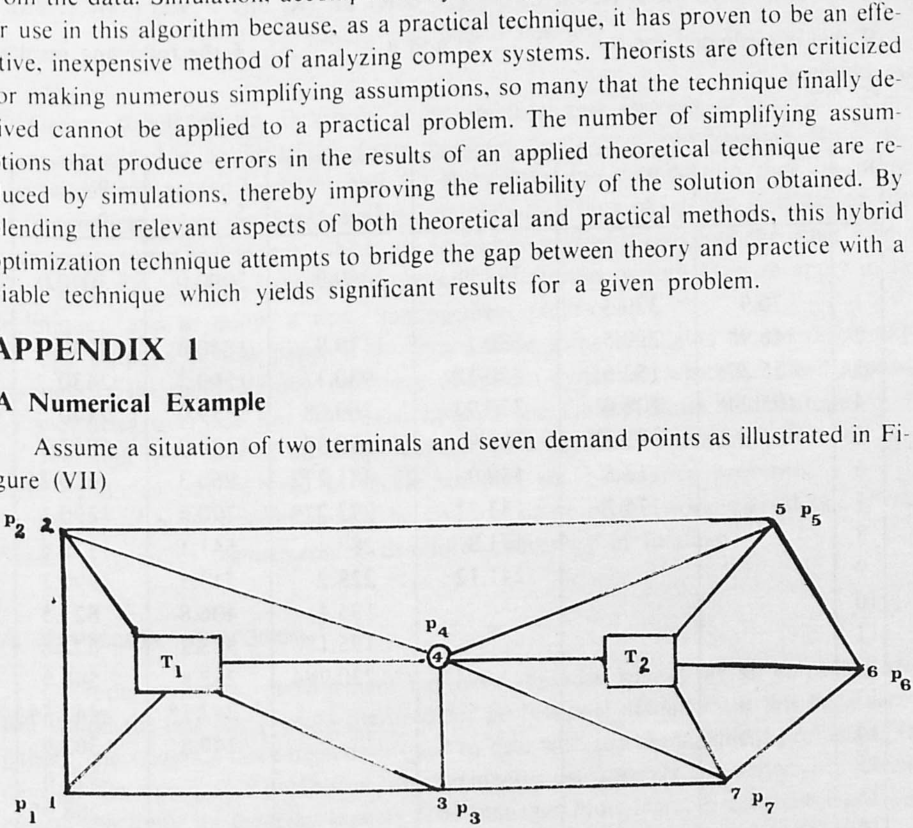

Figure VII

Let the loads on a route with n stops have a poisson distribution with n μ = 3n. The functions G_1 (L), G_2 (L) are assumed to be

$$G_1 (L) = 30 (L-R)^2$$
$$G_2 (L) = 10 (R-L)^2$$

Then

$$\text{Min E(cost)} = \text{Min}\ \{\sum_{L=0}^{R} 30\,(L-R)^2\,P\,(L,\,n\,\mu) + \sum_{R+1}^{\infty} 10\,(R-L)^2\,P\,(L,\,n\,\mu)\} =$$

$$\text{Min}\ \{30 \sum_{0}^{R} L^2 P\,(L,\,n\,\mu) + 30\,R^2 \sum_{0}^{R} P\,(L,\,n\,\mu) - 60R \sum_{0}^{R} LP\,(L,\,n\,\mu) +$$

$$+\ 10R^2 \sum_{R+1}^{\infty} LP\,(L,\,n\,\mu)\} + 10 \sum_{R+1}^{\infty} L^2\,P\,(L,\,n\,\mu) - 20R \sum_{R+1}^{\infty} LP\,(L,\,n\,\mu)\}$$

Letting $\mu = 3$, $\sum_{R}^{\infty} \mathbf{P}\,(L,\,n\,\mu) = P\,(R,\,n\,\mu)$ and collecting terms, we obtain

$$\text{Min E(cost)} = \text{Min}\ \{(90\text{-}180R)\cdot n\,[1\text{-}P\,(R,\,3n)] + 270n^2\,[1\text{-}P\,(R\text{-}1,\,3n)] + 30R^2\,[1\text{-}P\,(R+1\cdot 3n)] + 10\cdot R^2P\,(R+1,\,3n) + (30\text{-}60R)\,nP\,(R,\,3n) + 90n^2P\,(R\text{-}1,\,3n).$$

If this is evaluated for R= 1, 2 . . . 20 and n = 1,2, ... 6 the following results are obtained

R	n 1	n 2	n 3	n 4	n 5	n 5
			792.98	1329.9	2090.0	3070.0
1	70.9	320.3				
2	46.9*	220.5	580.1	1119.9	1840.0	2740.0
3	55.37	152.6	450.18	930.1	1590.3	2430
4	105.248	108.8	320.93	760.08	1359.0	2140
5		93.8*	188.9	610.35	1149.0	1869.2
6		113.5	150.9	481.272	960.3	1620.2
7		174.2	143.7*	373.278	790.6	1390.1
8			171.8	288.1	641.1	1179.8
9			241.12	228.2	513.5	990.3
10				195.4	406.8	821.5
11				195.1*	325.2	672.8
12				230.784	268.8	546.5
13					242.1*	442.44
14					247.8	363.9
15						311.6
16						288.9*
17						301.3
18						
19						
20	8760.0	6060.0	3900.6	2278.5	1183.9	570

The figures indicated by asterisks give the value of R minimizing the cost for the corresponding u.