VALUE ADDED TAX AND PRODUCTIVE EFFORT: THE CASE OF GREECE

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The substitution of VAT for the existing bundle of indirect taxes in Greece is apt to result in changes in relative prices of goods and services as some items, which escaped the tax in the pre - VAT situation, will now be subjected to its scope while other items, which were previously taxed with lower (higher) rates, will now bear higher (lower) rates. Consequently, factors' combinations and methods of production will have to be adjusted to the new conditions. The present paper records an attempt to delineate the post - VAT combinations of productive factors which will maximise profits or capital equipment in the context of an input - output table for the Greek economy.

The substitution of VAT for the existing indirect taxes in Greece, as a concomitant of the integration with EEC, is apt to have significant repercussions on the performance of the Greek economy. A meaningful analysis of the effects of the tax switch on investment and production should be based on a given structure of VAT. This is not however the case with Greece as fiscal authorities have not yet outlined the main characteristics of VAT. Thus, the conclusions to be reached here must be considered as tentative.

The usual postulate in evaluating indirect taxation is that producers shift the entire amount of either the existing indirect taxes or VAT forward onto consumers. Such a policy tends to burden private consumption excessively and may curb aggregate demand and production so that what seems to be an optimun strategy for each productive unit may ultimately retard economic growth. Perhaps a partial absorption of VAT by firms may stimulate domestic expenditure and promote sales by firms to an extent sufficient to cover the initial losses, provided that there is excess capacity.

In what follows the analysis of the effects of VAT is carried out in the context of a consolidated form of the Koutsoyianni's [1] input - output table of the Greek economy, 1960, as shown by table 1. In spite of the fact that the latter refers to a remote year and cannot reflect the contemporary tendencies in the Greek economy, we make use of it because this is the only available.

To check the hypothesis that varying patterns of tax shifting affect the level of output, we have constructed table 2 on the basis of table 1. In particular, sector (a) consists of agricultural products and food, sector (b) is composed of the subsectors tobacco - beverages, textile - leather, industry and construction - housing, and sector (c) includes fuel - power, services and trade.

Moreover, the distribution of an equal - revenue VAT (9.424 m. dr.) among the three sectors is made as follows :

The tax yield is divided by the value added by all sectors (wages, salaries, profit, rent etc.) ; thus, a uniform rate of VAT, equal to 8.62 %, is derived.

Subsequently, the value added by each sector is multiplied by this rate and the corresponding tax obligation is estimated.

The underlying assumption is that the intermediate input coefficients as well as the import coefficient remain constant, so that the effects of the substitution of VAT will manifest themselves either in consumer prices or in the rewards to productive factors, labour and capital (including land, depreciation etc: as indicated above).

We will treat each pattern of tax shifting as a discreet productive activity (which is defined as the column of input coefficients required to produce one unit of the particular commodity) because, depending on the nature and the degree of tax shifting, the flow matrix takes different forms each time. For instance, if VAT is completely passed forward, then the productive activity for sectors (a), (b) and (c) will be the one indicated by number 2 (in table 2) which is taken from the original table 1. The productive activity which characterises the Greek economy, under the present system of indirect taxes, is given by the first column of (a), (b), (c) in table 2 and is derived from table 1 when the latter is properly summarised. If we assume that the excess of VAT over existing indirect taxes [2] is completely shifted backward onto labour - productive activity three, then the wage bill falls from 16504 to 15569 m. drs. in sector (a) and from 21094 to 19061 m. drs. in sector

1. A. Koutsoyianni «Input-Output Table of the Greek Economy, Year 1960», CPER, Athens 1967.

2. Note that it is the excess of VAT (existing indirect taxes) over existing indirect taxes (VAT) which matters, since up to a certain amount of tax revenue (1558 m. dr. for Agriculture, 2865 m. dr. for industry and 2033 m. dr. for services) the incidence of the pre-VAT tax regime is supposedly given.

Table 1. Consolidated Input-Output table of Greece, 1960 (Source: A. Koutsoyianni's input-output table of the Greek Economy, 1960)

	1 Agric. Food	2 Tobacco Beverage	3 Text. Leather	4 Indus-try	5 Const. Housing	6 Fuel Power	7 Serv-ices	8 Trade	9 Gov. Cons.	10 Export	11 Private Cons.	12 Inv.	Total net output
1. Agriculture Food	x_{11} 0 a_{11} 0	x_{21} 2910 a_{21} 0.3716	x_{31} 1859 a_{31} 0.1134	x_{41} 1158 a_{41} 0.0370	x_{51} 59 a_{51} 0.0023	x_{61} 1 a_{61} 0.0002	x_{71} 1906 a_{71} 0.0483	x_{81} 188 a_{81} 0.0105	x_{T1} 252 a_{T1} 0.0267	x_{m1} 2116 a_{m1} 0.1239	x_{n1} 37.000 a_{n1} 0.7568	x_{p1} 756 a_{p1} 0.0125	X_1 48205
2. Tobacco Beverages	x_{12} 292 a_{12} 0.0061	x_{22} 0 a_{22} 0	x_{32} 0 a_{32} 0	x_{42} 14 a_{42} 0.0004	x_{52} 0 a_{52} 0	x_{62} 0 a_{62} 0	x_{72} 417 a_{72} 0.0106	x_{82} 0 a_{82} 0	x_{T2} 1 a_{T2} 0.0001	x_{m2} 2267 a_{m2} 0.1327	x_{n2} 4349 a_{n2} 0.0889	x_{p2} 492 a_{p2} 0.0081	X_2 7832
3. Textiles Leather	x_{13} 760 a_{13} 0.0158	x_{23} 23 a_{23} 0.0029	x_{33} 0 a_{33} 0	x_{43} 138 a_{43} 0.0044	x_{53} 0 a_{53} 0	x_{63} 0 a_{63} 0	x_{73} 369 a_{73} 0.0093	x_{83} 142 a_{83} 0.0079	x_{T3} 116 a_{T3} 0.0123	x_{m3} 749 a_{m3} 0.0439	x_{n3} 14084 a_{n3} 0.2881	x_{p3} 20 a_{p3} 0.0003	X_3 16401
4. Industry	x_{14} 2614 a_{14} 0.0542	x_{24} 214 a_{24} 0.0273	x_{34} 1553 a_{34} 0.0947	x_{44} 0 a_{44} 0	x_{54} 8109 a_{54} 0.3151	x_{64} 1336 a_{64} 0.2039	x_{74} 1603 a_{74} 0.0406	x_{84} 1096 a_{84} 0.0609	x_{T4} 800 a_{T4} 0.0849	x_{m4} 989 a_{m4} 0.0579	x_{n4} 9107 a_{n4} 0.1863	x_{p4} 3856 a_{p4} 0.0638	X_4 31277
5. Construction Housing	x_{15} 109 a_{15} 0.0023	x_{25} 17 a_{25} 0.0022	x_{35} 20 a_{35} 0.0012	x_{45} 68 a_{45} 0.0022	x_{55} 0 a_{55} 0	x_{65} 1 a_{65} 0.0002	x_{75} 108 a_{75} 0.0027	x_{85} 97 a_{85} 0.0054	x_{T5} 1572 a_{T5} 0.1668	x_{m5} 1 a_{m5} 0.0001	x_{n5} 9813 a_{n5} 0.2007	x_{p5} 13932 a_{p5} 0.2305	X_5 25738
6. Fuel Power	x_{16} 672 a_{16} 0.0139	x_{26} 34 a_{26} 0.0043	x_{36} 287 a_{36} 0.0175	x_{46} 768 a_{46} 0.0246	x_{56} 209 a_{56} 0.0081	x_{66} 0 a_{66} 0	x_{76} 1551 a_{76} 0.0393	x_{86} 361 a_{86} 0.0201	x_{T6} 312 a_{T6} 0.0331	x_{m6} 46 a_{m6} 0.0027	x_{n6} 2310 a_{n6} 0.0472	x_{p6} 0 a_{p6} 0	X_6 6550
7. Services	x_{17} 1904 a_{17} 0.0395	x_{27} 300 a_{27} 0.0383	x_{37} 363 a_{37} 0.0221	x_{47} 1801 a_{47} 0.0576	x_{57} 1117 a_{57} 0.0434	x_{67} 463 a_{67} 0.0707	x_{77} 0 a_{77} 0	x_{87} 2614 a_{87} 0.1453	x_{T7} 9470 a_{T7} 1.0049	x_{m7} 3876 a_{m7} 0.2270	x_{n7} 17562 a_{n7} 0.3592	x_{p7} 0 a_{p7} 0	X_7 39470
8. Trade	x_{18} 8642 a_{18} 0.1793	x_{28} 956 a_{28} 0.1221	x_{38} 3740 a_{38} 0.2280	x_{48} 4100 a_{48} 0.1311	x_{58} 0 a_{58} 0	x_{58} 545 a_{68} 0.0832	x_{78} 0 a_{78} 0	x_{88} 0 a_{88} 0	x_{T8} 0 a_{T8} 0	x_{m8} 0 a_{m8} 0	x_{n8} 0 a_{n8} 0	x_{p8} 0 a_{p8} 0	X_8 17983
9. Indirect taxes net of subsidies.	x_{1T} 1558 a_{1T} 0.0323	x_{2T} 2234 a_{2T} 0.2852	x_{3T} 777 a_{3T} 0.0474	x_{4T} 2253 a_{4T} 0.0720	x_{5T} 569 a_{5T} 0.0221	x_{6T} 1269 a_{6T} 0.1937	x_{7T} 694 a_{7T} 0.0176	x_{8T} 70 a_{8T} 0.0039	x_{TT} 0 a_{TT} 0	x_{mT} 0 a_{mT} 0	x_{nT} 0 a_{nT} 0	x_{pT} 0 a_{pT} 0	X_T 9424
10. Imports	x_{1m} 2722 a_{1m} 0.0564	x_{2m} 11 a_{2m} 0.0014	x_{3m} 1899 a_{3m} 0.1158	x_{4m} 10443 a_{4m} 0.3339	x_{5m} 0 a_{5m} 0	x_{6m} 484 a_{6m} 0.0739	x_{7m} 1519 a_{7m} 0.0385	x_{8m} 0 a_{8m} 0	x_{Tm} 0 a_{Tm} 0	x_{mm} 0 a_{mm} 0	x_{nm} 0 a_{nm} 0	x_{pm} 0 a_{pm} 0	X_m 17078
11. Wages Salaries (Households)	x_{1n} 16504 a_{1n} 0.3424	x_{2n} 767 a_{2n} 0.0979	x_{3n} 2324 a_{3n} 0.1417	x_{4n} 4602 a_{4n} 0.1471	x_{5n} 3602 a_{5n} 0.1399	x_{6n} 841 a_{6n} 0.1284	x_{7n} 18060 a_{7n} 0.4576	x_{8n} 2193 a_{8n} 0.1279	x_{Tn} 0 a_{Tn} 0	x_{mn} 0 a_{mn} 0	x_{nn} 0 a_{nn} 0	x_{pn} 0 a_{pn} 0	X_n 48393
12. Profit rent inter. depred. (Households)	x_{1p} 12428 a_{1p} 0.2578	x_{2p} 366 a_{2p} 0.0468	x_{3p} 3579 a_{3p} 0.2182	x_{4p} 5932 a_{4p} 0.1897	x_{5p} 12073 a_{5p} 0.4691	x_{6p} 1610 a_{6p} 0.2458	x_{7p} 13243 a_{7p} 0.3355	x_{8p} 11222 a_{8p} 0.6241	x_{Tp} 0 a_{Tp} 0	x_{mp} 0 a_{mp} 0	x_{np} 0 a_{np} 0	x_{pp} 0 a_{pp} 0	X_p 60453
Total net input	48205	7832	16401	31277	25738	6550	39470	17983	12523	10044	94225	19056	329304

(c). That is, by exactly the same amount by which the tax bill rises in each of these two sectors, i. e. by 935 and 2033 m. drs. respectively. In sector (b) we have the phenomenon of negative shifting on labour, i. e. the gains fron the tax reduction (5833 - 2865 = 2968 m. drs) are apparently allotted to the labour force (14.263 - — 11.295 = 2968 m. drs). By formulating the third productive activity in this way we succeed in two respects : firstly, we maintain the amount of the available resources unaltered at 48.893 m. drs. for labour and 60.453 m. drs. for the other gross value added ; secondly, we hold the algebraic sum of the technical - input coefficients equal to unity, so that no material change of the basic model occurs.

The same rationale lies behind re - casting of the initial input-output table in terms of the productive activity four ; the only difference is that the VAT increase (decrease) is reflected in lower (higher) rewards for capital. In the remaining three productive activities (five, six, seven) the excess of the VAT burden (gain) is shared by labour and capital in varying proportions.

Since there are many alternative tax - shifting proportions between labour and capital, we have experimented with various combinations. The selection of the three which are depicted on table 2 serves our purposes best as will be shown later on. The reader is free to choose any other combinations but bearing in mind the two restrictions set above, i. e. that the input coefficients should add vertically to unity and that available resources are taken as given.

Lastly, table 2 is treated as an open model, i. e. the sectors labour, capital, taxes, imports and the corresponding four components of final demand are considered to lie outside the technology matrix.

The basic question is which productive activity will be the proper one to maximise total output from a given endowment of labour and capital. When we say total output we mean a number of baskets of goods, each containing items from sectors (a), (b) and (c) in proportion to the final demand for their output.

Thus, one typical basket is composed of 0.2954 agricultural products $\left(=\frac{C_a}{C_a+C_b+C_c}\right)$, 0.4575 industrial products $\left(=\frac{C_b}{C_a+C_b+C_c}\right)$ and 0.2471 services $\left(=\frac{C_c}{C_a+C_b+C_c}\right)$, where C_a (C_b, C_c) represents the final demand (private consumption, government expenditure, investment, exports) for the output of sector a (b, c). The new input-output table will be of the form :

TABLE 3

A 3 x 3 input-output table for Greece, 1960.

sectors	a	b	c	Demand	Output
a	0	0.0842	0.0358	0.2954	X_a
b	0.0783	0	0.0884	0.4575	X_b
c	0.2327	0.1924	0	0.2471	X_c

From table 3 we obtain :

$$X_a - 0.0842\ X_b - 0.0358\ X_c = 0.2954$$

$$-0.0783\ X_a + X_b - 0.0884\ X_c = 0.4575$$

$$-0.2327\ X_a - 0.1924\ X_b + X_c = 0.2471$$

which is a system of three equations with three unknowns.

TABLE 4

Labour and Capital requirements for the first productive activity

	Output × Labour input coefficient		Amount of Labour	Output × Capital input coefficient		Amount of Capital
X_a	0.3549 × 0.3424	=	0.1215177	0.3549 × 0.2578	=	0.0914932
X_b	0.5233 × 0.1589	=	0.0831523	0.5233 × 0.3087	=	0.1615427
X_c	0.4304 × 0.3608	=	0.1552883	0.4304 × 0.4460	=	0.1919584
Total			0.3599583			0.4449943

By solving this system we find : $X_a = 0.3549$, $X_b = 0.5233$, and $X_c = 0.4304$. Since the entries of table 3 are common for all seven productive activities, we conclude that these values for output will hold whatever activity is ultimately chosen.

The production of each sector's output requires labour and capital inputs in varying proportions according to the particular activity which is pursued. As an example, let us suppose that the first activity is undertaken. Table 4 presents the labour and capital requirements for each sector separately and for all three sectors when the first activity (which is identical to the second one) is used. By changing only the input coefficients we can calculate, in a similar fashion, these requirements for the remaining sectors and simultaneously determine the corresponding labour-to-capital ratio (see table 5).

When the various labour-capital combinations are plotted in fig. 1 we can see that the corresponding points may lie on the same or different isoproduct curves according to whether the seven activities are capable of producing the same or different outputs.

To single out the optimum activity, two criteria must be employed; firstly, the preferable activity should maximise final output with the given endowment of resources ; secondly, such an activity should ensure full employment of the available factors of production.

In fig. 1 the output grows as we move to higher iso-product curves. In our case the highest curve is provided by the seventh activity; thus, the first require-

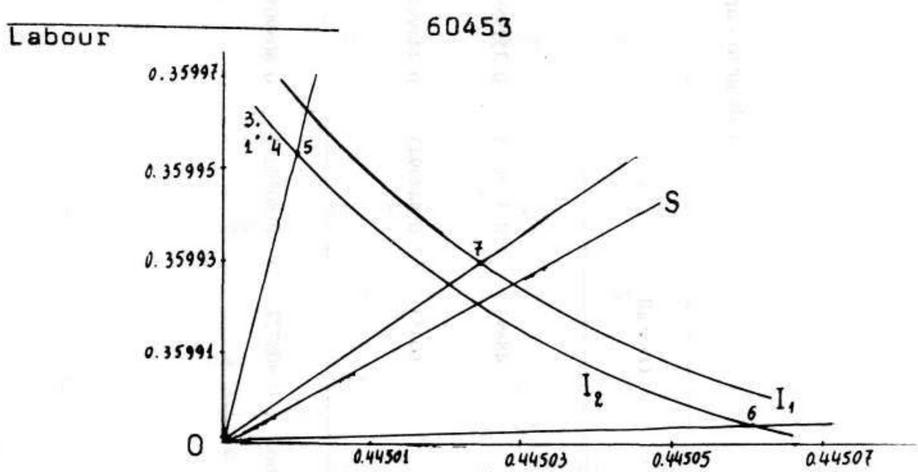

Fig. 1. Labour-capital combinations under various productive activities.

TABLE 5

Labour-to-Capital ratios for all productive activities.

	Over all	1	2	3	4	5	6	7
Labour	48893	0.3599583	0.3599583	0.3599600	0.3599583	0.3599546	0.3598931	0.3599299
Capital	60453	0.4449943	0.4449943	0.4449943	0.4449959	0.4449996	0.4450610	0.4450243
L/K ratio	0.808777	0.8089054	0.8089054	0.8089092	0.8089025	0.8088874	0.8086376	0.808787

ment is fulfilled. On the other hand, full employment of resources is achieved when labour and capital are utilised in the ratio indicated by the straight line OS which is drawn on the basis of the overall labour-to-capital ratio $\left(\frac{48893}{60453}=0.808777\right)$. The seventh activity is again the closest one to the line OS and must be preferred to the other six, even though point 7 does not coincide with OS, which in turn implies that some factor units will remain unemployed. If the seventh activity is not easily determined, we should resort to the second best set of activities which is made of the straddling activities five and six. In general, if the ratios of input coefficients in all activities differ from the overall ratio of resource supplies, then the two activities, which (i) lie on the highest possible iso-product curve and (ii) have ratios of input coefficients that straddle the overall factor-supply ratio with the smallest distance from the latter, should be identified. This means that a weighted sum of the two preferable activities ought to be used but we will return to this issue later.

The diagrammatic method of identifying the optimum activity gives an

TABLE 6

Output under various factor combinations

Productive Activity	Output	Capital input coefficient	Amount of Capital
1-2	$\frac{48893}{0.3599583} = 135829.62$	$\times\ 0.4449943$	$= 60443.41$
3	$\frac{48893}{0.35996} = 135828.98$	$\times\ 0.4449943$	$=60443.12$
4	$\frac{48893}{0.3599583} = 135829.62$	$\times\ 0.4449959$	$=60443.62$
5	$\frac{48893}{0.3599546} = 135831.01$	$\times\ 0.4449996$	$=60444.74$
6	$\frac{48893}{0.3598931} = 135854.23$	$\times\ 0.4450610$	$= 60463.42$
7	$\frac{48893}{0.3599299} = 135840.34$	$\times\ 0.4450243$	$=60452.25$

illustrative but oversimplified picture of the problems to be faced by the policy makers. However, it is too crude to be reliable. If we wish to make useful predictions, we must build our arguments on a sounder footing. Starting with, say, productive activity three , the available labour, if fully utilised, is capable of producing $\frac{48893}{0.35996} = 135828.98$ baskets of the given output.

Should this be produced, the employment of capital ($135828.98 \times 0.4449943 = 60443.12$) will fall short of the full employment standard (60453)[3]. Turning now to, say, the fourth activity (see table 6) we are able to increase the number of baskets of output and promote the employment of capital, and so on until all activities are tested. Any activity alone may be adopted except for the sixth one, which gives the greatest production, but it requires more capital input than the economy can offer, unless the sixth activity is combined with another activity. This will be considered below.

Table 6 reinforces the conclusions of fig. 1. It is the seventh activity which gives maximum production, full employment of labour and the best employment status for capital.

Analogous conclusions arise if we divide capital endowment by the respective capital - input coefficient and then proceed to estimate output produced and the amount of labour required by each activity.

Let us suppose that the seventh activity is not available. In this case, we wish to prove our contention that the second best alternative is the combination of activities five and six. The full employment requirement is fulfilled when the final outputs (produced by each of the complementary activities five and six) multiplied by the corresponding labour (capital) input coefficients produce the available stock of labour (capital) in the economy; that is, when :

$$0{,}3599546 . V + 0{,}3598931 . W = 48893$$

$$0{,}4449996 . V + 0{,}445061 . W = 60453$$

where V and W stand for the output generated by activities five and six respectively. Solving these linear equations, we obtain $V = 75803.57$ and $W = 60036.43$, whereas their algebraic sum, $V + W = 135840$, roughly equals the level of the maximum product achieved by the optimum activity seven alone.

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3. Note that the differences are quite small to justify significant (dis) advantages of one activity vis-a-vis the others.

An alternative way of reaching the same result is by working on the overall labour-to-capital ratio, viz :

$$\frac{48893}{60453} = \frac{0.3599546\ h + 0.3598931\ (1 - h)}{0.4449996\ h + 0.445061\ (1 - h)}$$

where h represents the portion of final demand to be satisfied by activity five. The solution of this equation gives $h = 0.558$, which implies that 55.8 % of demand will be satisfied by the fifth activity and the remaining 44.2 % by the sixth one.

Our findings imply that the assumption of complete forward shifting of VAT, which constitutes the core of the analysis and conforms with the nature of VAT and the realities of the Greek economy, does not necessarily lead to the maximisation of output and full employment. This is also the case under the premise of complete backward shifting of the VAT differential solely onto labour or solely onto capital. On the contrary, maximised output and full employment may be realised when approximately half the tax - burden (tax-benefit) differential is borne by wage-earners and half by profit-makers. The explanation of this outcome may be sought in two directions :

Firstly, under conditions of complete forward shifting of the VAT differential, the redistribution of the tax burden will affect productive activity indirectly through changing the pattern of consumer expenditure in favour of industrial products. By contrast, under conditions of complete backward shifting of the VAT differential, the alleviation of the tax burden on industrial capital and /or labour (sector b of table 2) and the corresponding tax increase on capital and /or labour of agriculture and services (sectors a and c) will reinforce the desirable tendency towards industrialization in a direct way.

Secondly, a more or less equiproportionate allocation of the VAT differential between labour and capital in each sector will leave relative factor shares unaltered, so that no factor gains at the expense of the other and no ground for a restriction of the factors' supply is provided, as might be the case with activities three or four of table 6. Of course, the reactions of the various groups to a prospective change in their factor rewards will not be uniform but the unit elasticity of substitution implied by the Leontief's model is sufficient to guarantee that nothing unusual occurs once activity seven (or five-six) is used. As a matter of fact, the shifting of the entire tax differential on labour in activity three (capital in activity four) tends to alter the net-of-tax price of labour (capital) relative to that of capital (labour), so that firms are given an incentive to change the labour-to-capital ratio.

Consequently, more or less labour and capital may be utilised than dictated by the given stock of resources and the technological requirements.

Such a disturbance is not likely to take place with activity seven (or five-six) because both factors absorb almost[4] equal portions of the VAT differential, so that the price ratio and the factor-use ratio will not sustain substantial readjustments. The above reasoning, though conceptually sound, is obscured by the fact that the tax-induced changes in labour and capital-input coefficients are too small to be captured by table 5 and the rounding-off process increases the unreliability of the figures. To disperse any doubts, therefore, we must use a more straightforward method for determining the optimum level of output.

By choosing arbitrarily one sector, (a), and two productive activities, five and six, we may define the cost function of the output of (a) from table 2, as follows :

$$P_a = 0.0783\,P_b + 0.2327P_c + 0.3327P_n + 0.2481P_k + 0.0517P_T + 0.0565P_m \quad \text{(in activity five)}$$

$$P_a = 0.0783P_b + 0.2327P_c + 0.3307P_n + 0.2501P_k + 0.0517P_T + 0.0565P_m \quad \text{(in activity six)}$$

The subtraction of the second equation from the first leaves the prices of labour and capital with non-zero values:

$$0.0020P_n = 0.0020P_k$$

and, taking the wage rate as numeraire, we find that $P_k = 1$ wage unit. Thus, we say that the shadow price of capital is one wage unit. The same unit value may be obtained for any sector and for any pair of productive activities from three

4. The word «almost» is of crucial importance. Note that the factor-use ratio with a perfectly equal distribution of the VAT burden (activity five) in table 5 is very close to the factor-use ratio with full forward shifting (activity two) whereas it differs significantly from the ratio of activity seven, in which the VAT burden on labour rises from 50% to 52 % only.

to seven, due to (i) the underlying assumptions of constant technical input coefficients and constant import coefficient, and (ii) the fact that labour and capital input coefficients in each activity add to the same total. This total is fixed by the difference between unity (the algebraic sum of all input coefficients) and the sum of the remaining (constant) input coefficients.

We can estimate the shadow price of tax by an analogous process, using the cost functions :

$$P_a = 0.0783P_b + 0.2327P_c + 0.3424P_n + 0.2578P_k + 0.0323P_T + 0.0565P_m$$

(in activity one)

$$P_a = 0.0783P_b + 0.2327P_c + 0.3230P_n + 0.2578P_k + 0.0517P_T + 0.0565P_m$$

(in activity three)

The subtraction of the second equation from the first leaves the prices of labour and tax with non-zero values, as follows :

$$0.0194P_n = 0.0194P_T$$

Again, taking P_n as the numeraire, we have : $P_T = 1$. In other words, the shadow prices of labour, capital and tax are all equal to one.

The unit shadow price for tax could also be found, should we use any other sector and any pair of activities-exclusive of the second activity — since labour and /or capital flows change by exactly the same amount by which tax changes and, as proved above, $P_n = P_k$.

The last step is to calculate the import shadow price. But, since the import coefficient is kept constant, we may simplify the analysis by incorporating the import price in the prices of the outputs produced by the three sectors. From the column of exports (table 2) we have :

$$P_m = 0.1239P_a + 0.2346P_b + 0.2296P_c$$

so that P_m is determined once output prices have been evaluated.

The assignment of shadow prices to labour, capital and taxes opens the way to a similar treatment of the prices of the three sectors. Taking any productive activity but the second (since the sum of its input coefficients is not unity), we may write the cost functions for products a, b and c as follows (if the sixth activity is chosen) :

$$P_a = 0.0783P_b + 0.2327P_c + 0.3307P_n + 0.2501P_k + 0.0517P_T + 0.0565P_m$$

$$P_b = 0.0842P_a + 0.1924P_c + 0.1839P_n + 0.3254P_k + 0.0403P_T + 0.1738P_m$$

$$P_c = 0.0358P_a + 0.0884P_b + 0.3399P_n + 0.4322P_k + 0.0695P_T + 0.0342P_m$$

By substituting the values of P_n, P_k, P_T and P_m in this system of linear equations and readjusting the terms we obtain :

$$0.9930P_a - 0.0916P_b - 0.2457P_c = 0.6325$$

$$-0.1057P_a + 0.9592P_b - 0.2323P_c = 0.5496$$

$$-0.0400P_a - 0.0964P_b + 0.9921P_c = 0.8416$$

The solution of the system gives output prices in terms of wage units as follows :

$$P_a = 0.9629, \ P_b = 0.9155, \ P_c = 0.9761$$

Thus, we have reached the final stage of our investigation.

The price of one basket is found if we multiply its three components by the corresponding prices, i.e.

$$X_aP_a + X_bP_b + X_cP_c = 0.2954 \times 0.9629 + 0.4575 \times 0.9155 + 0.2471 \times 0.9761 = 0.9445 \text{ wage units.}$$

The National Income at factor cost is :

$$LP_n + KP_k = 48893 \times 1 + 60453 \times 1 = 109346 \text{ wage units.}$$

We have already seen that 55.8 % of final demand will be satisfied by the fifth activity whose labour–input coefficient is 0.3599546 and the remaining 44.2 % by the sixth activity whose labour-input coefficient is 0.3598931, so that the weighted labour-input coefficient is :

$$0.5580 \times 0.3599546 + 0.4420 \times 0.3598931 = 0.3599273$$

The total number of baskets of goods produced is the ratio of the labour supply to the weighted labour-input coefficient :

$$\frac{48893}{0.3599273} = 135.841,32.$$

We note that the number of baskets so calculated is very close to the number obtained when the optimum activity seven is used instead (see table 6). Finally, the value of the gross national product is the product of the total number of baskets by the unit price, that is G.N.P. = 135841.32 X 0.9445 = 128302.12 wage units.

So far we have assumed that the main objective of economic policy is the maximisation of total output. This may not however be the case when the government has to tackle the more complicated problem of fixing a target-level of consumer-good production below what the economy would actually be able to produce so as to channel the released resources into sectors which supply the productive units with capital equipment and improve the capacity of the country for more future consumption. In this case, we must determine the criteria to be employed in the process of selecting the most suitable productive activity which will best exploit the restriction of current consumption through maximising the end-of-year capital stock.

TABLE 7

Labour and Capital-input coefficients under various productive activities

Activity		1-2	3	4	5	6	7
Consumer good	Labour	0.3424	0.3230	0.3424	0.3327	0.3307	0.3328
X_c	Capital	0.2578	0.2578	0.2384	0.2481	0.2501	0.2485
Capital Good	Labour	0.1589	0.2006	0.1589	0.1797	0.1839	0.1805
X_e	Capital	0.3087	0.3087	0.3504	0.3296	0.3254	0.3288
L/K ratio		0.5147	0.6498	0.4535	0.5452	0.5651	0.5490

To this end, we have arbitrarily decreased the optimum number of baskets of goods, which have already been found to amount to 135840 by, 35840 baskets, so that the target-level of consumption is now 100.000 baskets of a given composition. Moreover, we assume that the labour and capital input coefficients for the sectors (a) and (b) (see table 2) represent the potential combinations of factors which are capable of producing the consumer good, X_c, and the capital good, X_e, respectively, when both goods are of a specified composition. These coefficients are reproduced in table 7 which also contains the labour-capital ratio in the capital-good industry.

TABLE 8

Estimation of the optimum activity combination by using various levels of interest rate.

Activity	Cost function	for r = 10%, P	for r = 12%, P	for r = 8%, P
1-2	$P_e = 0.1589 + 0.03087P_e + 0.3087P_e . r$	0.1694	0.1705	0.1682
3	$P_e = 0.2006 + 0.03087P_e + 0.3087P_e . r$	0.2130	0.2152	0.2124
4	$P_e = 0.1589 + 0.03504P_e + 0.3504P_e . r$	0.1709	0.1722	0.1696
5	$P_e = 0.1847 + 0.03246Pe + 0.3246P_e . r$	0.1975	0.1994	0.1965
6	$P_e = 0.1839 + 0.03254P_e + 0.3254P_e . r$	0.1967	0.1981	0.1953
7	$P_e = 0.1901 + 0.03192P_e + 0.3192P_e . r$	0.2031	0.2045	0.2017
1-2	$Pc = 0.3424 + 0.02578P_e + 0.2578P_e . r$	0.3511	0.3521	0.3502
3	$Pc = 0.3230 + 0.02578P_e + 0.2578P_e . r$	0.3317	0.3327	0.3308
4	$Pc = 0.3424 + 0.02384P_e + 0.2384P_e . r$	0.3504	0.3513	0.3436
5	$P_c = 0.3303 + 0.02505P_e + 0.2505P_e . r$	0.3388	0.3397	0.3379
6	$P_c = 0.3307 + 0.02501P_e + 0.2501P_e . r$	0.3392	0.3401	0.3383
7	$Pc = 0.3278 + 0.02530P_e + 0.2530P_e . r$	0.3364	0.3373	0.3355

The available resources are again 48893 units of labour and 60453 units of capital equipment which is supposed to have a ten-year useful life and to depreciate evenly at the annual rate of 10 %. The question is now simpler ; which is the optimum activity combination that maximises X_e while keeping X_c at the level of 100.000 units, with the given endowment of resources ? Commencing with activity 3c we note that the production of 100.000 units of X_c would require 32300 units of labour and 25780 units of capital, thus leaving 16593 units of labour and 34673 units of capital for the production of X_e. Accordingly, the factor-use ratio in the capital good industry is :

$$\frac{16593}{34673} = 0.4786,$$ which is close to the labour-to-capital ratio of activity 4e.

Therefore, by adding 4e we can produce $\frac{34673}{0.3504} = 98952.62$ units of equipment which will absorb $98952.62 \times 0.1589 = 15723.57$ units of labour, leaving $16593 - 15723.57 = 869.43$ units of labour unemployed ; that is, the activity combination 3c - 4e alone fails to achieve full utilisation of resources and another, more efficient, combination should be sought. We will save time and effort if we experiment with activities whose labour-capital ratios are close to the factor-use ratio in the capital good industry, namely 0.4786. The next closer figure is that of activity 2e ; in order to distribute the remaining 16593 units of labour between 4e and 2e, we proceed as follows : If 16593. L labour units are to be hired by 4e, then the complementary employment of equipment in this activity may be defined as :

$$\frac{0.1589}{0.3504} = \frac{16593.L}{?} \rightarrow \frac{0.3504}{0.1589} \times 16593.L, \text{ where } 0 < L < 1$$

and the residual capital stock for 2e is therefore :

$$34673 - \frac{0.3504}{0.1589} \times 16593.\ L \qquad \text{(I)}$$

Turning to activity 2e, we easily determine the employment of labour :

$$16593\ (1 - L)$$

and the complementary employment of equipment :

$$\frac{0.1589}{0.3087} = \frac{16593\ (1 - L)}{?} \rightarrow \frac{0.3087}{0.1589}.\ 16593\ (1 - L) \qquad \text{(II)}$$

Since, under conditions of full utilisation of resources, the relations (I) and (II) are equal, we can solve for L :

$L = 0.5597$, that is 55.97 % of the residual labour force, i.e. $16593 \times 0.5597 = 9287.10$ units, will be absorbed by 4e, and 44.03 %, i.e. $16593 \times 0.4403 = 7305.9$ units, will be absorbed by 2e.

Consequently, the output of labour in 4e is $\frac{9287,10}{0.1589} = 58446.2$

whereas its output in 2e is $\frac{7305.9}{0.1589} = 45977.97$, so that the gross total of equipment produced by both activities, 4e-2e, amounts to 104424.17 units. Since the annual depreciation of the capital stock is : $60453 \times 0.10 = 6045.30$, we conclude that the net addition to the end-of-year stock equals 98378.87 units.

This is not, however, the whole story. We have computed the potential output of capital goods when activities 3c - 4e - 2e are combined, but we do not know as yet whether this output is the maximum one which may be attained with the given endowment of resources. To ascertain this, we must compare the output of 3c - 4e - 2e with what can be produced by the rest of the available combinations. However, we will not repeat the computations but will offer the following observations :

a) Whichever of the activities 1 (2)c, 4c, 5c, 6c or 7c is used alone, we find a labour-to-capital ratio for residuals that falls short of any factor-use ratio in activities 1 (2) e, 3e, 4e, 5e, 6e, 7e. But since the capital left for the capital-good industry is multiplied by the factor-use ratio to provide the labour requirements of the particular activity, we conclude that such requirements will exceed the residual labour, so that all activities in the consumer-good industry, exclusive of 3c, should be rejected as far as they are used alone.

b) By similar reasoning, 3c can be combined solely with 4e, because it is the only case where the labour-to-capital ratio for the residuals, i. e. $\frac{16593}{34673} = 0.4786$, exceeds the factor-use ratio, i.e. 0.4535 in 4e, thus leaving some labour units for 2e.

Extending our argument, we can say that activities 3c-4e can be further combined profitably only with 2e, since the remaining activities in Xe exhibit higher labour-capital ratios in the capital intensive industry of durables.

c) Lastly, we should attempt to combine 3c with the other activities availeble in the consumer-good industry.

Beginning with the activity combination 3c-5c, we assume that h baskets are produced by 3c and (1-h) baskets by 5c, where $0<h<1$. The amount of labour employed by the consumer-good industry equals the sum of the products of the outputs in 3c and 5c by the respective labour-input coefficients, i.e. $0.3230h + 0.3327(1-h)$, so that the residual labour for 4e is :

$$0.48893 - [0.3230h + 0.3327(1-h)]$$

In a similar fashion, the residual capital for 4e is :

$$0.60453 - [0.2578h + 0.2481(1-h)]$$

The ratio of these two residuals must be identical to the labour-capital ratio of 4e, as it appears on table 7 ; that is :

$$\frac{0.1589}{0.3504} = \frac{0.48893 - [0.3230h + 0.3327(1-h)]}{0.60453 - [0.2578h + 0.2481(1-h)]}$$

from which we obtain $h = 0.38335$. The meaning of h is that 38335 baskets will be produced by 3c and 61665 baskets by 5c. The capital required for this purpose is again the sum of the products of outputs in 3c and 5c by the respective capital-input coefficients, that is $38335 \times 0.2578 + 61665 \times 0.2481 = 25182$ units. The available equipment for 4e is thus the difference $60453 - 25182 = 35271$ units and the quantity of fixed capital which can be produced amounts to : $\frac{35271}{0.3504} = 100659.24$ units. In other words, the combination of activities 3c-5c-4e fails to reach the level of output of capital goods which can be obtained under the first combination 3c-4e-2e (104424.17 units).

we have experimented with most of the remaining combinations within the consumer-good industry and have discovered that either the values of h were inconsistent with our assumption, that is they were found to be $0 > h > 1$, or that output was smaller than that of 3c-4e-2e. We conclude therefore that the latter is the optimum combination which maximises the end-of- year capital stock.

Since we have repeatedly referred to terms like capital stock, depreciation and so on, it is reasonable to hypothesise the regulative role of the interest rate. To determine its magnitude we will make the simplified assumption that only labour and capital collaborate in the production of X_c and X_e in the proportions indicated by table 7.

The unit price of any commodity should cover labour costs, the interest paid on renting equipment — or on borrowing money for purchasing equipment, or the imputed interest on self-employed capital–and the capital cost incurred during the particular year in order to produce one unit of this commodity (since the annual rate of depreciation has been fixed at ten percent, the one tenth of the initial value of capital enters the cost calculations of the firm).

Taking the optimum activity combination 3c-4e-2e, we can define the shadow prices of consumer and capital goods as follows :

$$P_c = 0.3230P_n + 0.02578P_e + 0.2578P_e r \text{ (in 3c)}$$

$$P_e = 0.1589P_n + 0.035040P_e + 0.3504P_e . r \text{ (in 4e)}$$

$$P_e = 0.1589P_n + 0.03087P_e + 0.3087P_e . r \text{ (in 2e)}$$

where r stands for the interest rate, P_e for the price of equipment and $P_n = 1$ (numeraire) for the price of labour. By subtracting the third equation from the second we obtain :

$$0.00417P_e = -0.0417P_e . r \text{ and } r = 10\,\%.$$

Thus, the interest rate in our model coincides with the depreciation rate and may be used as an alternative way of determining the optimum activity combination.

To this end, we ascribe arbitrary values to the interest rate and solve for P_e in all the available methods of producing X_e. The prices of capital goods estimated in this manner are then introduced into the cost function of consumer goods so that the shadow prices of the latter can be readily computed. Table 8 presents part of this procedure.

We have picked out three levels of interest rate : 10 % as obtained above, a higher (than 10 %) and a lower rate.

Moreover, we have chosen the capital-good prices which correspond to the second activity as the cheapest set.

The first inference to be drawn from table 8 is that the lower the interest rate, the smaller will prices of consumer and capital goods be since the cost function is directly related to the interest rate, regardless of the activity which is ultimately chosen. The second and most important inference is that, at a given level of interest rate, the cheapest set of prices is attained when activities 3c for consumer goods and 2e-4e for capital goods are used. Since we have alread) shown that this activity combination succeeds in absorbing all available resources, we way definitely conclude that it is the general interest to abide by the technique suggested by activities 3c-4e-2e, although it is very doubtful whether the nature and structure of VAT system will allow backward shifting of the tax differential.