The Estimation and Testing of Generalised Linear Expenditure Systems for Greece (1953-1974)

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ABSTRACT

This paper estimates a system of five Generalised Linear Expenditure System (GLES)-type demand functions using Greek data for the period 1953-1974. Emphasis is given on the autocorrelation hypothesis which was found to be of considerable importance particularly in discriminating among competing versions of the GLES model. Two dynamic versions of the GLES were proposed for comparison with static versions of the GLES. It was found that one of them can be used alongside with an autocorrelated version of the GLES model. On the basis of the Greed experience it is suggested that dynamic models should be examined for further study.

1, Introduction

The family of the generalised linear expenditure systems (GLES) has been used to analyse consumer expenditures for many countries. These models are useful extensions of the linear expenditure system (LES), preserving all its plausible economic interpretations. Moreover they satisfy all the constraints assumed by the neoclassical theory and can be more or less easily estimated.

Nevertheless a complcation arises from the fact that their variance covariance matrix is singular. This problem was tackled in a succesful way by Barten (1969), by deleting one equation of the system and estimating the remaining ones. However, given that serial correlation is an important factor for the demand systems, the researcher has to follow Berndt's and Savin's (1975) proposals so that all properies

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for the non-autocorrelated case are preserved for the serial correlation case.

In this paper we investigate the behaviour of the parameter estimates of the GLES model when we assume (a) no serial correlation, (b) a misspecified case of serial autocorrelation (Parks, (1969)), (c) correct spesification of serial correlation and (d) we finally investigate the behaviour of two dynamic versions of the GLES model since we have evidence of dynamic demand systems.

Section 2 discusses the proposed versions fo the GLES model and provides two dynamic versions of it. Section 3 contains the stochastic part of the models and describes the restrictions imposed on the autocorrelation matrix. Sectiuon 4 describes the data. Section 5 contains the results associated with various forms of the GLES plus the dynamic versions of ait and finally section 6 presents some conclusions relating to the results obtained earlier.

2. The Models

The generalised linear expenditure system was derived by T. Gamaletsos (1970) who sublsequently estimated and compared it against other systems (1973) in a cross-country comparison of demand patterns. It differs from the LES model in two important points: (a) it allows the marginal budget shares to depend on prices, whereas LES assumes constant budget shares (b) it allows any negative value to be obtained by the compensated own-price elasticities, whereas the LES permits only values between 0 and −1. Furthermore the GLES model allows the «supernumerary elastictiy of substitution» to take any positive value in contrast with the LES which restricts it to unity.

The GLES model is based on a generalised Klein-Rubin (1974) utility function of the following form

$$U = \sum_{i=1}^{n} \delta_i^{(1-\rho)} (q_i - \gamma_i) \tag{1}$$

where δ_i, γ_i and ρ are parameters obeying to the restrictions

$$0<\rho<1, \quad 0<\delta_i<1, \quad \sum_{i=1}^{n} \delta_i = 1, \quad (q_i-\gamma_i)<0 \qquad (i=1,...n)$$

Gamaletsos (1973) has shown that two utility functions similar to (1) can give rise to the GLES model. Maximisation of (1) subject to the budget constraint

$$Y = \sum_{i=1}^{n} p_i \cdot q_i \tag{2}$$

where p_i's and q_i's are prices and quantities and Y is total expenditure respectively, leads to the following demand model (GLES)

TABLE 1

Maximum Likeligood Estimates of the Parameters for Five Different Spicifications of the GLES Model[0]

Commodity group	Model I (No Autocorrelation, R=0)			Model II (Autocorrelation with unequal diagonal elements)		
	δ	γ_i	r_i	δ	γ_i	r_i
Food	0.25210 a	3.6638	0.49513	0.24214	2.9817	0.69263
	(0.007249)	(Π.86560)	(0.076506)	(0.0013725)	(2.4247)	(0.14355)
Beverages &	0.061969	0.37607		0.0644730	0.12477	
Tobacco	(0.0017669)	(0.20897)		(0.0015985)	(0.60165)	
Services	0.26893	0.17223		0.26810	−0.73702	
	(0.0039968)	(0.90522)		(0.0020513)	(2.4817)	
Durables	0.084518	−0.23545		0.086835	−0.56751	
	(0.0031218)	(0.26606)		(0.0022272)	(0.78462)	
Rent	0.17583	0.96100		0.18219	0.25336	
	(0.0018593)	(0.57658)		(0.0013983)	(1.6963)	
Clothing &	0.15666	−0.059348		0.15601	−57232	
Footwear	(0.0035631)	(0.49767		(0.0015069)	(1.4076)	
R_{11}		0			1.0851	
					(0.12732)	
R_{22}		0			0.27085	
					(0.30914)	
R_{33}		0			0.47972	
					(0.13130)	
R_{44}		0			0.00575054	
					(0.19826)	
R_{55}		0			0.27777	
					(0.17620)	
R_{66}		0			0.57115	
					0.17729)	
2 Log-likelihood		336.82			357.08	

Estimated asymptotic standard errors in parenthses

Model III

(Autocorrelation with equal diagonal elements)

Model IV
(Psychological component model)
Model V
(Psysiological and Psychological component model)

δ	γ_i	r_i	δ	β_i	r_i	δ	β_i	γ_i	r_i
0.25252	2.6562	0.57287	0.16820	0.98523	−0.021516	0.16827	0.70182	1.50220	0.41930
(0.0030085)	(2.1113)	(0.13108)	(0.043673)	(0.014216)	(0.37192)	(0.037196)	(0.09259:)	(0.61233)	(0.24009)
2	4	0		0	9		)		
0.063585	0.11389		0.030677	1.0213		0.035884	0.85010	0.075107	
(0.0011538)	(0.52131)		(0.016659)	(0.026408)		(0.011756)	(0.096568)	(0.11268)	
0.26991	−0.94750		0.28556	0.92486		0.29935	0.52645	0.56726	
(0.0016497)	(2.1981)		(0.033146)	(0.023133)		(0.024277)	(0.10529)	(0.82060)	
0.084339	−0.55002		0.13472	0.84286		0.10503	0.48584	0.025399	
(0.0017220)	(0.68354)		(0.21749)	(0.035135)		(0.015968)	(0.092924)	(0.27165)	
0.17639	0.24453		0.17780	0.95847		0.17893	0.59629	0.63493	
(0.0008159)	(1.4150)		(0.018113)	(0.015553)		(0.012119)	(0.088880)	(0.49412)	
0.15325	−0.61616		0.20304	0.90146		0.21248	0.43793	0.25503	
(0.0020875)	(1.1980)		(0.038073)	(0.040346)		(0.028346)	(0.13441)	(0.55023)	
	0.60409			0			0		
	(0.084191)								
	0.60409			0			0		
	(0.084191)								
	0.60409			0			0		
	(0.084191)								
	0.60409			0			0		
	(0.084191)								
	0.60409			0			0		
	(0.084191)								
	0.60409			0			0		
	(0.084191)								
	342.04			334.35			358.33		

$$p_i q_i = p_i \gamma_i + \delta_i p_i^{\tau} \; (\sum_{j=1}^{n} \delta_j p_j^{\tau})^{-1} \quad (Y = \sum_{j=1}^{n} p_j \gamma_j) \tag{3}$$

$$\tau = \rho/\rho - 1, \; \sum_{i=1}^{n} \delta_i = 1, \; 0 < \delta_i < 1, \; (q_i - \gamma_i) > 0 \qquad (i=1,...,n)$$

Equation (3) implies that individuals purchase a minimum set of required quantities $(\gamma_1,...,\gamma_n)$, firstly, and then they allocate the remaining supernumerary income

$$(Y - \sum_{j=1}^{n} p_j \gamma_j)$$

over all comodities in price-dependent proportions

$$\delta_1 p_1^{\tau} \; (\sum_{j=1}^{n} \delta_j p_j^{\tau})^{-1} \; \ldots, \; \delta_n p_n^{\tau}) \; (\sum_{j=1}^{n} \delta_j p_j^{\tau})^{-1}$$

Model (3) can be extended to incorporate consumption habits in lines of Pollak's and Wales' suggestion (1969). If we assume that the minimum required quantities are not constant through time we may write them as follows

$$\gamma_{it} = \beta_i q_{it-1} \tag{4}$$

or

$$\gamma_{it} = \gamma_i^* + \beta_i q_{it-1} \tag{5}$$

where γ_i^* and $\beta_i q_{it-1}$ are respectively the physiologically and psychologically necessary parts of the minimum consumption. Thus model (3) becomes, after substitution of (4) and (5)

$$p_{it} q_{it} = \beta_i p_{it} q_{it-1} + \delta_i p^{\tau}_{it} \; (\sum_{j=1}^{n} \delta_j p_{jt}^{\tau})^{-1} \; (Y - \sum_{j=1}^{n} p_{jt} \beta_j q_{it-1}) \tag{6}$$

$$p_{it} q_{it} = p_{it} \gamma_i^* + p_{it} \beta_i q_{it-1} = \delta_i p_{it}^{\tau} \; (\sum_{j=1}^{n} \delta_j p_{jt}^{\tau})^{-1} \; (Y - \sum_{j=1}^{n} p_{jt} \; (\gamma_j^* + \beta_j q_{it-1})) \tag{7}$$

Model (3) is nested into (7) and similarly (6) is nested into (7).

Models (6) nad (7) can be directly derived from a utility maximisation. This utility function is fo the form

$$U = \sum_{i=1}^{n} \delta_i^{(1-\rho)} \; (q_i - \beta_i q_{it-1}) \qquad \text{or}$$

$$U = \sum_{i=1}^{n} \delta_i^{(1-\rho)} \; (q_i - (\gamma_i^* + \beta_i q_{it-1})) \tag{8}$$

Apart from these functional modifications of the GLES model we thought that the stochastic part of the model needed some attention too. Thus we have proposed a first order autocorrelation process i.e.

$$e_t = R\, e_{t-1} + u_t \qquad t=2,...,T \tag{9}$$

where R is an nxn matrix of autocorrelaion parameters, u_t $(\underset{\sim}{O},\Sigma)$ and e_t NID $(\underset{\sim}{O},\Omega)$ are the error terms of the models. Because of the additivity restriction,

$$\sum_{i=1}^{n} p_i q_i = Y$$

Ω is singular and thus (9) implies

$$i'R = K \tag{10}$$

where K is an unknown constant (Berndt, Savin (1975)), hence Σ is singular.

If we start with a diagonal matrix R (no cross-equation autocorrelation) then (10) implies that all the elements of R are constrained to be equal. If we do not attend to this we will have a misspecified situation like R. Parks (1969). In that case the resulting estimates are not asymptotically efficient and not invariant to the equation deleted.

3. Estimation Technique - Stochastic Part of the Models

For the case of no autocorrelation it was assumed that the GLES could be written as

$$p_{it}q_{it} = p_{it}\gamma_i + \delta_i p_{it}^{\tau} \quad \left(\sum_{j=1}^{n} \delta_j p_{jt}^{\tau}\right)^{-1} \left(Y - \sum_{j=1}^{n} p_{jt}\gamma_j\right) + e_{it}$$

where

$$E(e_t)=0,\ E(e_t e'_{t'}) = \delta_{tt'}\Omega\ ,\ \delta_{tt'}= \begin{cases} 1 & \text{if } t=t' \\ 0 & \text{if } t\neq t' \end{cases}$$

Here we assumed only contemporaneous variance-covariance matrix Ω.

Because of the adding-up restriction, we must have

$$\sum_{i=1}^{n} e_{it} = 0,$$

so it follows that Ω is a singular matrix. Barten (1969) has shown a way to get estimates for a singular system and in fact the resulting estimates have the best properties.

If we allow our model to exhibit serial correlation then the error term can be written as

$$e_t = Re_{t-1} + u_t \qquad t=2,...,T$$

where $u_2,...,u_T$ are independently and identically distributed random normal variates with mean zero and Σ contemporaneous variance-covariance mateix. Moreover R is an unknown matrix of order nxn. Correct specification of the Rmatrix implies that the sum of the elements of each column of R must add to the same unknown constant K. In our case we must ensure that the diagonal matrix R has equal elements.

The estimation technique used is a FIML[2] and avoids the problem of further modification for dealing with the autocorrelation case as it was the case with D. Hendry's (1971) technique.

4. Data

The demand systems estimated contained 6 equations covering 6 commodity groups: Food, Beverages & Tobacco, Services, Durables, Rent, Clothing & Footwear. Our original data covered 14 commodity groups and this level of aggregation was chosen due to computational difficulties and costs associated with the application of the FIML (see Appendix)

The estimates were obtained from Greek annual data covering annual price and expenditure series for the period 1953-1974. Expenditures ($q_{it}q_{it}$) were taken as per capita current drachmas personal expenditures. We also used implicit price indexes (1970=1) which were obtained by dividing expenditure in current drachmas by expenditure in constant (1970) drachmas). These implicit price indexes were used as prices. Income (Y) was measured using the per capita total expenditure in current drachmas. All these series were taken from the national Accounts of Greece Year 1975, (1976).

5. Results

We present the estimated parameters of the five specifications of the GLES model in Table 1. Tables 2 and 3 contain the mean income elasticities and the mean own price elasticities. Moreover Table 1 contains the estimates of the autocorrelation

2. We are grateful to Professor A.S. Deaton for providing us the FIML algorithm, NLFIML. Our debt also extends to the staff tof the Computer Laboratory of the University of Kent especially to Joan. Dobby, for her helpful advice during the computation of the estimates.

matrix plus twice the value of the logarithm of the likeligood function. Finally Table 4 contains the values of the coefficients of determination R^2 which measure the goodness of fit for these demand models. Parameter estimates are accurate up to the sixth decimal point (double precision).

5.1 Parameter Estimates

As we note from Table 1, all δ_i's are positive and less than one. Furthermore they are significantly different than zero with the exception of model IV in which we observe an almost insignificant δ_i. It is remarkable the stability of these estimates for the first three versions of the GLES. Interestingly the last two versions of the GLES (dynamic) give the lowest values for the δ_i's with respect to Food and Beverages & Tobacco.

Looking at the estimates of the γ_i's we note that all are characterised by high standard errors making many of them statistically insignifican. For instance model I implies insignificant amounts of minimum required quantities for all commodity groups with the only exception of the Food. Durables and Rent show negative signs (for the same model). Models II and III exhibit similar pattern but now Food becomes insignificant. Finally model V implies a significant physiological component with regard to the first good (Food).

Next we examine the importance of the parameter τ. Models I, II and III imply a statistically significant value of τ, hence the GLES does not collapse on the LES model. However for the last two versions (IV and V) this parameter begaves in an unstable way, particularly in the IV model. It seems that the latter model shouldn't be considered seriously due to the poor explanatory power (i.e. its likelihood is the lowest among all the five models and moreover is decisevly rejected in favour of model V). By the same type comparison model I should be rejected in favour of models III and V.

It should be emphasised that we got a value for the R parameter which is significantly different than zero (i.e. the autocorrelation parameter $R_1=R_2=R_3=R_4=R_5=R_6 \lesseqgtr =0.60409$ with an as. standard error 0.084191). This value compares favourably to the result of Green et al. (1978) for their LES model which was found to be 0.605. Model II is a misspecified version of the correct model III, and was included in order to assess any discrepancies arisen from using such a model (Parks' case, (1969)).

Finally we note that models IV and V confirm Pollaks' and Wales' habit formation hypothsis depicting ignificant psychological components (i.e. significant β_i's). Those β_i's which correspond to commodities such as Benerages and Food are higher than those β_i's which correspond to the rest of the groups. This can be explained by the observation that people are more habit prone for perishable goods rather than luxury goods. In this way the most luxurious goods (Durables and Clothing) are characterised by the lowest β_i's.

In conclusion we have experienced dynamic behaviour in the static GLES model through the acceptance of the autocorrelation gypothisis (model III), and through the validity of the habit model V. It seems that the GLES rightly extends the LES model, giving values for the τ parameter very close to that reported by T. Gamaletsos (1973) in his coss-country study. There is a considerable stability in the estimates of the δ_i parameters, whereas there is considerable uncertainty associated with the size and sign of the γ_i parameters, which was experienced by others ((Lluch and Williams (1975), |Green et al.(1978)).

TABLE 2

Income Elasticities Estimates[α]

Commodity group	Model I	Model II	Model III	ModelIV	Model V
Food	0.625907	0.602461	0.627753	0.416293	0.417477
Beverages& Tobacco	0.908167	0.944157	0.931192	0.458219	0.527751
Services	1.342868	1.334220	1.346715	1.442965	1.497221
Durables	1.905055	1.953887	1.893759	2.992787	2.350909
Rent	0.990309	1.025779	0.993759	1.004685	1.007981
Clothing & Footwear	1.462258	1.464987	1.433352	1.861512	1.975419

α. Evaluated at the mean point of the sample $(\bar{Y}, \bar{p}_1, ... \bar{p}_6)$

TABLE 3a

Uncompensated Own-Price Elasticities[α]

Commodity group	ModelI	ModelII	Model III	Model IV	Model V
Food	−0.430431	−0.375783	−0.450089	−0.180733	−0.340113
Beverages & Tobacco	−0.386095	−0.321186	−0.424783	−0.009610	−0.160844
Services	−0.619356	−0.623286	−0.659682	−0.309796	−0.558968
Durables	−0.643563	−0.663127	−0.726695	−0.273578	−0.495014
Rent	−0.469142	−0.413611	−0.501540	−0.212708	−0.430565
Clothing & Footwear	−0.596577	−0.593673	−0.629430	−0.283140	−0.557753

α. Evaluated at the mean point of the sample $(\bar{Y}, \bar{p}_1, ..., \bar{p}_6)$

TABLE 3b

Compensated Own-Price Elasticities[a]

Commodity group	Model I	Model II	Model III	Model IV	Model V
Food	−0.177561	−0.132388	−0.196476	−0.012550	−0.171452
Beverages & Tobacco	−0.325248	−0.257927	−0.362393	0.021090	−0.125485
Services	−0.3534468	−0.359110	−0.393032	−0.024089	−0.262519
Durables	−0.557835	−0.575202	−0.641447	−0.138902	−0.389223
Rent	−0.293857	−0.232048	−0.325645	−0.074879	−0.252152
Clothing & Footwear	−0.437191	−0.433989	−0.473194	−0.080236	−0.342432

α. Evaluated at the mean point of the sample $(\bar{Y}, \bar{p}_1, ..., \bar{p}_6)$

5.2. Elasticity Estimates

Tables 2 and 3 present the income and price elasticities for the five models.

Looking at Table 2 we note that ther is a relative stability in the values of the income elasticity estimates. Models I,II,III give almost identical estimates, whereas the dynamic models IV and V tend to portray necessities as more necessities and luxuries as more luxuries. No dramatic changes appear to happen with the estimates. Clearly Food and B everages are necessities and Durables, Services and Clothing are Luxuries. The Rent is a unitary income elasticity good.

Table 3a presents the uncompensated own-price elasticities for all our specifications. Again we note the difference in the size of the estimated elasticities between the first three models and the dynamic ones. Food and Beverages are characterised by lower values of the price elasticities. All these differences can be attributed to the relatively high values of the commited quantities γ_i, with the highest those corresponding to the dynamic models. Table 3b presents the compensated own-price elasticities for the five GLES models. It appears that model IV gives exceptionally low values of price elasticities with one notable violation of the law of consumer demand (Beverage case), due to the very small value of the respective uncompensated own-price elasticity. Interestingly model V gives higher values for the uncompensated own-price elasticities which can be compared favourably to those obtained in cases I, II, and III.

5.3 Goodness of fit

Table 4 presents the determination coefficients for the five GLES family models. All models are characterised by very high values of R^2. This was anticipated because of the use of variables in levels such as expenditures as left hand variables. Model III fares slightly better than model I, whereas the comparison of models II and III is inconclusive. Model IV does not appear to fare better than model III, but model V seems to exhibit the best fit. In that respect one can possibly pick model V for further use alongside model III. It is difficult to discriminate among the above five models solely relying to the R^2 criterion. Other criteria such as information inaccuracy or predictive performance tests can alongside the R^2 assist us in discriminating among these models.

TABLE 4

Coefficients of Determination (R^2s) for the GLES Models

Variants of the GLES	Commodities					
	Food	Beverages & Tobacco	Services	Durables	Rent	Clothing & Footwear
Model I	0.996854	0.993715	0.998676	0.985360	0.999494	0.995061
Model II	0.998649	0.992447	0.999141	0.985535	0.999499	0.995202
Model III	0.998708	0.993659	0.999038	0.989392	0.999567	0.995103
Model IV	0.998694	0.993257	0.999002	0.990844	0.999581	0.993158
Model V	0.998888	0.995207	0.999080	0.991529	0.999646	0.995349

5.4. A Discussion of the Model Restrictions

Parks (1969) in his attempt to assess the implications of serial correlation used the LES model in a similar version as our model II. We have shown that corrected action results to estimate model III. Although the parameter estimates are comparable for these two models we note a very significant difference among the estimated values of the R matrix. Three values of that matrix appear to be insignificant whereas one is higher than 1 (Food) and significant. This variability is due to the inefficient estimation against which Berndt and Savin (1975) warn. When corrected action is taken the value of the autocorrelation coefficient becomes less than one, significant and reasonable. On that basis model II should be abandoned in favour of model III.

6. Conclusions. Some Further Comments

Five GLES-type models have been estimated and compared using fit, elasticity and autocorrelation criteria.

Berndt's and Savin's (1975) suggestion on treating the singularity problem in case of serial correlation has been taken into account so that model III was duly estimated. Our result supports Green's (1978) findings and suggest the existance of dynamic elements in the GLES model. Hence model III was picked for further use instead of I and II.

All the models showed a remarkably good fit probably due to the estimation in levels. Therefore it is difficult to choose among demand models solely relying upon the R^2 criterion. In aour case the autocorrelation criterion provides a better means to discriminate among competing demand models.

Elasticity estimates were more or less stable with the exception of model IV. There was more variation in the price elasticities than in the income elasticities, due to more variation in the estimated γ_i's rather than the δ_i's. A similar situation was experienced in the estimation of the γ_i's of the LES model by other researchers as Lluch and Williams (1975), J. Mackinnon (1976), N. Kiefer and J. Mackinnon (1976) and Green (1978).

Finally we can suggest in lines with L. Phlips (1974) the use of dynamic models. These models are partly justified by the acceptance of the autocorrelation hypothesis and partly by the acceptance of the dynamic version V of the GLES. Therefore Greek experience suggests the use of either model III or model V for any practical purpose.

APPENDIX

The National Accounts of Greece (1976) provide data for 14 commodity groups whish gave been aggregated into 6 broad groups. In what follows we present the original 14 commodity grouping and our 6 broad commodity aggregation.

1. Food
2. Beverages
   a. Non Alcoholic
   b. Alcoholic
3. Tobacco
4. Clothing-Footwear
   a. Clothing
   b. Footwear
   c. Other personal Effects
5. Rent and water charges
6. Fuel and Light
7. Furniture furnishings & household equipment
   a. Furniture and furnishings
   b. Household equipment

8. Household operation
9. Personal care & health expenses
10. Transportation
    a. Personal transport equipment
    b. Operation of personal transport
    c. Purchased transport
11. Communications
12. Recreation and entertainment
13. Education
14. Miscellaneous services

A. Food	(1)
B. Beverages and Tobacco	(2)+(3)
C. Servises	(14)+(13)+(12)+(11)+(10b)+(10c)+(9)
D. Durables	(7)+(10a)
E. Rent	(5)+(6)+(8)
F. Clothing and Footwear	(4)

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