A TWO-SECTOR GROWTH MODEL FOR A DEVELOPING ECONOMY*

By Dr. A.V. KATOS,
Lecturer στὸ Πανεπιστήμιο τοῦ Keele.

1. Introduction

The usual procedure in constructing a theory of economic development of a two-sector economy is to view growth as a movement by the economy through a sequence of short-run equilibrium positions in which sectoral wages and rental rates are equated. In other words, these theories are based on a fundamental proposition of the theory of general equilibrium and the theory of the competitive market according to which the discrepancies in factor payments lead to appropriate re-allocations of productive factors until equilibrium is restored. In these models we assume that capital and labour adjust instantaneously to any differentials between sectors and that there are no costs associated with the transfer.

Undoubtedly, from the empirical data, we can see that in the real world there exist sectoral differentials in wages and in rental rates (1, 2, 3, 4, 5, 6). The persistence of such differentials has led Higgins, Bruton, and others to argue that economic development is not an equilibrium process, but rather a process of change that is subject to continuous disequilibrating forces (7, 8). A variety of hypotheses have been advanced to explain why discrepancies in sectoral incomes and factor prices may exist over extended periods. Most of these are basedon the notion of slugish adjustments in factor allocation in response to changes in the industrial output mix and labour demand (9). The slow adjustment of the capital and labour markets is attributed to a variety of factors that include the existence of high transactions cost, imperfect information, risk and uncertainty, and sectoral capital specificity (10).

The aim of this article is to construct a two-sector growth model for a developing economy by taking into account wage differentials in the labour market.

We use the notion of dualistic growth (11, 12, 13, 14) as a suitable basis for our

*This paper is a revised version of a chapter of a doctoral dissertation submitted to the University of Southampton in 1976. The author wishes to express his gratitude to Professor A.K. Dixit, to Mr. A. Ingham and to Mr. P.J. Simmons, for their valuable comments and criticism.

model but at the same time we avoid the assymmetry in the two production functions assuming that the two sectors use the same inputs, capital and labour, in the production of their own outputs but combined in different proportions (10, 22). This is due to the historical evidence that capital in the agricultural sector was an important element in the successful development of many economies and because in introducing capital into the agricultural sector we add another policy dimension to these models.

Under the above assumptions we try to present the structure of a developing economy, to find the behavioural and technical sufficient conditions for the properties of existence and uniqueness of the momentary solution and stability of balanced growth of our model and finally to analyse the structural change and the growth process through time.

2. The Model

We shall consider the case of a closed economy consisting of an agricultural and a non-agricultural sector. The output of the agricultural sector can be used only as a consumption good whether the output of the non-agricultural sector can be used as a consumption good or as a capital good or as both. The models for the dual economies of Lewis (11), Fei and Ranis (12), and Jorgenson (13) also use the assumption that the output of the "advanced" sector (or "industrial" or "non-agricultural") can be consumed, invested or both. Uzawa and others assume that the output of the first sector is all consumed while the output of the second sector is all invested. We must note here that the latter models refer to advanced economies with one consumption-good and another capital-good sector while the previous models to developing economies with the two sectors being the "backward" and the "advanced" sector.

Both agricultural and non-agricultural goods are assumed to be composed of homogeneous quantities and to be produced by two homogeneous factors of production, labour and capital. In each sector, production is subject to constant returns to scale and diminishing marginal rates of substitution. Joint products and external economies (diseconomies) do not exist.

The quantities of the two outputs, $Y_i(t)$, $(i=1, 2)$, produced at time t are related to the quantities of labour, $L_i(t)$, and capital, $K_i(t)$, by the corresponding production functions, F_i:

$$Y_i(t) = F_i(A_i(t) K_i(t), B_i(t) L_i(t)), \; i=1, 2 \tag{2.1}$$

where $A_i(t) > 0$ are variables showing the technological progress of capital and labour respectively. It is

$$\mathring{A}_i(t) = \lambda_{1i} A_i(t), \qquad i = 1, 2 \tag{2.2}$$

and

$$\dot{B}_i(t) = \lambda_{2i} B_i(t) \qquad i = 1, 2 \tag{2.3}$$

where λ_{1i} and λ_{2i}, $(i = 1, 2)$, denote the constant rates of the technological change of the two factors of production, in the two sectors respectively. We denote

$$\overline{K}_i(t) = A_i(t)\ K_i(t), \qquad i = 1, 2 \tag{2.4}$$

and

$$\overline{L}_i(t) = B_i(t)\ L_i(t), \qquad i = 1, 2 \tag{2.5}$$

and we call $\overline{K}_i(t)$, the capital in efficiency units or "efficiency capital" and $\overline{L}_i(t)$ the labour in efficiency units or "efficiency labour". Hence, the production functions are written:

$$Y_i(t) = F_i(\overline{K}_i(t), \overline{L}_i(t)), \qquad i = 1, 2 \tag{2.6}$$

The grross national product in terms of the second sector goods, Y(t), is given by

$$Y(t) = p(t)\ Y_1(t) + Y_2(t) \tag{2.7}$$

where p(t) is the relative price (ratio of the price of the first-sector over the second-sector goods).

We assume that labour and capital are fully employed, namely

$$K_1(t) + K_2(t) = K(t) \tag{2.8}$$

and

$$L_1(t) + L_2(t) = L(t) \tag{2.9}$$

where K(t) and L(t) are the respective stocks of capital and labour available for employment in the economy at time t.

We also assume that total labour force grows at a constant (or variable) rate, η, i.e.

$$\dot{L}(t) = \eta L(t) \tag{2.10}$$

and the capital stock depreciates at a constant rate, δ, i.e.

$$\dot{K}(t) = I(t) - \delta K(t) \tag{2.11}$$

where I(t) is the total gross investment in the economy at time t.

The allocation of capital existing at any moment of time is assumed perfectly competitive, so that in each sector the rentals, $r_i(t)$, of capital goods are equal to the marginal product of capital, i.e.

$$r_i(t) = \frac{\vartheta Y_i(t)}{\vartheta K_i(t)}, \qquad i=1, 2 \tag{2.12}$$

In terms of the second sector goods, it is

$$r_1(t) = p(t)\ r_1'(t) \tag{2.13}$$

and

$$r_2(t) = r_2'(t) \tag{2.14}$$

We assume that

$$r_1(t) = r_2(t) = r(t) \tag{2.15}$$

where r(t) is the common rate of return on capital in both sectors and in terms of the second sector goods.

For the allocation of labour existing at any moment in time we assume that there is not perfect mobility of labour and there are transfer costs and other similar imperfections according to which the allocation of labour takes place. Since the proposed model refers to those developing economies in which the wage rate is above the subsistence level, we assume that the prevailing wage rate is equal to the marginal product of labour, $w_i'(t)$, in each sector, i.e.

$$w_i'(t) = \partial Y_i(t) / \partial L_i(t), \qquad i=1, 2 \tag{2.16}$$

$$w_1(t) = p(t)\ w_1'(t) \tag{2.17}$$

$$w_2(t) = w_2'(t) \tag{2.18}$$

The relation between the two wage rates is found as follows:
We assume that direct migration costs are small in many developing regions and so the major determinant of costs is the loss of income during the migration process itself (10), and the "phychic" costs (32) of the individuals. We assume that the costs of migration are given by

$$c(t) = \beta(t)\ w_1(t) \tag{2.19}$$

where $\beta(t) \geqslant 0$ is a decreasing function of time showing that as the economy becomes more developed the migration process becomes quicker, and the "physic" costs become less strong.

We define P(t) as the probability of an individual finding a job in the non-agricultural sector. If we assume that employment and unemployment in the non-agricultural sector are two mutually exclusive events, then the individual who migrates to the urban area has a probability of $1-P(t)$ of not finding a job, or e-quivalently, of not earning any wage. Then, adopting an individual's utility function of the form

$$u(x) = \frac{1}{1-v}\, x^{1-v}, \qquad 0 \leqslant v < 1 \tag{2.20}$$

the individual in aggriculture migrates to the non-agricultural sector if, and only if, he asumes that he will gain greater utility than by staying in the agricultural sector.

Hence, the decision to migrate is given by the expression

$$(1-P(t))\ u(0) + P(t)\ u\ (2\xi(t) - c(t)) > u(w_1(t)) \tag{2.21}$$

From (2.21) with the help of (2.20) we obtain that

$$< \quad \frac{1}{\qquad} \quad \frac{w_1(t)}{w_2(t)} \quad (\frac{1}{P})^{\frac{1}{1\mp v}}\beta \tag{2.22}$$

From (2.22) we see that, at time t, given the probability of somebody finding a job in the non-agricultural sector and the costs of migration, an individual in the agricultural sector migrates to the non-agricultural sector if, and only if,

$$\frac{w_1(t)}{w_2(t)} < \mu(t) \tag{2.23}$$

where

$$\mu(t) = \frac{1}{(\frac{1}{P})^{\frac{1}{1-v}} + \beta} \tag{2.24}$$

In the case where $w_1(t)/w_2(t) > \mu(t)$, nobody is willing to migrate. Thus, the lowest wage ratio for the migration process is:

$$\frac{w_1(t)}{w_2(t)} = \mu(t) \tag{2.25}$$

$\mu(t)$ always lies between zero and one. We obtain this by giving to P(t) and $\beta(t)$ extreme values. Thus, for $P(t) = 0$ and $0 \leqslant \beta(t) \leqslant \infty$ we have that $\mu(t) = 0$; for $P(t) = 1$ and $\beta(t) = 0$ we have that $\mu(t) = 1$; for $P(t)>0$ and $\beta(t)>0$ it is $0<\mu(t)<1$.

In the Harris and Todaro[16], Harberger[17] and Lal[18] the equilibrium value of P(t) is determined by the equilibrium migration condition which the above authors found to be

$$P_e(t) = \frac{w_1(t)}{w_2(t)} \tag{2.26}$$

In our formulation this equilibrium migration condition is the (2.25) or, equivalently:

$$P_e(t) = (\frac{w_2}{w_1} - \beta)^{v-1} \tag{2.27}$$

which is more general than (2.26).

But in (2.8) and (2.9) we assumed full employment of the resources, i.e. $P(t) = 1$, and therefore,

$$\mu(t) = \frac{w_1(t)}{w_2(t)} = \frac{1}{1 + \beta(t)} \tag{2.28}$$

The introduction to our economy of one common rental rate and two different wage rates show that in this economy there exist three classes of income levels; the capitalists in both the sectors, the workers in the agricultural sector and the workers in the non-agricultural sector. Therefore, it is quite reasonable to assume that the propensities to invest (consume) out of income are different between these three classes. If we denote by s_1, s_2 and s_3 the propensities to invest out of income of the workers of the first sector, the workers of the second sector and the capitalists respectively, then the total investment of the economy, $I(t)$, which equals ex ante savings, is

$$I(t) = s_3\, r(t)\, K(t) + s_2\, w_2(t)\, L_2(t) + s_1\, w_1(t)\, L_1(t) \tag{2.29}$$

Suppose now that all individuals belonging in the same income class have the same utility function $V_j(t)$, $j = 1, 2, 3$, given for the hth individual by

$$V_{jh}(t) = \Gamma_{jh}\, C_{1jh}(t)^{\gamma_{jh}}\, C_{2jh}(t)^{\varepsilon_{jh}} \qquad j=1, 2, 3 \qquad h=1, \ldots\ldots \tag{2.30}$$

where $C_{ijh}(t)$ for $i=1, 2$; $j=1, 2, 3$; $h=1, 2, \ldots\ldots$ is the amount of good i which is consumed by the hth individual of the jth income class and Γ_{jh}, γ_{jh} and ε_{jh} are parameters such that $\Gamma_{jh} > 0$, $0 < \gamma_{jh} < 1$ and $0 < \varepsilon_{jh} < 1$.

Assuming that the individuals belonging to the same income class behave in exactly the same way, ($\gamma_{jh} = \gamma_j$ and $\varepsilon_{jh} = \varepsilon_j$) we maximize (2.30) subject to each individual's budget constraint and summing up for all the individuals belonging in the same income class, we obtain the following demand functions for each of the two outputs and for each of the three income classes:

$$C_{1j} = m_{1j}\,(1-s_j)\,\frac{w_j(t)}{p(t)}\,L_j(t), \qquad j=1, 2 \tag{2.31}$$

$$C_{2j} = m_{2j}\,(1-s_j)\,w_j(t)\,L_j(t), \qquad j=1, 2 \tag{2.32}$$

for the two worker classes, and

$$C_{13} = m_{13}\,(1-s_3)\,\frac{r(t)}{p(t)}\,K(t) \tag{2.33}$$

$$C_{23} = m_{23}\,(1-s_3)\,r(t)\,K(t) \tag{2.34}$$

for the capitalists, where $m_{1j} = \gamma_j / (\gamma_j + \varepsilon_j)$ and $m_{2j} = \varepsilon_j / (\gamma_j + \varepsilon_j)$*.

Under our assumptions that the output of the first sector is all consumed and the output of the second sector is partly consumed and partly invested, we obtain the e-quilibrium conditions in the goods market as

$$Y_1(t) = C_1(t) \tag{2.35}$$

$$Y_2(t) = C_2(t) + I(t) \tag{2.35}$$

where $C_i(t)$, i=1, 2, is the total consumption of the ith good.

Having now completed the structure of the growth model we find that the static model is a system of 18 equations in 17 endogenous variables. The behaviour of the static economy is completely described by the 17 equations, ignoring one of the com-odity market clearance equations (10), so the model becomes a system of 17 equa-tions in 17 endogenous variables. These endogenous variables are $Y_i(t)$, $K_i(t)$, $L_i(t)$, $w_i(t)$, $r(t)$, $p(t)$, $C_{ij}(t)$ and $I(t)$ for i=1, 2 and j=1, 2, 3; the exogenous variables are $K(t)$ and $L(t)$.

3. The Static Economy (Existence and Uniqueness)

In this section, we shall discuss under which conditions the momentary solution of quantities and prices, given the capital stock, $K(t)$, and the available labour force, $L(t)$, exists and is uniquely determined.

The static model can be stated as follows:

$$y_i = B_i f_i \left(\frac{A_i}{B_i} k_i \right), \qquad i=1, 2 \tag{3.1}$$

$$l_1 + l_2 = 1 \tag{3.2}$$

$$k = k_1 l_1 + k_2 l_2 \tag{3.3}$$

$$\omega_i = \frac{B_i}{A_i} \left[\frac{f_i(\frac{A_i}{B_i} k_i)}{f_i'(\frac{A_i}{B_i} k_i)} - \frac{A_i}{B_i} k_i \right], \qquad i=1, 2 \tag{3.4}$$

$$\omega_1 = \mu \omega_2 \tag{3.5}$$

* Assuming that $\gamma_j = \gamma$ and $\varepsilon_j = \varepsilon \forall j$ (or equivalently, $m_{1j} = m_1$ and $m_{2j} = m_2$) we have that $C_1(t) = m_1 p(t)^{-1} X(t)$ and $C_2(t) = m_2 X(t)$, where $m_1 + m_2 = 1$, and $X(t)$ is the total ouput of the economy which is consumed. These expressions are of the usual Graham (15) type functions.

$$p = \frac{A_2}{A_1} \quad \frac{f_2'\,(\frac{A_2}{B_2}\;k_2)}{f_1'\,(\frac{A_1}{B_1}\;k_1)} \tag{3.6}$$

$$c_{ij} = m_{ij}\;(1 - s_j)\;f_i\;(\frac{A_i}{B_i}\;k_i)\;A_i\,\omega_j\,, \qquad i=1,\,2 \tag{3.7}$$

$$c_{i3} = m_{i3}\;(1- s_3)\;f_i\;(\frac{A_i}{B_i}\;\;k_i)\;A_i\;k, \qquad i=1,\,2 \tag{3.8}$$

$$i = A_2\;f_2'\;(\frac{A_2}{B_2}\;k_2)\;(s_3\,k + s_2\,\omega_2\,l_2 + s_1\,\omega_1\,l_1) \tag{3.9}$$

$$c_1 = B_1\,l_1\,f_1\;(\frac{A_1}{B_1}\;k_1) \tag{3.10}$$

$$c_2 = B_2\,l_2\,f_2\;(\frac{A_2}{B_2}\;\;k_2) - i \tag{3.11}$$

where $f_i\;(\frac{A_i}{B_i}\;k_i) = F_i\;(\frac{A_i}{B_i}\;k_i,\;1)$

obeys the Inada derivative conditions, $y_i = Y_i\,/\,L_i$ is the sectoral output/labour ratio, $k=K/L$ is overall capital/labour ratio, $k_i = K_i\,/\,L_i$ is the sectoral capital/labour ratio, $l_i = L_i\,/\,L$ is the proportion of labour engaged in each sector, $\omega_i = w_i\,/\,r$ is the sectoral wage/tentals ratio, $c_i = C_i\,/\,L$ is the overall per capita consumption of the ith good, $c_{ij} = C_{ij}\,/\,L_j$ is the per capita consumption of the ith good by individuals of the jth income class and $i=I/L$ is the overall investment/labour ratio.

This model is a system of 17 equations in 16 endogenous variables and can be reduced into a system of 16 equations in 16 endogenous variables, ignoring as in section 2, one of the commodity market clearance equations. The endogenous variables are y_i, k_i, l_i, ω_i, p, c_{ij}, i for $i=1, 2$ and $j=1, 2, 3$; while k is the exogenous variable.

Adding up (3.7) and (3.8) for $i=1$ and $j=1, 2, 3$ and using (3.1), (3.4) and (3.7) we find that:

$$m_{11}\;(1-s_1)\omega_1\,l_1 + m_{12}\,(1-s_2)\omega_2\,l_2 + m_{13}\,(1-s_3)k = l_1\,(\omega_1 + k_1) \tag{3.12}$$

Solving the system of (3.2) and (3.3) for l_1 and l_2, under the assumption that $k_1 \neq k_2$, we obtain that

$$l_1 = \frac{k - k_2}{k_1 - k_2} \qquad \text{and} \qquad l_2 = \frac{k_1 - k}{k_1 - k_2} \tag{3.13}$$

Substituting (3.13) into (3.12) and solving for k we find that

$$k = \frac{(m_{11}\,(1-s_1)-1)\omega_1\,k_2 - (m_{12}\,(1-s_2)\omega_2 + k_2)k_1}{(m_{11}\,(1-s_1)-1)\omega_1 - m_{12}\,(1-s_2)\omega_2 + m_{13}\,(1-s_3)\,(k_1 - k_2)-k_1} \tag{3.14}$$

The numerator, N, in (3.14) is always negative. For k to be positive the demoninator, D, in (3.14) must be negative. A sufficient condition for the denominator to be negative, in other words for the existence of k, is $k_1 - k_2 < 0$, or the second sector (non-agricultural) must be more capital intensive than the first sector (agricultural). In the case where $s_3 = 1$ and $s_1 = s_2 = 0$ (classical saving function) it can be seen from (3.14) that k is always positive without imposing any restriction on the capital intensities of the two sectors.

Equation (3.14) uniquely determines the wage/rental ratio ω_2, ($\omega_1 = \mu\omega_2$). To see this we differentiate (3.14) with respect to ω_2, i.e.

$$\frac{dk}{d\omega_2} = D^{-2} \cdot [(m_{11}(1-s_1)-1)^2 \sigma_2 k_2 \omega_2 - m_{13}(1-s_3)(m_{11}(1-s_1)-1)\mu k_2^2 -$$
$$-(m_{13}(1-s_3)-1)m_{12}(1-s_2)k_1^2 - (m_{11}(1-s_1)-1)m_{12}(1-s_2)\sigma_2 k_2 \omega_1 -$$
$$-(m_{13}(1-s_3)-1)\sigma_2 \frac{k_1^2 k_2}{\omega_2} + m_{13}(1-s_3)\sigma_1 \frac{k_1 k_2^2}{\omega_2} +$$
$$+(m_{11}(1-s_1)-1)\mu(1-\sigma_1-\sigma_2)k_1 k_2 + m_{12}(1-s_2)(\sigma_1+\sigma_2-1)k_1 k_2 -$$
$$-m_{12}(1-\sigma_2)\sigma_1 k_1 \omega_2 (m_{12}(1-s_2) + \mu(m_{11}(1-s_1)-1)) +$$
$$+(m_{12}(1-s_2)m_{13}(1-s_3) + (m_{11}(1-s_1)-1)(m_{13}(1-s_3)-1)\mu)k_1 k_2 +$$
$$+(m_{12}(1-s_2)m_{13}(1-s_3) - (m_{11}(1-s_1)-1)(m_{13}(1-s_3)-1)\mu)(\sigma_1-\sigma_2)k_1 k_2] \qquad (3.15)$$

where σ_i, i = 1, 2, is the sectoral elasticity of factor substitution which is equal to

$$\sigma_i = \frac{dk_i}{d\omega_i} \quad \frac{\omega_i}{k_i} \quad i=1, 2 \qquad (3.16)$$

Sufficient conditions for (3.15) to be positive, or in other words for the monotonicity of $k(\omega_2)$ are the following:

(i) $\sigma_1 + \sigma_2 \geqslant 1$

(ii) $\mu \geqslant \dfrac{m_{12}(1-s_2)}{1-m_{11}(1-s_1)}$ (3.17)

(iii)
a) $\sigma_1 - \sigma_2 > 0$ with $\mu \leqslant \dfrac{m_{12}(1-s_2)}{1-m_{11}(1-s_1)} \cdot \dfrac{m_{13}(1-s_3)}{1-m_{13}(1-s_3)}$
b) $\sigma_1 - \sigma_2 < 0$ with $\mu \geqslant \dfrac{m_{12}(1-s_2)}{1-m_{11}(1-s_1)} \cdot \dfrac{m_{13}(1-s_3)}{1-m_{13}(1-s_3)}$
c) $\sigma_1 - \sigma_2 = 0$ with $\forall\, \mu$

Therefore, sufficient conditions for the unique determination of $k(\omega_2)$ are (3.17 i and ii) in combination with any of (3.17 iii). A closer examination of (3.17) shows that in (3.17 iii, c) μ must obey (3.17 ii). Also, the coefficients of consumption of agricultural goods for the two worker classes, m_{11} and m_{12}, together with the propencities to consume out of income for the two worker classes, put a lower limit in the wages ratio $\mu(t)$ over time as shown in (3.17 ii).

From the above exposition we conclude that in the case of non-classical saving function ($0 \leqslant s_1 \leqslant s_2 \leqslant 1$) for any positive capital/labour ratio, k, the wage/rental ratio $\omega_2 = \omega_2(k)$ is uniquely determined by solving the equation (3.14), under the purely behavioural and technical sufficient conditions of capital intensities, $k_1 - k_2 < 0$, and that of (3.17), respectively.

4. Properties of the Static Model (Comparative Statics)

In the static model, equations (3.1) – (3.11), all the endogenous variables can be expressed in terms of the wage/rental ratios, which as we have already shown in section 3 can be uniquely determined under some sufficient conditions. Below we will analyse in more detail some of their properties.

(i) From (3.4) and the Inada derivative conditions it can be shown that

$$\frac{d\omega_i}{dk_i} = - \frac{f_i(\bar{k}_i)\, f_i''(\bar{k}_i)}{[f_i'(\bar{k}_i)]^2} > 0, \qquad i=1, 2 \tag{4.1}$$

where $\bar{k}_i = \bar{K}_i / \bar{L}_i$ is the capital/labour ratio in efficiency units and the derivatives of f_i are with respect to $\bar{k}_i$, which proves the monotinicity of $\omega_i(k_i)$. Thus, we conclude that, for any wage/rental ratios, ω_i, the optimal capital/labour ratios k_i in each sector are uniquely determined by the relations (3.4) and therefore we can invert (3.4) to obtain the capital per worker in either sector as a function of the wage/rentals ratios, respectively.

$$k_i = k_i(\omega_i), \qquad k_i'(\omega_i) > 0, \qquad i=1, 2 \tag{4.2}$$

where the assumptions on the production functions imply that

$$k_i(0) = 0, \qquad k_i(\infty) = \infty, \qquad i=1, 2^* \tag{4.3}$$

(ii) Differentiating (3.6), taking into account (3.4) and (4.1), we have for the terms of trade that

$$\frac{1}{p} \cdot \frac{dp}{d\omega_2} = \frac{\mu k_2(\omega_2) - k_1(\omega_1)}{[k_2(\omega_2) + \omega_2]\,[k_1(\omega_1) + \omega_1]} \tag{4.4}$$

(4.4) is positive, zero or negative according to whether $\mu k_2 - k_1$ is greater, equal or less than zero.

---

* There are of course cases where although k_i can assume any value between zero and infinity, the range of θ_i may not include all positive numbers[19].

In the case where $\mu k_2 - k_1 > 0$ or $k_1 / k_2 < \mu$ we see from (4.4) that as ω_2 increases or as labour becomes relatively more expensive than capital, then the agricultural goods become relatively more expensive than the non-agricultural goods. Because μ is always less than one for $0 \leqslant t < \infty$ we see that $k_1 / k_2 < 1$ which means that the second sector is more capital intensive, which does not contradict with the sufficient conditions of existence and uniqueness of section 3.

In the case where $\mu k_2 - k_1 < 0$ or $k_1 / k_2 > \mu$ we see that as ω_2 increases the agricultural goods become relatively less expensive than the non-agricultural goods. In this case it is possible for k_1 / k_2 to be greater than one but that contradicts with the capital intensity hypothesis; so we only consider the range $1 > k_1 / k_2 > \mu$.

For the case where $\mu k_2 - k_1 = 0$ or $k_1 / k_2 = \mu$ we see that the relative positions of the prices of the two goods remain the same with a change in ω_2.

From the above discussion we conclude that the value of μ is of critical importance because it determines the behaviour of the terms of trade.

(iii) For the percentages of urbanization, $l_i(\omega_i)$, we have from (3.13) that

$$\frac{dl_1}{d\omega_1} = \frac{-1}{k_1 - k_2} \left(\frac{1}{\mu} \frac{dk_2}{d\omega_2} l_2 + \frac{dk_1}{d\omega_1} l_1 \right) \lesseqgtr 0 \text{ if } k_1 - k_2 \lesseqgtr 0 \tag{4.5}$$

and

$$\frac{dl_2}{d\omega_2} = \frac{\mu}{k_1 - k_2} \left(\frac{1}{\mu} \frac{dk_2}{d\omega_2} l_2 + \frac{dk_1}{d\omega_1} l_1 \right) \lesseqgtr 0 \text{ if } k_1 - k_2 \lesseqgtr 0 \tag{4.6}$$

Thus, in the case where the second sector is more capital intensive, $k_1 - k_2 < 0$, we find that as labour becomes relatively more expensive than capital, urbanization decreses (l_2 decreases, l_1 increases), which means that substitution of labour for capital takes place.

(iv) For the per capita demand of the workers in both sectors and for both goods we have from (3.7) with the help of (3.4) and (4.1) that

$$\frac{dc_{ij}}{d\omega_j} = m_{ij}(1-s_j) A_i f_i(\overline{k}_i) \frac{k_i}{\omega_i + k_i}, \quad \begin{matrix} i=1,\ 2 \\ j=1,\ 2 \end{matrix} \tag{4.7}$$

which is always positive and means that an increase in the wage/rental ratio implies an increase in each wage-earner's income and so increase in the demand for all consumption goods.

In the opposite case, for the per capita demand of the capitalists for both goods, we have from (3.8) with the help of (3.4) and (4.1) that

$$\frac{dc_{i_3}}{d\omega_i} = -m_{i_3}(1-s_3) A_i f_i(\overline{k}_i) \frac{k}{\omega_i + k_i}, \quad i=1, 2 \tag{4.8}$$

which is always negative and means that an increase in the wage/rental ratio implies a decrease in the capitalist's income and so a decrease in the demand for all consumption goods.

(v) From (3.9) with the help of (4.5) and (4.6) we have

$$\frac{di}{d\omega_2} = \frac{A_2}{\omega_2 + k_2} f_2'(\bar{k}_2)\,[k_2(s_1 \mu l_1 + s_2 l_2) - s_3 k] +$$

$$+ \frac{A_2\,\omega_2}{k_1 - k_2} f_2'(\bar{k}_2)(s_2 \mu - s_1)\left(\frac{1}{\mu}\frac{dk_2}{d\omega_2} l_2 + \frac{dk_1}{d\omega_1} l_1\right) \tag{4.9}$$

In our model we assume that all income classes invest part of their incomes. Threrefore, we cannot be so sure that an increase in the wage/rental ratios will decrease the per capita investment. We need further behavioural or technical restrictions. Thus, sufficient conditions for (4.9) to be negative, under the general capital intensity condition of $k_1 - k_2 < 0$, are

$$\text{(i)} \qquad \mu s_2 - s_1 \geqslant 0$$

$$\text{(ii)} \qquad \frac{k}{k_2} \geqslant \frac{s_1 \mu l_1 + s_2 l_2}{s_3} \tag{4.10}$$

(4.10) does not only show that $s_1 / s_2 \leqslant 1$, which we have already assumed in the previous section, but that the relative level of s_1 compared with that of s_2 must be less than or equal to $\mu<1$. (4.10 ii) shows that under the capital intensity hypothesis it is $s_1 \mu l_1 + s_2 l_2 \leqslant s_3$. Of course, in the usual case of the Uzawa type models where $\mu=1$, $s_1 = s_2 = s_w \leqslant s_3 = s_r$, (4.10) relations reduce to the very simple one

$$1 \geqslant \frac{k}{k_2} \geqslant \frac{s_w}{s_r} \tag{4.11}$$

If we start from the per capita investment of the form

$$i=A_2 f_2'(\bar{k}_2)\,[l_2 k_2 - m_{21}(1-s_1)\omega_1 l_1 + (1-m_{22}(1-s_2))\omega_2 l_2 - m_{23}(1-s_3)k] \tag{4.12}$$

which is equivalent to that of (3.9), we have that

$$\frac{di}{d\omega_2} = -\frac{i}{\omega_2 + k_2} + A_2 f_2'(\bar{k}_2)\,[(1-m_{22}(1-s_2))\omega_2 \frac{dl_2}{d\omega_2} -$$

$$- m_{21}(1-s_1)\omega_2 \frac{dl_1}{d\omega_1} + ((1-m_{22}(1-s_2))l_2 - m_{21}(1-s_1)\mu l_1) +$$

$$+ \frac{1}{k_1 - k_2}(\mu k_2 l_1 \frac{dk_1}{d\omega_1} + k_1 l_2 \frac{dk_2}{d\omega_2}))] \tag{4.13}$$

Under the capital intensity condition, a sufficient condition for (4.13) to be negative is

$$\frac{l_1}{l_2} \geqslant \frac{1 - m_{22}(1-s_2)}{m_{21}(1-s_1)\,\mu} \tag{4.14}$$

(vi) The effect of a change in the wage/rental ratios upon the sectoral gross output per head, $q_i = Y_i / L$, i=1, 2, can be found by differentiation of

$$q_i = y_i\, l_i\,, \qquad i=1,\ 2 \tag{4.15}$$

i.e.

$$\frac{1}{q_1}\,\frac{dq_1}{d\omega_1} = \frac{-(\omega_1 + k_2)}{(\omega_1 + k_1)(k_1 - k_2)}\,\frac{dk_1}{d\omega_1} + \frac{1}{\mu}\,\frac{k - k_1}{(k-k_2)(k_1 - k_2)}\,\frac{dk_2}{d\omega_2} \tag{4.16}$$

$$\frac{1}{q_2}\,\frac{dq_2}{d\omega_2} = \mu\,\frac{k - k_2}{(k_1 - k)(k_1 - k_2)}\,\frac{dk_1}{d\omega_1} + \frac{\omega_2 + k_1}{(\omega_2 + k_2)(k_1 - k_2)}\,\frac{dk_2}{d\omega_2} \tag{4.17}$$

From the above expressions, under the capital intensity hypothesis ($k_1 < k < k_2$), we have

$$\frac{1}{q_1}\,\frac{dq_1}{d\omega_1} > 0 \qquad \text{and} \qquad \frac{1}{q_2}\,\frac{dq_2}{d\omega_2} < 0 \tag{4.18}$$

i.e. the sectoral output per head moves in opposite directions with an increase in the wage/rental ratios.

(vii) Finally, for the overall gross saving ratio, s, where

$$s = \frac{s_3\, k + s_1\, \omega_1\, l_1 + s_2\, \omega_2\, l_2}{k + \omega_1\, l_1 + \omega_2\, l_2} \tag{4.19}$$

we obtain

$$\begin{aligned}\frac{ds}{d\omega_2} = {} & (k + \omega_1\, l_1 + \omega_2\, l_2)^{-2}\ [k\,[\mu l_1 (s_1 - s_3) + l_2 (s_2 - s_3)]\ + \\ & + \mu\omega_1\ \frac{dl_1}{d\omega_1}\,[k\,(s_1 - s_3) + \omega_2\, l_2 (s_1 - s_2)] + \\ & + \omega_2\ \frac{dl_2}{d\omega_2}\,[k\,(s_2 - s_3) + \omega_1\, l_1 (s_2 - s_1)]\end{aligned} \tag{4.20}$$

A sufficient condition for (4.20) to be negative, under the capital intensity hypothesis, is

$$\frac{k}{\omega_1\, l_1} \leqslant \frac{s_2 - s_1}{s_3 - s_2} \tag{4.21}$$

i.e. under (4.21) an increase in the wage/rental ratio ω_2 causes a decrease in the gross saving ratio, s.

5. The Dynamic Economy (Stability)

The dynamic equations of our model are (2.10) and (2.11) from which we can derive the fundamental dynamic equation

$$\dot{k} = i-(\delta + n)k = i-\lambda k \tag{5.1}$$

where $\lambda=\delta+n$, or the equivalent

$$\frac{\dot{k}}{k} = \frac{i}{k} - \lambda \tag{5.2}$$

Let k^* be a trajectory of capital/labour ratio such that

$$\frac{i(\omega_2{}^*)}{k^*} = \lambda \tag{5.3}$$

where $\omega_2{}^*$ is the appropriate trajectory of the wage/rental ratio given by (3.14) for $k=k^*$. Such a k^* may be referred to as the trajectory of the "balanced capital/labour ratio".

We will see now under which conditions this trajectory of the balanced capital/labour ratio is uniquely determined. We define

$$\Phi(\omega_2) = i(\omega_2)/k \tag{5.4}$$

where $i(\omega_2)$ is given in (3.9) (or in (4.12)) and k in (3.14). Differentiating (5.4), after transforming it into logs, we have

$$\frac{1}{\Phi}\frac{d\Phi}{d\omega_2} = \frac{1}{i}\frac{di}{d\omega_2} - \frac{1}{k}\frac{dk}{d\omega_2} \tag{5.5}$$

In (5.5) $-\, dk/d\omega_2$ is negative under the sufficient conditions of (3.17) and $di/d\omega_2$ is negative under the sufficient condition of (4.10) or (4.14). Therefore, under these sufficient conditions, we have that

$$\frac{d\Phi(\omega_2)}{d\omega_2} < 0, \qquad \forall\ \omega_2 \tag{5.6}$$

Hence, under the sufficient conditions (3.17) and (4.10) or (4.14), for any λ, there exists one and only one trajectory, $\omega_2{}^*$ for which $\Phi(\omega_2{}^*) = \lambda$ and the corresponding trajectory $k^* = k(\omega_2{}^*)$ is uniquely determined by λ. Also, under the same sufficient conditions the relations (5.6) and (3.15) yield the following stability theorem, which is that along an arbitrary path of growth of K(t) and L(t), the capital/labour ratio k(t) asymptotically approaches the uniquely determined trajectory of the balanced capital/labour ratio, k^*. The approach of k(t) to k^* is monotonic, i.e. if k(0) is less than k^*, k(t) increasingly approaches k^*. Of course, this stability theorem applies to all the endogenous variables of the system..

In section 4.(vii) the overall gross saving ratio s can take multiple values with the same overall capital/labour ratio. This multiplicity of s causes multiplicity of i, which in turn is possible to produce causal indeterminancy in the model (multiple $\dot{k}$ for the same k). Therefore, the Inada derivative conditions which guarantee the existence of at least one nontrivial balanced growth path cannot assure the uniqueness of it without additional assumptions. For our model these additional assumptions are the (3.17) and (4.10) or (4.14). Of course, indeterminacy rules out, if we adopt the keynesian saving assumption, workers and capitalists save the same proportion of income, since under that assumption the gross saving ratio is unique and given.

6. Structural Change and the Growth Process

In this section we will study the viability of our economy considering (i) the per capita agricultural output, (ii) the composition of output among the two sectors and (iii) the labour redistribution[21].

(i) The per capita agricultural output is given by

$$q_1(t) = B_1(t)\ f_1(\bar{k}_1(t))\ l_1(t) \tag{6.1}$$

By differentiating (6.1) we obtain

$$\frac{\dot{q}_1(t)}{q_1(t)} = \pi_1(t) + [\varepsilon_{11}(t)\sigma_1(t) + \frac{1}{k_2(t)-k_1(t)}(\sigma_2(t)k_2(t)l_2(t) + \tag{6.2}$$

$$+ \sigma_1(t)k_1(t)l_1(t))]\frac{\dot{\omega}_1(t)}{\omega_1(t)} \tag{6.2}$$

where $\sigma_i(t)$ is the elasticity of substitution between factors;

$$\sigma_i(t) = \frac{dk_i}{d\omega_i}\ \frac{\omega_i}{k_i} = -\frac{f_i'(f_i - \bar{k}_i f_i')}{\bar{k}_i f_i f_i''} > 0, \quad i=1, 2, \tag{6.3}$$

$\varepsilon_{1i}(t)$ and $\varepsilon_{2i}(t)$ the current elasticity of output with respect to capital and labour respectively in the ith sector;

$$\varepsilon_{1i}(t) = \frac{\partial Y_i}{\partial K_i}\ \frac{K_i}{Y_i} = \frac{k_i}{\omega_i + k_i}, \quad i=1, 2, \tag{6.4}$$

and
$$\varepsilon_{2i}(t) = \frac{\partial Y_i}{\partial L_i}\ \frac{L_i}{Y_i} = \frac{\omega_i}{\omega_i + k_i}, \quad i=1, 2 \tag{6.5}$$

with
$$\varepsilon_{1i}(t) + \varepsilon_{2i}(t) = 1, \quad i=1, 2 \tag{6.6}$$

and $\pi_i(t)$ the increase in output due only to the passage of time and not to any changes in the quantity of the two inputs, capital and labour;

$$\pi_i(t) = \frac{1}{Y_i(t)} \frac{dY_i(t)}{dt} = \lambda_{1i}\,\varepsilon_{1i}(t), + \lambda_{2i}\,\varepsilon_{2i}(t), \quad i=1, 2 \tag{6.7}$$

According to Hicks[20], the factor-saving bias, $\lambda_i(t)$, is defined by the proportionate rate of change in the marginal rate of factor substitution in the ith sector;

$$\lambda_i(t) = \frac{\partial(\frac{\partial F_i}{\partial K_i})}{\partial_t} \frac{1}{\frac{\partial F_i}{\partial K_i}} - \frac{\partial(\frac{\partial F_i}{\partial L_i})}{\partial_t} \frac{1}{\frac{\partial F_i}{\partial L_i}} =$$

$$= (\lambda_{1i} - \lambda_{2i}) \left(\frac{\sigma_i(t)-1}{\sigma_i(t)}\right), \quad i=1, 2 \tag{6.8}$$

Turning now to (6.2) we see that it is positive, under the capital intensity condition, the condition of increase in the wage/rental ratio and the assumption that $\lambda_{11} \geqslant 0$ and $\lambda_{21} \geqslant 0$. Therefore, the per capita agricultural output is increasing with the passage of time.

(ii) The composition of output among the two sectors is defined by

$$v(t) = \frac{Y_2(t)}{p(t)Y_1(t) + Y_2(t)} = \frac{Y_2(t)}{Y(t)} \tag{6.9}$$

from which we obtain

$$\frac{\dot{v}}{v} = (1-v)\left[\frac{\dot{Y}_2}{Y_2} - \frac{\dot{Y}_1}{Y_1} - \frac{\dot{p}}{p}\right] \tag{6.10}$$

From (3.6) we obtain that the price effect in (6.10) is

$$\frac{\dot{p}}{p} = (\lambda_{12} - \lambda_{11}) + (\varepsilon_{21} - \varepsilon_{22}) \frac{\dot{\omega}_2}{\omega_2} + \varepsilon_{21} \frac{\dot{\mu}}{\mu} \tag{6.11}$$

The sign of (6.11) is ambiguous. In an equilibrium model where $\omega_1 = \omega_2$ and where also $\lambda_{12} = \lambda_{11}$ (same technological bias between sectors) [10], (6.11) is written as

$$\frac{\dot{p}}{p} = (\varepsilon_{12} - \varepsilon_{11}) \frac{\dot{\omega}_2}{\omega_2} \tag{6.12}$$

Since the non-agricultural sector is more capital intensive than the agricultural sector, $\varepsilon_{11} < \varepsilon_{12}$ and thus, with rising wage/rental ratios the relative price of agricultural to non-agricultural products is always rising. In our more general case of (6.11) we have the same result like (6.12) of [10] if we assume that $\lambda_{12} \geqslant \lambda_{11}$ i.e. if we assume that the rate of growth of technological progress of capital is greater in the non-agricultural sector than that of the agricultural sector.

For the quantity effect in (6.10) we have that

$$\frac{\dot{Y}_2}{Y_2} - \frac{\dot{Y}_1}{Y_1} = (\pi_2 - \pi_1) + (\varepsilon_{12}\sigma_2 - \varepsilon_{11}\sigma_1)\frac{\dot{\omega}_2}{\omega_2} - \varepsilon_{11}\sigma_1\frac{\dot{\mu}}{\mu} + \left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}_1}{L_1}\right) \quad (6.13)$$

The sign of (6.13) is ambiguous and so in the sign of the (6.10), or the corresponding

$$\frac{\dot{v}}{v} = (1-v)\,[(\lambda_{22} - \lambda_{12})\varepsilon_{22} + (\lambda_{11} - \lambda_{21})\varepsilon_{21} +$$

$$+ [\varepsilon_{12}(\sigma_2 - 1) - \varepsilon_{11}(\sigma_1 - 1)]\frac{\dot{\omega}_2}{\omega_2} - [\varepsilon_{11}(\sigma_1 - 1) + 1]\frac{\dot{\mu}}{\mu} + \left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}_1}{L_1}\right) \quad (6.14)$$

and therefore we cannot draw general conclusions about the composition of output among the two sectors. Even in the simpler case of [10] the sign of (6.14) is still ambiguous.

(iii) For the redistribution of labour between the two sectors let us define with $m_1(t) = M_1(t)/L_1(t)$ the rate of migration from the first sector into the second, where $M_1(t)$ is the absolute volume of migration, and $\eta_i(t)$, i=1, 2, the natural (or physiological) rate of increase of labour in the ith sector. Then, we have that

$$\frac{\dot{L}_1(t)}{L_1(t)} = \eta_1(t) - m_1(t) \quad (6.15)$$

and

$$\frac{\dot{L}_2(t)}{L_2(t)} = \eta_2(t) + m_1(t)\frac{l_1(t)}{l_2(t)} \quad (6.16)$$

With the help of (6.15), (6.16) and

$\frac{\dot{\omega}_1}{\omega_1} = \frac{\dot{\mu}}{\mu} + \frac{\dot{\omega}_2}{\omega_2}$ where $\frac{\dot{\omega}_i}{\omega_i} = \frac{1}{\sigma_i}\frac{\dot{k}_i}{k_i}$, i=1, 2 we obtain

$$m_1(t) = l_2(t)\,\frac{\dfrac{\dot{\mu}(t)}{\mu(t)} + \dfrac{1}{\sigma_2(t)}\left(\dfrac{\dot{K}_2(t)}{K_2(t)} - \eta_2(t)\right) - \dfrac{1}{\sigma_1(t)}\left(\dfrac{\dot{K}_1(t)}{K_1(t)} - \eta_1(t)\right)}{\dfrac{l_2(t)}{\sigma_1(t)} + \dfrac{l_1(t)}{\sigma_2(t)}} \quad (6.17)$$

Substituting $\dot{\mu}/\mu$ from (6.11) into (6.17) we obtain the equivalent expression,

$$m_1(t) = l_2(t)\,\frac{\dfrac{\dot{p}(t)}{p(t)} + (\lambda_{11} - \lambda_{12}) + \dfrac{\varepsilon_{22}(t)}{\sigma_2(t)}\left(\dfrac{\dot{K}_2(t)}{K_2(t)} - \eta_2(t)\right) - \dfrac{\varepsilon_{21}(t)}{\sigma_1(t)}\left(\dfrac{\dot{K}_1(t)}{K_1(t)} - \eta_1(t)\right)}{\dfrac{\varepsilon_{21}(t)}{\sigma_1(t)}\,l_2(t) + \dfrac{\varepsilon_{22}(t)}{\sigma_2(t)}\,\lambda^\star(t)} \quad (6.18)$$

(6.17) and (6.18) are two alternative expressions of the rate of migration from the agricultural sector. In (6.18) we see that the rate of migration depends upon the existing labour allocation, the elasticities of factor substitution, the elasticities of outputs with respect to inputs, the rates of technological progress, the rates of exogenous physiological labour change and the rates of change of the relative price and capital in the two sectors. All these factors influence the rate of migration with the correct sign. For example, we see that an increase in the capital of the second sector produces a positive effect on migration whilst on the contrary, an increase in the capital of the first sector produces a negative effect on migration; increases in the physiological rates of fertility of labour produce positive and negative effects for the first and the second sector respectively; and so on. Finally, using (6.17) we see that as $t \to \infty$ or as the economy becomes more developed, $\dot{\mu}/\mu \to 0$, which means that the effect of wages' differential on migration is decreasing over time.

7. Conclusions

The basic characteristics of our model is the introduction of capital into the agricultural sector, the notion of wage differentials between the two sectors which causes labour reallocation, the different savings' behaviour between the three income classes, namely the two worker classes and the capitalists, and the introduction of the technological progress of different rates into general production functions.

Kelley, Williamson and Cheetham[10], argue that the industrial sector must be more capital intensive than the agricultural sector. Our model with its sufficient consdition $k_1 - k_2 < 0$, for existence of the momentary solution agrees with the above authors.

Echaus[23], has argued that in underdeveloped economies the production process in the secondary industry is more capital intensive than in agriculture, which, together with the differences in elasticities of factor substitution, gives rise to the phenomenon of technological dualism. The above authors assume that $0<\sigma_2<1$ and $1 \leqslant \sigma_1 < \infty$ so that $\sigma_1 > \sigma_2$. In our model the sufficient conditions for uniqueness (3.17a) agree with the above writers. Moreover, in our model it is possible to be $\sigma_1 \leqslant \sigma_2$ but at the same time we must add some restrictions which can be seen in (3.17).

At the same sufficient conditions, (3.17), we have that $\sigma_1 + \sigma_2 \geqq 1$. This is a result obtained by Hahn[24] in his two sector growth model of one consumption good and one capital good sectors with classical saving function. In similarity with the latter model, Uzawa[25], Shinkai[26] and Inada[27] found that the consumption good sector must be more capital intensive than the capital good sector and that Takayama[28], Drandakis[29] and Amano[30] that as long as the elasticity of factor substitution in at least one sector is equal to or greater than unity, the two-sector growth model is globally stable. Of course, these results are different from ours but we must take in to account the different structure of our model concerning the commodity market.

The introduction of the wage differential or of the wages ratio, $\mu(t)$, into our model appears relevant, as was expected, to all the sufficient conditions with labour allocation, (4.14) savings behaviour, (3.17, ii) and (4.10, i) and consequently capital allocation (4.10, ii), and trade of goods between the two sectors, (4.4). Thus, from (4.14) we have that the ratio $L_1(t) / L_2(t)$ must have a lower limit; from (3.17, ii) and (4.10, i) we have that $s_2 > s_1$ which is true if we remember that the workers of the second sector are in higher income level than the workers in the first sector; from (4.10, ii) we see that the allocation of capital enters both the savings behaviour and the wages differential which constitute the determinants of investment; and finally from (4.4) we see the critical importance of $\mu(t)$ in determining the behaviour of the terms of trade.

In analysing the structural change and the growth process through time of our model we concluded that our economy is viable in the case where the non-agricultural sector is more capital intensive than the agricultural sector and the intensity of innovation $\pi_1(t)$ is positive, with increasing the wage/rental ratios through time. Also, we found that the migration rate $m_1(t)$ depends on most of the variables and parameters of our model, with the expected sign, and that $m_1(t)$ is a decreasing function of the wage-differential as $t \rightarrow \infty$.

If our model we put that $\mu(t)$ is constant and that there is no capital in the agricultural sector, then our model, after the appropriate changes in the savings function, behaves as the 'conventional' (Niho[22]) theories of dual development. If now, we put that $\mu(t) = 1$ and assume that the output of the second sector is completely invested in both sectors, then our model behaves as the two-sector growth models for the 'advanced' economies (Sato[31]). Therefore, in spite of the more complicated results of our model compared with those of other writers, we believe that the present model is more general and tries to bridge the gap between the models for the developed and less-developed economies.

8. References

1. Borts, G.H. and Stein, J.L. (1964). Economic Growth in a Free Market, New York, Columbia University Press.

2. Watanabe, T. (1965). Economic Aspects of Dualism in the Industrial Development of Japan, Economic Development and Cultural Change.

3. Williamson, J.G. (1965). Regional Inequality and the Process of National Development, Economic Development and Cultural Change.

4. Reynolds, L.G. (1965). Wages and Employment in a Labour Surplus Economy, A.E.R.

5. Taira, K. (1966), International Labour Review.

6. Ohkawa, K. and Rosovsky, H. (1972). Japanese Economic Growth.

7. Higgins, B.J. (1959). Economic Development, London, Constable.

8. Brutton, H.J. (1965). Principles of Development Economics, Englewood Cliffs, N.J., Prentice-Hall.

9. Nourse, H.O. (1968). Regional Economics: A Study in the Economic Structure, Stability, and Growth of Regions, N.Y., McGraw-Hill.

10. Kelley, A.C., Williamson, J.G., Cheetham, R.J. (1972). Dualistic Economic Development, The University of Chicago Press, Chicago and London.

11. Lewis, W.A. (1954), Economic Development with Unlimited Supplies of Labour, Manchester School.

12. Fei, J.C.H. and Ranis, G. (1964). Development of the Labour Surplus Economy, Richard D. Irwin, Homewood, Illinois.

13. Jorgenson, D.W. (1961). The Development of a Dual Economy, Economic Journal.

14. Dixit, A. (1973). Models of Dual Economies, in J.A. Mirrlees and N.H. Stern (ed.), Models of Economic Growth, MacMillan, London.

15. Graham, F.D. (1923). The Theory of International Values Re-examined, Quart. Journal, Econ.

16. Harris, J.R. Todaro, M.P. (1970). Migration, Unemployment and Development: A Two Sector Analysis, American Economic Review.

17. Harberger, A.C., (1971). On measuring the Social Opportunity Cost of Labour, International Labour Review.

18. Lal, D., (1973). Disutility of Effort, Migration, and Shadow Wage-rate, Oxford Economic Papers.

19. Burmeister, E., Dobell, A.R., (1972). Mathematical Theories of Economic Growth, The MacMillan Co., London.

20. Hicks, J.R., (1932). Theory of Wages, Macmillan and Co., London.

21. Zarembka, P. (1972). Toward a Theory of Economic Development. Holden-Day, San Francisco.

22. Niho, Y. (1974). Population Growth, Agricultural Capital and the Development of a Dual Economy, A.E.R.

23. Eckaus, R.S. (1955). The Factor Proportion problem in Underdeveloped Areas, A.E.R.

24. Hahn, F.H. (1965). On Two-Sector Growth Models, Review of Economic Studies.

25. Uzawa, H. (1961). On a Two-Sector Model of Economic Growth, Review of Economic Studies.

26. Shinkai, Y. (1960). On Equilibrium Growth of Capital and Labour, International Economic Review.

27. Inada, K. (1963). On a Two-Sector Model of Economic Growth; Comments and a Generalisation, Review of Economic Studies.

28. Takayama, A. (1963). On a Two-Sector Model of Economic Growth; A Comparative Static Analysis, Review of Economic Studies.

29. Drandakis, E.M. (1963). Factor Substitution in the Two-Sector Growth Model, Review of Economic Studies.

30. Amano, A. (1965). Factor Substitution in a Two-Sector Growth Model, Review of Economic Studies.

31. Sato, R. (1969). Stability Conditions in Two-Sector Models of Economic Growth. Journal of Economic Theory.

32. Sjaastad, L.A. (1962). The Costs and Returns of Human Migration, Journal of Political Economy.