RESIDUAL INCOME AND RETURN ON INVESTMENT AS CRITERIA FOR INVENTORY MODELS

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1. INTRODUCTION

Inventory management is an important area of decision making for the firm. One criterion proposed for inventory models is the criterion of maximizing residual income from investment in inventory, which results in the familiar Economic Order Quantity, (EOQ) model. Another criterion is the maximization of return on investment (ROI). The inventory model developed from the ROI concept is called the ROQ model [5]. This paper explores relationships between these two approaches and demonstrates, in a general way, the magnitude and limits of the differences in results obtained from the application of the ROQ model versus the EOQ model.

The concept of residual income as a criterion for investment decisions has also been presented [4, pp. 710-712]. Residual Income (RI), is defined as the cash inflow generated by the asset investment less the imputed interest on the asset determined at the opportunity cost of capital, and thus, maximization of residual income is equivalent to maximization of net profits. There is a similarity between the residual income criterion and the present value capital budgeting model, as well as a similarity between the ROI criterion and the internal rate of return approach to capital budgeting.

2. COMPARISON OF RI AND ROI INVENTORY MODELS

Given a choice in circumstances where inventory models based on either cost minimization or maximization of return on investment (ROI) would be applicable, a decision maker's initial reaction would likely be in favor of the ROI model. Where income could be affected by the inventory decision, it would seem appropriate to use a criterion which accounts for income. The ROI represents only one of the alternatives which utilize income. As pointed out in the previous section, residual income is an alternative measure.

Since a criterion of maximizing residual income is consistent with shareholder wealth maximization [6], it is compared in this paper to the criterion of maximization of ROI.

The residual income measure (RI) is defined as:

$$RI = PS - \left[\frac{KS}{Q} + CS + \frac{(I' + i)\,CQ}{2} \right]$$

or

$$RI = (P-C)S - \frac{KS}{Q} - \frac{(I' + i)\,CQ}{2}$$

or

$$RI = MS - \frac{KS}{Q} - \frac{(I' + i)\,CQ}{2} \qquad (1)$$

where:

S = annual demand rate, units

P = selling price, dollars/unit

K = fixed order and set up costs, dollars/lot

C = unit cost, dollars/unit

M = gross profit margin = (P-C), dollars/unit

Q = fixed order quantity or lot size, units

i = annual carrying cost expressed as a fraction of the unit cost (C), exclusive of opportunity costs of funds tied up in inventory.

I′ = opportunity cost of capital.

Taking the first derivative of RI with respect to Q and setting it equal to zero yields

$$\frac{d\,(RI)}{dQ} = \frac{KS}{Q^2} - \frac{(I' + i)\,C}{2} = 0$$

solving for Q, the order quantity is defined by

$$Q = EOQ = \sqrt{\frac{2KS}{(I' + i)\,C}} \qquad (2)$$

The second derivative of RI with respect to Q is

$$\frac{d^2\,(RI)}{dQ} = -\frac{2KS}{Q^3}$$

and therefore the Q in equation (2), which is the familiar Wilson EOQ, maximizes residual income when K, S and Q are greater than or equal to zero.

2.1 Residual Income Using EOQ Versus ROQ

The lot size which maximizes ROI [5] has been derived as

$$ROQ = 2K/M \tag{3}$$

Substituting equation (2) into (1), RI at the EOQ can be expressed as

$$RI_{(EOQ)} = MS - \sqrt{2CKS\,(I' + i)} \tag{4}$$

Substituting equation (3) into (1), RI at the ROQ can be expressed as

$$RI_{(ROQ)} = \frac{MS}{2} - \frac{(I' + i)\,CK}{M} \tag{5}$$

The difference between the two residual incomes expressed in the form of equations (4) and (5) is not readily apparent. Additional comparisons which will be made involve even greater complexity than is involved in equations (4) and (5). In order to provide generalizations, the following transformations are made.

$$I' = h(M^2S/2CK) \tag{6}$$

and

$$i = g(M^2S/2CK) \tag{7}$$

where h and g are initially restricted to $h=0$ and $g=0$. These transformations are developed from equations (2) and (3) utilizing the fact that when $I' + i = M^2S/2CK$, the ROQ and EOQ are equal.

It should be noted that $M^2S/2CK$ is assumed constant for the decision to be made on any particular product, and the use of h or g in any equation is equivalent to using I' or i respectively since h and g are one to one transformation of I' and i.

Substituting equations (6) and (7) into (4) and (5) one obtains,

$$RI_{(EOQ)} = MS\ (1 - \sqrt{h+g}) \tag{8}$$

$$RI_{(ROQ)} = \frac{MS}{2}(1 - (h+g)) \tag{9}$$

Clearly, when $h+g>1$, RI with either the EOQ or ROQ becomes negative. Therefore, $RI \geqslant 0$ only when

$$h+g \leqslant 1 \tag{10}$$

Multiplying equation (10) by $M^2S/2CK$, yields

$$h\,(M^2S/2CK) + g\,(M^2S/2CK) \leqslant M^2S/2CK \tag{11}$$

Using equations (6) and (7), equation (11) becomes

$$I' + i \leqslant M^2S/2CK$$

or

$$I' \leqslant (M^2S/2CK) - i \qquad (12)$$

Schroeder and Krishnan [5] have shown that using the ROQ, the maximum return on investment for a given product is

$$ROI_{max} = (M^2S/2CK) - i \qquad (13)$$

Equations (12) and (13) imply that the opportunity cost of capital must be less than or equal to ROI_{max} to obtain a residual income which is greater than or equal to zero. This rather intuitive result indicates that a product should be selected for inventory only when the maximum rate of return for the product is greater than the opportunity cost of capital.

According to equations (8) and (9), this rule is applicable, regardless of whether the EOQ or the ROQ is used.

The ratio of equation (8) to equation (9) is

$$\frac{RI_{(EOQ)}}{RI_{(ROQ)}} = \frac{2 - 2\sqrt{h+g}}{1 - (h+g)} \qquad (14)$$

This ratio is greater than one for all $0 \leqslant (h+g) < 1$, undefined for $h+g=1$ (where both RI's are zero), and less than one for $h+g>1$ (for negative RI's).
When $h+g<1$, the EOQ earns a greater residual income than the ROQ. It can be shown that this condition occurs whenever the opportunity cost of capital is less than the maximum rate of return that can be earned on the product ($I' < ROI_{max}$). When $h+g=1$, the residual incomes for both the ROQ and EOQ approaches are zero. This condition occurs whenever $I' = ROI_{max}$. When $h+g>1$, the ROQ creates a larger residual loss than does the EOQ. This condition occurs whenever $I' > ROI_{max}$. Thus, the EOQ is always superior to the ROQ with respect to residual income.

2.2 Comparison of the EOQ and ROQ

A comparison of the magnitudes of the EOQ and ROQ is also facilitated by transforming I′ and i. When the transformed values of I′ and i from equations (6) and (7) are substituted into equation (2), the following expression for the economic order quantity is obtained

$$EOQ = \frac{2K}{M} \frac{1}{\sqrt{h+g}} \qquad (15)$$

Using equations (15) and (3), the ratio of EOQ/ROQ can be expressed as

$$\frac{EOQ}{ROQ} = \frac{1}{\sqrt{h+g}} \qquad (16)$$

When h+g<1 ($I' < ROI_{max}$), the EOQ is greater than the ROQ. When h+g=1 ($I' = ROI_{max}$), the EOQ and ROQ are equal. Thus, results equivalent to the ROQ approach are obtained by using ROI_{max} as the opportunity cost of capital in the EOQ formula. When h+g>1 ($I' > ROI_{max}$), the product is not acceptable for stocking. For all acceptable products, the EOQ is always greater than or equal to the ROQ. This is an intuitive result since, as long as additional investment earns at a rate which is above the opportunity cost of capital, the residual income approach generally encourages greater investment than does a ROI approach.

2.3 Rate of Return on Differential Investment

The differencial investment between the EOQ and ROQ, denoted by ΔINV, is

$$\Delta INV = \frac{C\ EOQ}{2} - \frac{C\ ROQ}{2} \qquad (17)$$

Substituting, the values of EOQ and ROQ given by equations (15) and (3) into equation (17), the differential investment can be expressed as

$$\Delta INV = \frac{CK}{M} \cdot \frac{1-\sqrt{h+g}}{\sqrt{h+g}} \qquad (18)$$

The value of ΔINV, as defined in equation (18), is always positive for all h+g<1. This relationship implies that the EOQ always requires a larger dollar investment in inventory than does the ROQ for all products where $I' < ROI_{max}$. When h+g=1, the EOQ equals the ROQ and the investment amounts are also equal.
Defining gross income (GI) as

$$GI = MS - \frac{KS}{Q} - \frac{iCQ}{2}, \qquad (19)$$

and using equations (3), (7) and (15) the differential gross income (ΔGI) between the EOQ and ROQ approaches is

$$\Delta GI = \frac{MS}{2} \cdot \frac{\sqrt{h+g} + g\sqrt{h+g} - h - 2g}{\sqrt{h+g}} \qquad (20)$$

Equation (20) indicates that differential gross income is greater than or equal to zero for all $0<h+g\leqslant 1$. Thus, the EOQ approach always leads to a gross income which is

greater than or equal to the gross income from the ROQ approach for all acceptable products with non-zero carrying costs.

The gross rate of return on the differential investment (RDI) when using the EOQ is

$$RDI = \frac{\Delta GI}{\Delta INV} = \frac{M^2S}{2CK} \; \frac{\sqrt{h+g} + g\sqrt{h+g} - h - 2g}{1-\sqrt{h+g}} \qquad (21)$$

Using equation (6), the ratio of the gross rate of return on the differential investment to the opportunity cost of capital is

$$\frac{RDI}{I'} = \frac{\sqrt{h+g} + g\sqrt{h+g} - h - 2g}{h - h\sqrt{h+g}} \qquad (22)$$

which is greater than one for all $h+g<1$, and undefined for $h+g=1$.
This result implies that the additional investment suggested by the EOQ earns at a rate which is above the opportunity cost of capital.

2.4 The Influence of Sales Volume

Examination of equations (2) and (3) indicates that although the EOQ is a function of the level of sales (S), the ROQ is independent of sales level.

The consequences of non-responsiveness to sales demand can be measured by using equation (1) for residual income. The following data depict a reasonable sales level and set of cost parameters for an inventoried item:

S = 1000 units
P = \$15.00/unit
C = \$12.50/unit
K = \$5.00/order
$I' + i = 0.10$

The data is taken from an example in [2, p. 378] with the exception of the selling price which was appended to the example.

Using equations (4) and (5), the residual incomes, using the EOQ and ROQ approaches, are:

$$RI_{(EOQ)} = \$2388.20, \qquad RI_{(ROQ)} = \$1247.50$$

In addition to substantial differences in residual income, the ROQ may create unrealistic order frequencies. In the example the EOQ equals 89 units and the ROQ equals 4 units. This results in an order frequency of one order every 32 days for the EOQ model and one order every 1.5 days for the ROQ model.

Noting that the order frequency is determined by S/Q, it can be further demonstrated that

$$\lim_{S \to \infty} \frac{S/ROQ}{S/EOQ} = \infty$$

In the previous example, although the ratio of the ROQ and EOQ order frequencies is low (21.3) relative to its limit, the ROQ order frequency is high enough to make the ROQ impractical.

3. CONCLUSIONS

The EOQ inventory model has been shown to be superior to the ROQ model in terms of residual income. Although this benefit is obtained at the expense of an additional investment above the required for the ROQ, the additional investment always earns at a rate which is greater than the opportunity cost of capital.

Therefore, the ROQ approach does not take advantage of the additional investment which would result in maximization of shareholder wealth. Finally, the ROQ may lead to unrealistic order frequencies.

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