NOTES ON AN IS- LM MODEL

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1 In a paper presented in 1957, Hudson (1957), consructed trade cycle model in which the interaction of monetary and real factors were emphasised. This model which is very close to Kaldor's model of trade cycle (Kaldor, 1940), has an interest of its own, because it can be used to illustrate the simpler form of elementary catastrophes, the fold catastrophe (Thorn 1969). In this note, we first introduce Hudson's model, and then we use it to illustrate the fold catastrophe.

2. Hudson (1957) (a linear version of Hudson's model is given in Harcourt, 1978) introduces a model of the IS - LM type in which investment is made a function of the real output, and the capital stock as well as the rate of interest. And the period to period changes in the capital stock are taken into account. These assumptions allow IS curves to haves positive slopes. In Hudson's model the stability conditions require that the IS schedule slopes upward less steeply than the LM schedule. It is seen therefore that Hudson was aware of the possibility that the IS curves may slope upward as well as of the stability conditions for an IS - LM model in this case. The systematic analysis of IS-LM models with positivelty sloped IS curves was introduced later in macroeconomic theory (Silber, 1971, Cebula 1976, Yannacopoulos, forthcoming).

3. If we allow IS curves to have positive and negative slopes, then given the LM curve we can get multiple equilibria (Fig. 1). Equilibrium points A and C are stable, while the eguilibrium point B is unstable (under the assumptions given in the previous section). A similar figure can be found in Varian (1978, p. 121). In Hudson's model the cycle is generated as follows : Suppose that the economy is in equilibrium at the point A.

Assume that realised investment per period is less than depreciation. Capital stock will fall from period to period and the IS curve will rise (be-

cause investment, cet. par., is greater the smaller the value of capital stock K, and vice versa) until it becomes tangent to the LM at the point D (Fig. 2).

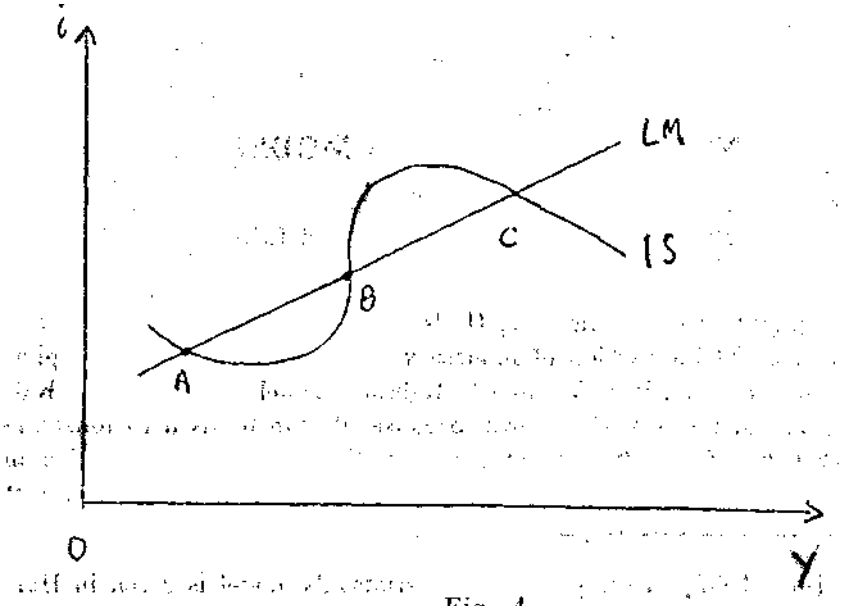

Fig. 1

Point D is unstable and the economy will jump to the boom position E. At E it may supposed that gross capital formation is greater than depreciation. The IS curve therefore falls (because capital stock rises), until the IS curve become? tangent to the LM at the point Z. The economy will then jump to the slump position F (see Fig 2). This may be an interpretation of the "sudden collapse of the marginal efficiency of capital" about which Kenyes (1936, p. 315) spoke [1].

4. If we draw a graph of the short run equilibrium values of y against the values of K we get the Fig. 3, where the graph Y = 0 presents the equilibrium path of Y when a single parameter (in this case the capital stock K) varies. The dotted part of the Y = 0 presents the unstable (and unobservable) part of the curve while the heavy drawn parts the stable (and observable) parts.

(1) «But I suggest that a nore typical, and often the predominant, explanation of the cirsis IS, not primarily a rise in the rate of interest, but a sudden collapse in the marginal efficiency of capital», (Keynes 1936, p. 315).

The cyclical behaviour of this model is as follows : an increase in the capital stock (i. e. a decline in the level of investment) leads to a gradual

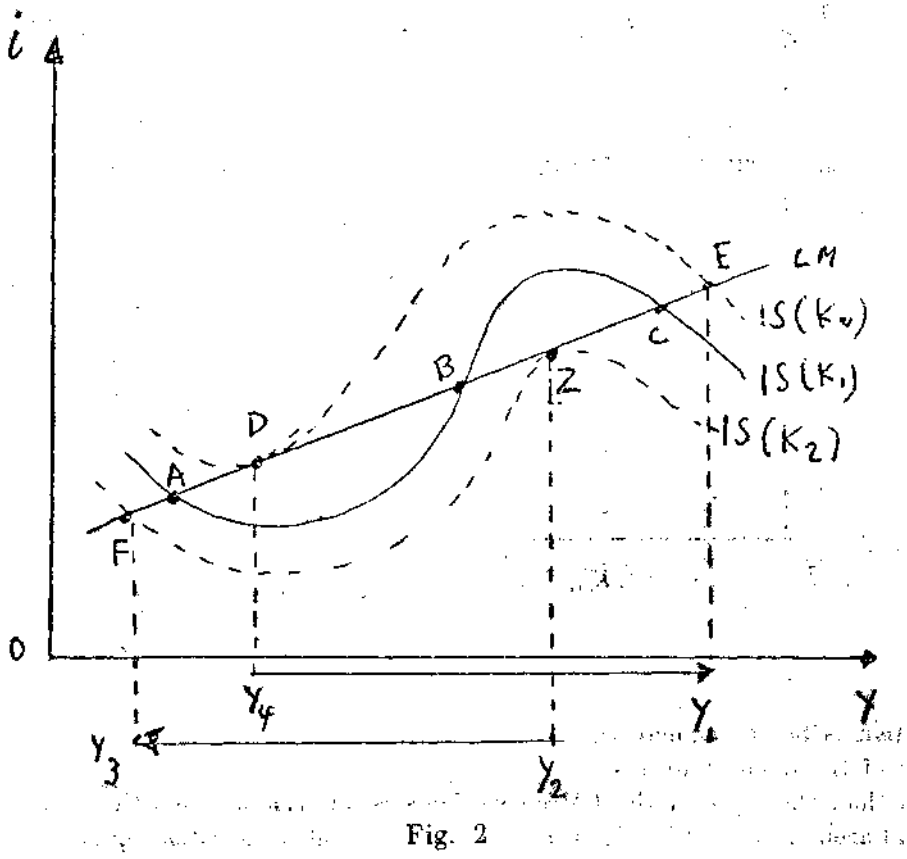

Fig. 2

decline of income. The system however is locally stable until the capital stock reaches its critical points K_2. At this point income jumps catastrophically down to Y_3, followed by a gradual recover and finally a sudden boom (see Fig. 3).

This movement is an example of the fold catastrophe (see also Varian 1979, and Goodwin 1980). It can be shown that in a system with one control parameter (in this case the capital stock) and a single behaviour variable (in this case the national income), this is essentially the only kind of catastrophe that can ocour.

5. The cyclical behaviour of Hudson's model depends, as it can be easily-verified, on the special assumption on which the stability of its IS - LM

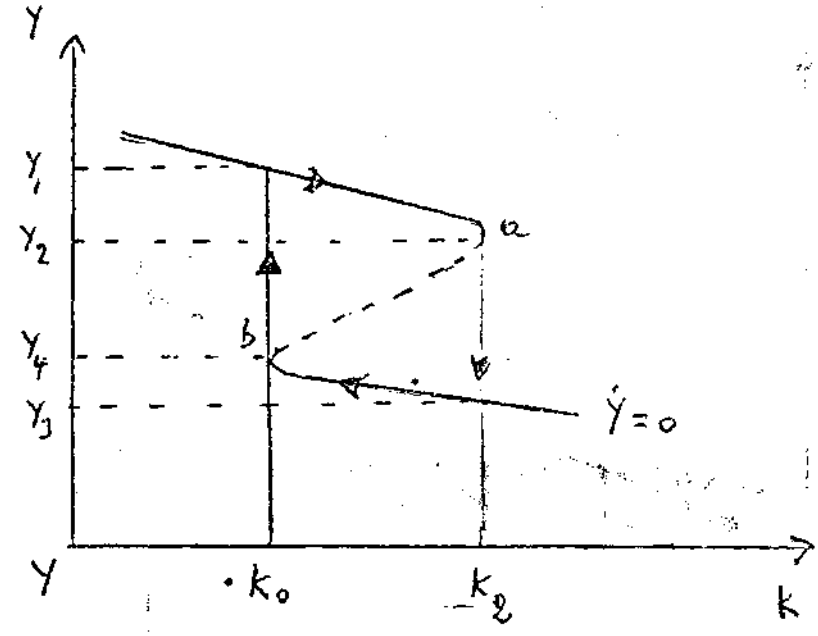

Fig. 3

system is based, namely that with positively sloped IS curves the stability of the model requires that the slope of the upward sloping IS curve is less than the slope of the LM curve. This assumption renders the neibourhoud around point B in Fig. 1 unstable and generates the catastrophic jumps described in Fig. 3. Cebula (1976), however, suggested that under special economic assumptions a system with positively sloped IS curves may be stable even if the IS schedule slopes upward m o r e steeply than the LM schedule. It is obvious that in the last case (i.e. if Cebula's stability conditions prevail) the economic system will generate no "catastrophic" jumps.

6. The formalism of catastrophe theory was used by Goodwin (1980) in his well known model, which generates the same kind of cyclical behaviour with the one described in this note and by Varian(1979), and George (1981) in their analysis of Kaldor's model of trade cycle (Kaldor, 1940). Varian and George employed the same diagram with that in Fig. 3, but later in

their analysis they have increased the control dimensions of the system by adding wealth as a control parameter, associating thus Kaldor's trade cycle model with the cusp catastrophe. The similarity of Hudson's model (viewed from the point of view of catastrophe theory), and Kaldor's model (as developed by Varian and George) owed to be expected, given that Hudson's model is a generalisation of the Kaldor's model. In fact while Kaldor's model ignores the monetary factor (it sresses the importance of the real factors **for** the trade cycle), in Hudson's model the interaction of monetarty **and real** factors are emphasised.

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