

ON THE ANALYSIS OF PORTFOLIO DECISIONS

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1. Introduction

The intention of this article is to analyse the concepts of portfolio decisions, to describe the basis of two monetary theories, and in particular to consider the critical aspects. The global concept of portfolio selection shall be replaced by a detailed approach to the theory of portfolio decisions. One possibility is provided by Multidimensional Portfolio Decision Analysis (MPD-Analysis), which is based on the assumption of an individually conditioned rational decision-making process.

In each economic period the individuals are confronted with the problem of planning their budgets for the following period. Holding a definite portfolio, divided into different assets, and expecting current receipts for the planning period, the individuals have to decide on the composition of their portfolio investment which can be varied by debt-financing and on the destination of incoming funds. All these decisions have to be made simultaneously.

In economic literature there are several approaches for analysing the process of portfolio decision. All these approaches are based on (the general) decision theory, that implies rational acting. According to the different models of decision theory, several concepts can be classified to explain the process of portfolio allocation. Based on stock-variables and/or on flows, all these concepts rest on the individuals' goals in order to determine their portfolio composition. The different theories of portfolio allocation consider different goals, which lead to various results.

In literature on monetary theory and -policy the explanation of the effects of monetary impulses on the non-financial sector rests on the analysis of portfolio allocation. There are two fundamental concepts of monetary transmission mechanism. First, there is the approach of Macroeconomic Portfolio Theory, represented by Post-Keynesians, especially by Tobin¹. Second, there is the Monetarist's approach, which among others was developed by Friedman², Brunner³

1. See J. Tobin (1961), (1963).

2. See M. Friedman (1969), (1970).

3. See K. Brunner (1970), (1973).

and Meltzer⁴. Based on the fundamental hypothesis that the individuals aim at optimum portfolio composition and thus acting rationally, both schools of monetary theory derive the transmission of monetary impulses on the non-financial sector. The explications of this transmission mechanism in the two concepts ground on different interpretations of the individuals' portfolio decisions, which lead to the well-known different results.

2. The different Concepts to analyse Portfolio Decisions

Since in economic literature the approaches of Monetarism and Portfolio Theory have been discussed in detail, in this article a short description should be sufficient. MPD-Analysis, however, demands a more detailed explanation.

a) The Approaches of Monetarism and Portfolio Theory

The approaches of Macroeconomic Portfolio Theory and Monetarism are used to analyse the transmission mechanism of monetary impulses on the non-financial sector. Under microeconomic aspects, Portfolio Theory is employed to find out the optimum portfolio, especially in banking business⁵. In this article the macroeconomic approach of Portfolio Theory⁶ shall be described as well as the Monetarists' concept⁷.

Both approaches maintain, that every individual aims at an optimum portfolio and acts rationally. Facing decision the individual holds a portfolio of fixed amount and structure. At the same time he has definite notions of the optimum portfolio for the planning period. If there is a discrepancy between the desired and the actual portfolio the individual has to adjust his portfolio accordingly. The concept of this general adjustment-model is specified differently in the two approaches. The Monetarists maintain a definition of wealth, which contains money, financial assets (papers, credit) and real capital (buildings, equipment and inventories, consumer's capital) and human capital, whereas in the definition of Portfolio Theory consumer's and human capital are not included. The Monetarists assume, that the individuals aim at maximum profits. In Portfolio Theory this intention is enlarged by explicitly considering the risk of a portfolio, which leads to the following procedure: Maximum profits with a definite risk, or a minimum risk with definite profits.

The mathematical version of the Monetarists' approach is:

4. See K. Brunner, A.H. Meltzer (1963).

5. See H.M. Markowitz (1952), (1959); W.F. Sharpe (1970).

6. See J. Tobin (1963).

7. See M. Friedman (1969), (1970); K. Brunner (1970), (1973); A.H. Meltzer (1963).

$$(1) PV_i = \sum_{h=1}^H pv_h \cdot a_{ih} \rightarrow \max! i=1, \dots, I; \sum_{h=1}^H a_h = 1;$$

with

PV_i : = value of portfolio i , $i=1, \dots, I$

a_{ih} : share of the asset h , $h=1, \dots, H$ in the portfolio i

pv_h : value of the asset h (= pecuniary returns on the asset).

In Portfolio Theory we have:

$$(2) E(pv)_{ih} = \sum_{l=1}^{\Lambda} pv_{ihl} \cdot p_{ihl} \quad h=1, \dots, H; \sum_{l=1}^{\Lambda} p_{ihl} = 1;$$

with

$E(pv)_{ih}$: = expected value of the asset h

l : = index of the possible values of the asset h

p_{ihl} : = probability of realization of the value pv_{ihl} , $l = 1, \dots, \Lambda$.

Equation (2) determines the expected value of the asset h . In addition Portfolio Theory includes the risk of the realization of this expected value by stating the variance δ^2 of the expected value:

$$(3) \delta_{ih}^2 = \sum_{l=1}^{\Lambda} (pv_{ihl} - E(pv)_{ih})^2 p_{ihl} \quad h=1, \dots, H.$$

Adding up the expected pecuniary returns on the H assets h and their risks we get for the expected value of the portfolio i :

$$(4) E(PV)_i = \sum_{h=1}^H E(pv)_{ih} \cdot a_{ih} \rightarrow \max! i=1, \dots, I;$$

with the risk (variance)

$$(5) \delta_i^2 = \sum_{h=1}^H \sum_{g=1}^H \delta_{ih} \cdot \delta_{ig} \cdot \rho_{ihg} \cdot$$

$\cdot a_{ih} \cdot a_{ig} \quad i=1, \dots, I; h \neq g.$

with

ρ_{ihg} = coefficient of correlation between the assets h and g .

Comparing (4) and (5) with (1) it becomes obvious, that Portfolio Theory is an extension of the Monetarist Theory of relative prices. (4) is the objective function, which is to be maximized without exceeding a definite risk.

(6) $\delta_i^2 \leq R$ R : = definite risk.

Equation (1) is just a maximization approach under security without constraint. Following this rule, the optimum portfolio is determined, which the given portfolio is to be adjusted to⁸, if necessary.

b) The Approach of MPD-Analysis

(1) The Theoretical Concept

A theoretical approach to determine portfolio decisions has to consider the results of decision theory, which evaluates alternative actions in order to realize an objective function, assuming rational acting. The objective function can be one-dimensional or multidimensional. The one-dimensional approach maintains that only one goal in the bundle of goals is dominant and therefore important for the evaluation of the alternative. Only if the alternatives referring to this goal are equivalent, the next important goal is chosen as decision criterion. In special cases there is only one goal. In reality, the individuals decide under simultaneous consideration of several goals. In decision theory this fact is represented by a multi-dimensional objective function.⁹

MPD-Analysis, which is described in this article, is a individually conditioned rational decision model, based on a multi-dimensional objective function¹⁰. Directing the portfolio decisions towards a bundle of goals means rational acting in relation to this subjectively determined bundle of goals. According to this an individual acts rationally, if he tries to realize his goals with apt means, even if goals seem to be irrational from a different subjective point of view. That means that MPD-Analysis takes into account the psychological and social situation of the deciding individuals. Irrational acting, which means not pursuing the goal, is excluded in this approach.

In order to evaluate the alternatives MPD-Analysis distributes the multi-dimensional objective function into J one-dimensional objective functions. Referring to these, the alternatives are evaluated in separate steps. Then these separate values are aggregated to a total value. As for MPD-Analysis, various proceedings with different measuring levels are available for evaluation and synthesis of values. Generally nominal, ordinal and cardinal MPD-Analysis' can be performed. The following equations describe the procedure of MPD-Analysis¹¹:

8. The costs of information and transformation do not change the general procedure of the decision process, see K. Brunner (1970) p. 7.

9. See R.L. Keeney, H. Raiffa (1976).

10. As for the different definitions of rationality, see D. Bott (1962), p. 1-38.

11. As for the practicable methods for MPD-Analysis, see F. Sixtl (1968) p.7-11; W. Gutjahr (1972); B. Orth (1974).

$$(7) PV_i = f(k_{j1}, k_{j2}, \dots, k_{jJ}) \quad i=1, \dots, I.$$

This is the multi-dimensional objective function, according to which the I alternative portfolios are evaluated. This means that the value PV_i of the portfolio i is found out by means of the decision criterions k_j , $j=1, \dots, J$. The values PV are fixed, based on the individuals' preferences PR , referring to the realization of the separate decision criterions k_j ¹²:

$$(8) PV_i = PR(k_{i1}, k_{i2}, \dots, k_{iJ}) \quad i=1, \dots, I.$$

This preference function is dissected

$$(9) PV_i = PR(PR_1(k_{i1}), PR_2(k_{i2}), \dots, PR_J(k_{iJ})) \\ i=1, \dots, I.$$

and evaluated to receive the values of assets pv_{ij} :

$$(10) PV_i = PR(pv_{i1}, pv_{i2}, \dots, pv_{iJ}) \quad i=1, \dots, I.$$

with

$$(11) PR_j(k_{ij}) = pv_{ij} \quad \begin{matrix} i=1, \dots, I. \\ j=1, \dots, J. \end{matrix}$$

In order to aggregate the values of assets, the decision rule DR is formulated:

$$(12) PV_i = DR\{w_j; pv_{ij}\} \quad \begin{matrix} w_j = \text{weight of } pv_j; \\ i=1, \dots, I; \\ j=1, \dots, J. \end{matrix}$$

General forms of the decision rule are the additive, the multiplicative and the additive-multiplicative synthesis of values¹³:

$$(13) PV_i = \sum_{j=1}^J w_j \cdot pv_{ij} \quad i=1, \dots, I; \quad \sum_{j=1}^J w_j = 1; \\ \text{(additive synthesis)} \quad w_j \geq 0; 0 \leq pv_{ij} \leq 1$$

$$(14) PV_i = \prod_{j=1}^J pv_{ij}^{w_j} \quad i=1, \dots, I; \\ \text{(multiplicative synthesis)} \quad w_j \geq 0; 0 \leq pv_{ij} \leq 1$$

12. As for the assumptions of preference function, see W. Krelle (1968), p. 7-11; H. Schneeweiss (1963), p. 178-220; F. Alt (1939), p. 161-169.

13. As for additive synthesis of values, see H.S. Houthakker (1960), p. 244-257; P.C. Fishburn (1966), p. 500-503.

$$(15) PV_i = \sum_{j=1}^J w_j pv_{ij} + \prod_{j=1}^J pv_{ij}^{w_j} \quad i=1, \dots, I;$$

(additive-multiplicative synthesis) $w_j \geq 0$; $0 \leq pv_{ij} \leq 1$

The additive synthesis of values, that implies linearity and independence of the values of assets, is already applied to a couple of economic questions. In simulation attempts it has been proved to be a good approximation¹⁴ and therefore is mostly used in practice.

(2) The Application of MPD-Analysis

The precondition for practical application of MPD-Analysis is the definition of the assets. In economic literature there are different proposals. As for MPD-Analysis we use a definition that includes money, financial assets, non-financial capital (including consumer's capital) and human capital. Portfolio allocation theory usually is a static analysis with stock-variables, whereas economic life is determined by variations. Therefore, a dynamic contemplation of portfolio decisions, based on dynamic variables is the appropriate instrument for analysis¹⁵. This procedure applies to asset-flows, amount and structure of which are to be fixed.

In order to specify the general approaches of MPD-Analysis, as shown in equations (13) to (15), the weights and decision criteria are to be determined, which means finding a bundle of goals, which the portfolio decisions are conformed to. This bundle of goals has to be complete and consistent. In general, it consists of elements, which are placed on different levels in a hierarchy of goals. The lowest level contains the decision criteria. According to the assumptions of MPD-Analysis, the decision criteria have to be independent, in order to be able to apply the additive synthesis of values. This is a condition which is seldom fulfilled in practice. The transgression of this condition, however, can be accepted. For example, Moore¹⁶ found out by simulations, that the additive synthesis of values leads to results, which are at least as good as others', like the multiplicative synthesis of values, even if the condition of independence is transgressed.

Furthermore considering pecuniary returns and their risks, it is necessary to include the non-monetary goals of portfolio behavior in formulating the objective function. Thus it becomes evident, that the known theories of portfolio allocation are

14. As for these simulation attempts, see J.R. Moore Jr. (1968), p. 98-112; as for linearity, see J.C.T. Mao (1969), p. 92, 268.

15. See R. Pohl (1975), p. 234; as for critical aspects of portfolio allocation theory, see W. Neubauer (1972), p. 58.

16. See J.R. Moore Jr., (1968), p. 98-112.

special cases of the developed general approach. Adjusting the optimum portfolio only to the goal "maximization of yield" the Monetarist Theory maintains the special case of an one-dimensional objective function. A modification of this concept is represented by the approach of Portfolio Theory, which contains the constraint of a maximum risk.

The determination of a bundle of goals for portfolio decisions may be based on a general social hierarchy of goals. These scopes of goals can be found in principle in the developed general social bundle of goals¹⁷. The decision elements, which are relevant to the portfolio decisions, are not explicitly considered by those bundles of goals till now, they also are not applied to that problem. In this connection there is appropriate research work to be done, which cannot be the purpose of this article.

The decision elements of the bundle of goals concerning the portfolio decisions, which are specific to the sectors, can be determined by various methods. In general all methods of priority determination¹⁸ as for example sample surveys among the individuals of the relevant sectors, brainstorming, Delphi etc. are practicable. With these methods not only the selection of decision criterions is to be done, but also their contributions to the total goal and to the derivated weights of the decision criterions.

According to the above mentioned methods for each economic sector a bundle of goals has to be determined, on the lowest level containing measurable decision criterions. For each of these criterions a weight is to be found, which indicates the importance of the criterion for the sector X relative to the total evaluation. Referring to these decision criterions, the alternative portfolios must be evaluated. These portfolios are different in amount and structure, as the economic sectors fix the volume of the income-flows especially by debts and by distributing the total flow. The alternative portfolios, the individuals have a choice between, are to be given.

The values of assets are determined by analysing the separate components of each portfolio. Each separate asset must be investigated in order to find out its share to the contribution of the separate decision criterions. These goal-shares cb of the separate assets h , $h=1, \dots, H$, are added up to get the values of assets pv_{ij} according to their shares of the total portfolio a_{ih} . This leads to equation:

$$(16) \quad pv_{ij} = \sum_{h=1}^H a_{ih} \cdot cb_{ijh} \quad \sum_{h=1}^H a_{ih} = 1; \quad 0 \leq a_{ih} \leq 1;$$

$$i=1, \dots, I; \quad j=1, \dots, J.$$

17. See H.H. Koelle (1974), p. 30; W. Zapf (ed.) (1974 a.o.).

18. See Bundesminister für Bildung und Wissenschaft (ed.) (1971).

The values of assets pv_{ij} then are weighted corresponding to the concept of MPD-Analysis and aggregated to the total value. The portfolio with the highest value is realized.

The aggregation of the values of assets is only possible, if all values of assets have the same unit of measure. Adequate to the decision criterions there are monetary and non-monetary values of assets. In order to make an aggregation possible, three general solutions can be given:

- (1) Transformation of the monetary and non-monetary values of assets into a scale without a dimension,
- (2) transformation of the monetary values into non-monetary values, and
- (3) transformation of the non-monetary values into monetary values.

Since portfolio decisions are dominated by monetary aspects, the third solution should be preferred. In economic theory the necessary transformation is made by a transformation curve. Having uncertain data, the application of transformation curves is problematical, because in this case the transformation curves are to be given normatively.

The following three transformation curves are often used:

$$(17) \text{ curve I: } p\tilde{v}_{ij} = \frac{1}{a} \cdot pv_{ij} \quad a > 0 \text{ and constant}$$

$$(18) \text{ curve II: } p\tilde{v}_{ij} = \sqrt[3]{-\frac{1}{a} \ln 1 - pv_{ij}} \quad a > 0 \text{ and constant}$$

$$(19) \text{ curve III: } p\tilde{v}_{ij} = -a \ln 1 - pv_{ij} \quad a > 0 \text{ and constant.}$$

The general approach of MPD-Analysis may lead to an optimum portfolio, which cannot be realized in practice because of internal and/or external restrictions. These restrictions may be missing divisibility or disposability, like credit terms or legal restrictions on investment. Transforming the approach into a linear program with appropriate constraints, those restrictions can be considered. The constraints may be decision criterions of the determined bundle of goals, but for external restrictions it may be necessary to formulate new goals as constraints. This leads to:

$$(20) PV_i = \sum_{j=1}^J w_j \cdot pv_{ij} \rightarrow \max! \quad i=1, \dots, I.$$

$$\frac{d PV_i}{d pv_{ij}} = 0$$

with the constraints

$$w_j \geq 0$$

$$0 \leq pv_{ij} \leq 1$$

$$1 \geq pv_{ij} \geq \min_j$$

$$\text{or } 0 \leq pv_{ij} \leq \max_j$$

Min_j and max_j are the minimum and maximum requirements of effectiveness of goals to the constraints of decision criterions k_j .

3. Comparison of the Concepts of Portfolio Decisions

All the three described concepts of portfolio decision may be used in order to analyse the transmission mechanism of monetary impulses on the non-financial sector. The quality essentially depends on reality nearness of the respective approach. Most approaches of portfolio allocation theory maintain decisions on stocks, which has often been criticized in literature. MPD-Analysis in contrast assumes, that the individuals decide on flows.

In all three concepts the individuals make their portfolio decision according to their goals, but the concepts differ in the considered goals. The Monetarists only define an one-dimensional objective function, which in Portfolio Theory is enlarged by determining a constraint. As both approaches require maximum profits, the evaluation criterions of the alternative portfolios are the pecuniary returns of the assets and additionally in Portfolio Theory the risks of profits. Thus Portfolio Theory includes a two-dimensional objective function. MPD-Analysis is based on a multidimensional objective function, which can be specified according to the pretensions of the decision-makers concerning the dimensions and constraints, which allows the consideration of monetary goals as well as of non-monetary goals. For all goals that are relevant for the analysis indicators have to be found in order to determine the effectiveness of goals-attaining of the assets and of the alternative portfolios.

Considering the goals it becomes obvious, that MPD-Analysis is the general case of portfolio allocation theory and contains the approaches of Monetarism and Portfolio Theory as special cases, which can easily be shown by the comparison of the mathematical representation of the three concepts (compare equations (1), (4), (6) and (12)-(15).

4. Concluding Remarks

The values of assets in MPD-Analysis replace the relative prices in the transmission mechanism concepts.

In addition to the pecuniary returns and their risk MPD-Analysis also considers non-monetary goals of the decision-maker. This takes into account, that portfolio decisions are not only influenced by the principle of profit maximization, but also by non-monetary or even non-economic goals. With the help of this approach the portfolio decisions of the economic sectors can be explained by an extensive goal-system and the relations of substitution between the separate assets, which are controversial in literature, can be determined.

As a consequence for economic policy there are several rudiments of influencing the portfolio decisions of private households, financial and non-financial enterprises. The concept of Monetarists' assumes a variation of the quantity of money; the approach of Tobin underlines the importance of deficit-spending. In both concepts the profits, which the portfolio decisions are based on, can be varied by political actions. The success of such policy of influencing the economic activities will be lessened, if the portfolio decisions of the economic sectors only particularly depend on pecuniary returns. The shown approach of MPD-Analysis includes the consideration of non-monetary returns, and therefore may improve the analysis of actions of monetary and fiscal policy and moreover raise the efficiency of economic policy. MPD-Analysis can generally be applied on all sectors, on public as well as on private, which allows the analysis of interactions between the portfolio decisions of all sectors – public on private and private on public.

Whether the shown approach of the analysis of general economic connections leads to acceptable results, depends on the complete registration of the bundle of goals, spezified to the sector, and on the empirical funding of the conversion of non-monetary into monetary values.

Summary

Analysing portfolio decisions portfolio allocation theory considers pecuniary returns. In the approach of Tobin risk is included. Non-monetary decision criterions are neglected, which leads to an incomplete analysis of portfolio decisions as a basis of theoretical transmission mechanism concepts.

MPD-Analysis tries to remove this deficit. Based on modern methods of decision theory an approach is shown, which also considers non-monetary elements. Resting on bundles of goals spezified to the economic sectors, decision criterions are derived to evaluate alternative portfolios. In order to convert the non-monetary decision criterions into monetary, transformation curves, kown from other spheres of economic theory, are given. The derivation of bundles of goals is based on the general economic spheres of goals, which are also used for bundles of goals in other scopes of economic research. For MPD-Analysis adequate of goals are to be developed. How far the approach of MPD-Analysis, applied to all economic sectors, is able to explain general economic connections and to analyse actions of economic policy, could just shortly be described.

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