AGGREGATE PRODUCTION PLANNING AND MARKETING STRATEGIES APPLIED TO EITHER AN OLIGOPOLISTIC OR A DUOPOLISTIC MARKET

(A price change decision model)

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1. INTRODUCTION

In the last two or three decades much research effort has been expended in trying to provide manufacturing organizations with models which, when applied to forecast demand for the firm's product or products in future periods, would yield such values of the decision variables as would ensure minimum overall costs and/or maximum profits.

A number of models have been presented by Hanssman and Hess [1], Tuite [2], Peterson [3], [4], Taubert [5] and Leitch [6]. The model which laid the foundation for much subsequent work in this area was that which appeared in 1960 under the joint authorship of Holt, Modigliani, Muth and Simon [7] and which is the basis for the work reported herein. In 1977 and 1978 Kioulafas [8], [9] presented a model, the Variable Price Model, (VPM), which attempted to merge managerial decisions involving aggregate production scheduling and pricing policy. Formally this model is an extension of the decision model developed by Holt et. al., consisting of the treatment of selling price as an endogenous variable. To this effect an opportunity cost is introduced which represents the revenue differential attributed to a change of selling price.

By treating price as an endogenous variable of the model a lower overall cost and greater profitability is achieved. This claim is validated by comparison of results given by the model presented with those of previous models especially the ones by Holt et al, Tuite, Leitch, Peterson and Taubert using the same data (see Kioulafas, pp. 98 — 239, [8]).

Furthermore an application with real data from a Greek kernel oil producer is presented by Kioulafas, pp. 240 - 264, [8] and in [9]. But the main assumption of this model is that its firm enjoys a monopoly position.

Several models have been published recently relating to pricing decisions.

The focus of these papers has been on four pricing decision areas : new product decisions [10], [11], [12], [13], [14], [15], [16] ; price change decisions [17], [18], [19], [20], [21], price structure decisions and product-line pricing decisions [22], [23], [24], [25].

Much attention in the pricing literature has also been devoted to the use of price changes for the cases of mature products and price promotions, [26], [27], [28], [29].

A major limiting aspect of the current pricing models is the lack of dynamism Only a few models recognize the need to develop pricing strategies over a relevant time period and to allow for market dynamics, [30].

In this paper we will extend the HMMS model by introducing a dynamic pricing policy for the duopolistisc and oligopolistic markets in concomitance with other marketing variables under the assumption that the firm we are dealing with produces differentiated products, and it is the leader of the duopoly or oligopoly.

2. THE MODEL

Several authors have considered the problem of using price changes as a competitive tool.

Factors which influence consideration of a price reduction are price elasticity of demand and unit cost reductions brought about by increased volume. Manufacturers tend to set the selling price of their products by adding to the total costs of production and distribution some predetermined mark up. An alternative method would be to set the selling price in accordance with the strength of the market for the product. This latter approach would then relate price directly to demand or to forecast demand. We think that setting the selling price in accordance with the strength of the market may have quite considerable advantages for the manufacturer concerned with smoothing his production process so as to maximize profit or minimize costs. We also reserve the right, as per Peterson [4] and Kioulafas [8] to indulge in over shipment s or undershipments per period as may be necessary.

The introduction of these new variables has the following consequences to the original HMMS model.

(a) The modification of the sales expression to include the selling price.

(b) The modification of the inventory connected cost expression to include the variables related to overshipment and undershipment.

(c) The addition of two more cost items, as follows :

(i) The opportunity cost, which the manufacturer must bear in virtue of his using a policy of price variation.

(ii) The cost of overshipment and undershipment.

2.1. Extending the AMMS model for the cases of duopoly and oligopoly

Because a price decrease often results in a similar response from one's competitors, for a price reduction to be profitable three conditions are necessary :

(i) Industry demand must be price elastic.

(ii) The firm's demand must be price elastic.

(iii) Revenues gained from the price reduction must be greater that the costs of producing and selling additional units.

There will be a distinct demand curve for each individual producer of a differentiated product in a duopolistic market and his sales will be a function, first of his own selling price, and second of his competitors' selling prices.

The demand function may be constructed to describe a situation in which price is the independent variable for one seller and quantity for the other, when we have a duopolistic market or for the others in the case of oligopoly. Therefore our firm's demand function will be expressed in the inverse form as follows :

$St = O_t = a — btpt\text{-}1 \quad tq_1(t)$ (Case of duopoly) (2.1)

$S_t = O_b = a — b_tpt\text{-}m_tq_2(t)$ (Case of oligopoly) (2.2)

where :

Ot = orders placed with manufacturers during period t,

St = quantity shipped in period t,

qi(t) = the quantity sold by our competitor for the case of duopoly at time t,

$q_{2(t)}$	=	the total quantity sold by our competitors in the market, at time t, for the case of oligopoly,
l_t	=	the coefficient expressing the relationship existing between our firm's sales and our competitor's sales, for the case of duopoly,
m_t	=	the coefficient expressing the relationship existing between our firm's sales and our competitor's sales, for the case of oligopoly,
pt	=	dynamic selling price in period t,
a	=	a market constraint constant, and
bt	=	measure of change in demand per unit change in price.

There are at least three ways to evaluate the coefficients 'a'. b_t, l_t and mt. (i) Time series analysis method, (ii) Simulation method, and (iii) Quantification of subjective data method, [31], [32], [33], [34], [35], [36].

Kohn and Plessiner [37] have presented a demand function and they have evaluated the above mentioned coefficient mt using real data in their paper : . . . they presented a marketing model designed to find the revenue maximising allocation of a set of interrelated products over space and time, when demand, represented by a relation in which price at time t is a function of quantities in the same period, involves prices . . . ' [37].

2.2. The O ver shipment and Undershipment Policy

Normally single decision marketing models ignore the interactive, joint effects of other marketing variables. According to K.B. Monroe and A.O. Bitta [30] : *. .. Indeed, because the authors purposely exclude marketing mix models, the market reaction to other marketing variables is included in only two models. . . Nevertheless, at this stage of model development the complexities of including several decision variables probably outweigh the gain in realism.

In order to be able to discuss the effects of discrepancies caused because of under - and over - shipments we define two variables Z_{1t} and Z_2t as follows :

$$\left.\begin{aligned} Z_{1t} &= \text{shipments - orders} \\ &= St-Ot \\ Z_{2t} &= 0 \end{aligned}\right\} \quad \text{when orders} < \text{shipments} \qquad (2.3)$$

$$\left.\begin{aligned} Z_{2t} &= \text{Orders - shipments} \\ &= O_t - S_t \\ Z_{1t} &= 0 \end{aligned}\right\} \quad \text{when orders} > \text{shipments} \qquad (2.4)$$

$$Z_{1t} = Z_{2t} = 0 \qquad \text{when orders = shipments} \qquad (2.5.)$$

so that both are non negative variables. We use these variables to represent the costs associated with overshipments and undershipments respectively and we take our expressions in quadratic form. We take, as per R.F. Peterson [4], the cost of overshipment in period t as :

$$q(Z_{1t}) = d_t Z_{1t}^2 \qquad (2.6)$$

and of undershipment in period t as :

$$h(Z_{at}) = e_t Z_{at}^2 \qquad (2.7)$$

Both dt and e_L would vary with time and would only existwhen the corresponding variable, as defined in (2.3) and (2.4) was positive.

The sales in period t would then be given by :

$$S_t = a - b_t P_t + Z_{jt} - Z_{2t}$$

By introducing the variables Z,t, Z_2t, we accept the existence of interactive and joint effects of pricing and other marketing variables.

Furthermore, overshipment is defined here as the quantity sold because of advertising effort.

2.3 Introduction of Pricing and Shipment Policy into HMMS Model

The HMMS model consists of the following main components of cost :

(a) Regular payroll cost $= C_j W_t$

(b) Hiring and lay-off costs $= c_2(W_t - W_{t.i} - c_u)^2 + c_{13}$ (2.8)

(c) Overtime and idle - time costs $= c_3(X_t - c_4 W_t)^2 + c_5 X_t - c_e W_t$

(d) Inventory related costs $= c_7(I_t - C8 - c_9 O_t)^2$

where :

Wt = work - force,

Xt = production rate at time t,

It = inventory at time t, and

Ot = orders at time t.

Combining the forms (2.1), (2.3), (2.4), (2.5), (2.6), (2.7) and (2.8) we develop the new model which, in its complete form, will be as follows :

$$CTOT = \sum_{t=1}^{T} \{ (c_1-c_6)Wt+c_2(W_t-Wt._1-c_u)2+c_3(Xt-c_4Wt)2+ c_5X_t+ c_{12}X_t W_t + c_{13} +$$

$$+c_7[I_t—c_8—c_9(a—btpt-ltqit+Z_1t-Z_3t)]^2-pt(a-b_tpt-ltqii+Z_1t-Z_{21})+dtZ_1t^2+etZ_2t^2\} +$$
$$+P_CQ.$$

(Case of duopoly) (2.9)

where :

(a) P_c = Constant Selling Price

(b) Q = Total quantity that would have been sold over the planning horizon of T periods if the price had been maintained at a constant level P_c

(c) Regular payroll cost = CjWt

(d) Hiring and lay off costs = $c_2(W_t-W_{t-1-}c_4)^2+c_{13}$

(e) Overtime and idle-time costs = $c_3(Xt—c_4W_t)^2+c_5Xt—c_eWt$

(f) Inventory related costs = $c_7[It-C8-c_9(a-btpt-ltgit+S_1i-Z_2t)]^2$

(g) $PcQ-pt(a-btpt+Z_{1t}-Z_2t-ltqit)$ = Opportunity Cost

(h) dtZ_2t^2, etZ_{1t}^2 = Over and undershipment costs.

Subject to the constraint :

$$a-btpt - ltq_{1t}+Z_{1t} - Z_{2t} = Xt+It-It-j \qquad (2.10)$$

In the same way we can develop the model for the case of oligopoly combining the forms (2.2), (2.3), (2.4), (2.5), (2.6) and (2.7).

In Figures 2 and 3 below we present the structure of the model in comparison to the HMMS model.

Figure 2

VARIABLES OF THE MODEL

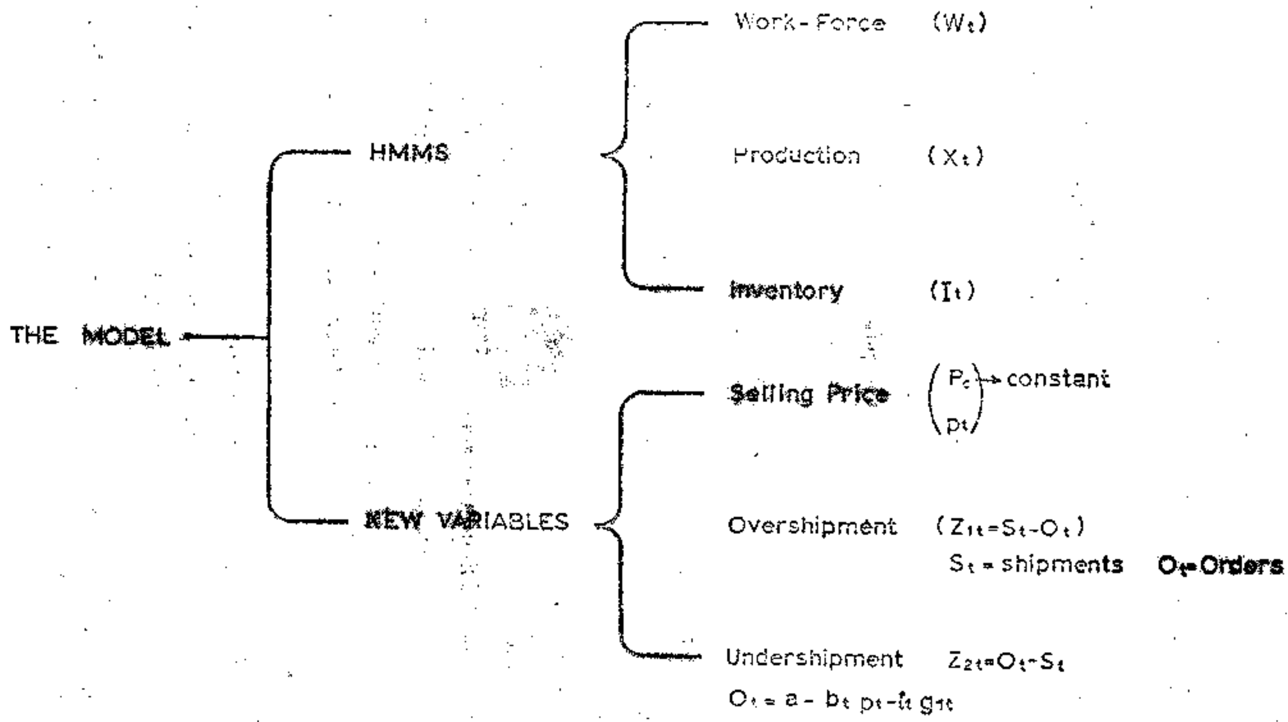
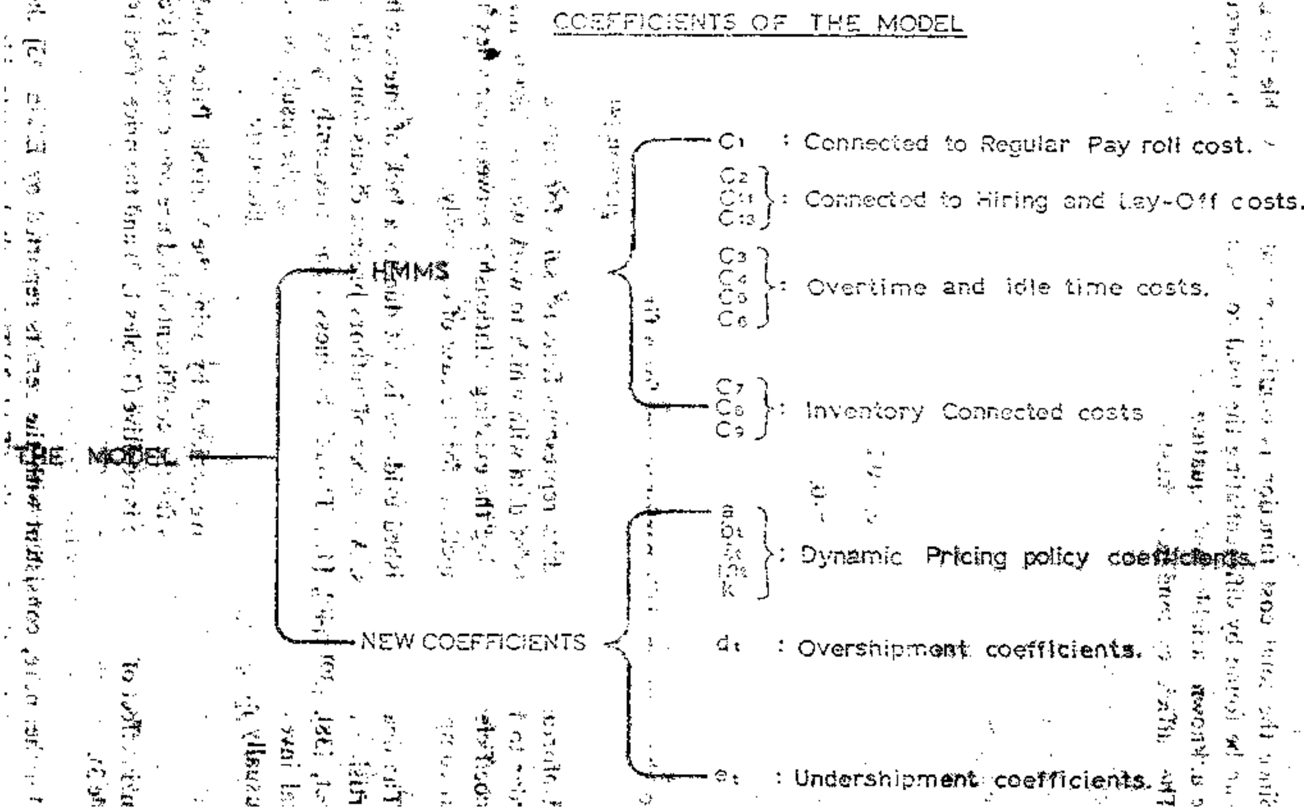


Figure 3


COEFFICIENTS OF THE MODEL
THE MODEL
HMMS
NEW COEFFICIENTS
C1 : Connected to Regular Pay roll cost.
C2 C11 C13 : Connected to Hiring and Lay-Off costs.
C3 C4 C5 C6 : Overtime and idle time costs.
C7 C8 C9 : Inventory Connected costs
Dynamic Pricing policy coefficients.
dt : Overshipment coefficients.
et : Undershipment coefficients.

2.4 The Solution

Since the total cost function is continuous and differentiable the minimum cost can be found by differentiating the total cost expression with respect to each of the unknown variables and equating the resulting expression to zero.

The sufficiency conditions must be considered and these are :

(i) $c_2 + c_3 c_4 2 > 0$

(ii) $c_2 c_s > 0$

(iii) $c_2 c_3 c_7 > 0$

(iv) $c_2 c_3 c_7 bt > 0$

(v) $b_t d_t > 0.25$, $b_t et > 0.25$

(vi) $c_{1(} \ldots \ldots \ldots c_{13} > 0$.

3. RESULTS

To check our model's performance for a firm in a duopolistic and oligopolistic market we have used data given by Taubert [5], Peterson [4] and HMMS [7].

Unfortunately, this data represents forecast sales for only a single firm and in order to have two sets of data with which to work we have had to introduce a new coefficient K expressing the existing relationship between our sales and those of our competior or competitors for the case of oligopoly.

The dearth of published evidence is not due to a lack of interest in price advertising relationships. A number of authors present discussions related to this subject, [38], [39], [40], [41]. Though business firms frequently sponsor experimental investigations to specify these relationships [42] the results of such work are usually proprietory and are not published in the literature.

From the results we have achieved by using the Variable Price Model under a number of assumptions as far the coefficients used are concerned it is clear that the main effect of advertising is positive (Tables 1, 2) and the price effect is negative (Table 3).

Furthermore, consistent with the results reported by Eskin [43] the price advertising interaction is negative (Table 2.) The result also indicates higher responsiveness of sales to increases in advertising under the lower price condition than under the higher price condition. All these results are also consistent with the empirical evidence presented,by G. Eskin and P. Baron [44].

By carrying out sensitivity analysis in terms of the new coefficients used by the new model we obtained the following operative range of values for the coefficients used by the New model we obtained the following operative range of values for the coefficients used by the New Model.

$0 < K < 1$

$0 < l_t < 1$ (case of duopoly)

$0 < m_t < 1$ (case of oligopoly)

$1500 < a < 2500$

$0.027 < d_t <$ very large number

$32.0 < e_t <$ very large number

Comparing the results we have obtained for the three market situations (Tables 4-9) we note the following :

As expected the model suggested fewer sales than in the case of monopoly because we have a single competitor in the duopolistic market and several competitors in the oligopolistic market. The existence of competitors decreases the effectiveness of our pricing policy. The reduction of sales causes a reduction in production, a reduction in work - force, and finally, a reduction in inventory.

Comparing the results for the case of duopoly to those for the case of oligopoly the model suggested a greater decrease in sales, production and work - force, for this second case.

As a result of changes in selling price the model suggested, for the cases of duopoly and oligopoly, greater cost and less revenue than was the case for monopoly. By introducing such policy we have managed to decrease the opportunity cost which has become negative and thus achieved a decrease in the total cost associated with the running of the system with a consequent increase in revenue.

Finally, comparing the results for all these three market situations with the results given by Taubert and HMMS, we may say that we are getting greater profitability and better smoothed work - force, production and inventory planning.

4 CONCLUSIONS

In this paper we have presented a single product deterministic model which is also an interactive marketing mix model which includes price decisions. More precisely the model does not ignore the interactive joint effects of advertising and other promotional efforts. A major aspect of this current pricing model is the existence of dynamism. The model recognizes the need to develop pricing strategies over a relevant time period and to allow for market dynamics while at the same time work - force, production, inventory, sales and advertising strategies are introduced in the model. This is an important cotribution because it is true that the need for correct pricing decision is becoming increasingly more important as today's pricing environment places intensive pressure for better, faster and more frequent pricing decisions in coordination with the other main decision variables of a firm.

In this paper we have further extended the range of application of the HMMS by applying it in duopolistic and oligopolistic markets. The results we obtained tended to smooth out quite effectively the fluctuations in sales, production and inventory and to be more effective from the point of view of both costs and revenue than those obtained by all the other aforementioned models.

Another important contribution of this" paper, apart from the reduction of private costs and the increase of private profit, is that it proposes a model which, in significantly reducing fluctuation in production, work - force and inventories, could provide considerable macroeconomic social benefits. We feel, theorefore, that if the model were to find wide application the consequent reduction of fluctuations in aggregate inventories would exert a major stabilizing effect on the whole Economy.

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APPENDIX A

Symbols that have been used by the VPM are presented alphabetically

a	A market constraint constant
bt	Measure of change in demand per unit change in price
ci (i = 1,. · .,13)	Original parameters
dt	Coefficient of z_{lt}^2 in CT representing the cost of overshipment in period t
jet	Coefficient of $Zgt^{\cdot 2}$ in ex representing the cost of undershipment in period t
$_0$	Net inventory at the end of period 0
I_t	Net inventory at the end of period t
I_t	The coefficient expressing the relationship existing between our firm's sales and our competitor's sales for the case of duopoly
m_t	The coefficient expressing the relationship existing between our firm's sales and our competitors' sales for the case of oligopoly
Ot	Orders placed with manufacturer during period t
P_c	Constant Selling Price
p_t	Dynamic Selling Price in period t
Q	Total quantity that would have been sold over the planning horizon of T periods if the price had been maintained at a constant level Pc
St	Quantity shipped in period t
Wt	Strength of the work - force in period t
X_t	Aggregate production rate in period t
$Z_x t$	Overshipped quantity
Z_{2t}	Undershipped quantity

Table 1 : Totals of Inventory, Production, Overshipment and Sales, predicted by the VPM for the lower bound of dt values and 'a' = 2000, based on Taubert's data for the case of duopoly (Planning horizon – 2 years)

Pc	Inventory	Production	Overshipment	Sales	Overshipment / Sales × 100
101.20	7,559	14,036	4,019	13,975	28.7
126.50	7,464	16,239	4,729	16,177	29.2
132.80	7,444	16,688	4,875	16,625	29.3
189.70	7,295	19,607	5,848	19,539	29.8

Table 1a, Totals of overshipment predicted by the VPM tfor various values of dt, for a = 2000 and Pc = 101.20, based on Taubert's data for the case of duopoly (Planning Horizon 2 years).

dt	Overshipment
0.064	4,019
0.160	3,822
0.800	3,543
1.600	3,132
3.200	2,765
5.600	1,182
16.000	373

Table 2 : Overshipment Cost (Advertising Cost) as a Percentage of Turnover for the Variable Price Model (VPM) working on Taubert's data with 'a' = 2000, and the lower bound of dt values for the case of duopoly (Planning horizon— 2 years)

Constant Selling Price (Pc)	Overshipment	Overshipment Cost	Revenue	Overshipment Cost / Overshipment	Overshipment Cost / Revenue × 100
101.20	4,019	81,181	1,493,204	20.1	5.4
126.50	4,729	140,505	2,063,966	29.7	6.8
132.83	4,875	156,822	2,209,197	32.1	7.1
189.75	5,848	322,213	3,516,735	55.0	9.7

Table 3 : Totals of Sales predicted by the VPM for various values of Pc keeping all the remaining coefficients constant, based on Taubert's data for the case of duopoly (Planning horizon—2 years) 'a' = 2000

P_c	Sales
100	14500
101.20	12985
105	10400
110	10088

Table 4 : Aggregated results predicted by the VPM based on data presented by Taubert for the case of oligopoly in comparison to those obtained by the same model for the case of monopoly, and for the following set of coefficients : 'a' = 2000, P_c = 101.20, d_t = 0.08, e_t = 32.00

Market Situation	Coefficient Values		Inventory	Production	Over-shipment	Sales	Total Cost associated with the running of the Ne New Model
	K	m_t					
Monopoly	0.0	0.000	7,579	13,403	2,574	13,346	646,970
Oligopoly	0.2	0.125	7,586	13,331	2,465	13,274	648,572
	0.2	0.135	7,581	13,328	2,463	13,270	648,566
	0.2	0.145	7,588	13,271	2,463	13,213	647,667
	0.2	1.000	7,593	13,021	2,459	13,043	646,834
Taubert's Model Results	–	–	7,970	11,619	–	11,428	734,982
HMMS	–	–	7,859	11,615	–	11,428	734,186

Table 5 : Aggregated results predicted by the VPM based on data presented by Taubert for the case of duopoly in comparison with those obtained by the same model for the case of monopoly and for the following set of coefficients :

'a' = 2000, P_c = 101.20, d_t = 0.08, e_t = 32.00, K = 0.2

Market Situation	Coefficient Values K	l_t	Inventory	Production	Over–shipment	Sales	Total Cost associated with the running otf the New Model
Monopoly	0.0	0.00	7,579	13,403	2,575	13,346	646,971
Duopoly	0.2	0.05	7,585	13,355	2,464	13,297	648,616
	0.2	0.10	7,585	13,339	2,463	13,282	648,588
	0.2	0.20	7,586	13,307	2,460	13,250	648,520
	0.2	0.50	7,590	13,212	2,451	13,154	648,236
	0.2	1.00	7,595	13,052	2,436	12,993	647,457
Taubert's Model	–	–	7,970	11,619	–	11,428	734,982
HMMS	–	–	7,859	11,621	–	11,428	734,186

Table 6 : Aggregated results predicted by the VPM based on data presented by Taubert for the case of oligopoly in comparison with those obtained by the same model for the case of monopoly, and for the following set of coefficients :

'a' = 2000, P_c = 101.20, d_t = 0.08, e_t = 32.00

Market Situation	Values K	m_t	Inventory	Production	Over–shipment	Sale	Total Cost associated with the running of the New Model
Monopoly	0.00	0.000	7,579	13,403	2,575	13,346	646,971
Oligopoly	0.10	0.135	7,585	13,349	2,464	13,292	648,607
	0.17	0.135	7,585	13,334	2,462	13,277	648,579
	0.20	0.135	7,586	13,328	2,462	13,271	648,566
	0.25	0.135	7,586	13,317	2,460	13,260	648,543
	0.40	0.135	7,587	13,285	2,458	13,228	648,465
Taubert's Model	–	–	7,970	11,619	–	11,428	734,982
HMMS	–	–	7,859	11,619	–	11,428	734,186

Table 7 : Maximum variations in Selling Pcice, Work – force, Sales and Production, predicted by the VPM for various values of lt and K = 0.2 using Taubert's data in monopoly and duopoly markets, for the following set of coefficients :

'a' = 2000, P_c = 101.20, d_t = 0,08, e_t = 32.00

Market Situation	Coefficient Values		Selling Price	Work – force	Sales	Production
	K	l_t				
Monopoly	0.0	0.00	19.0	16	150	86
Duopoly	0.2	0.05	18.0	15	143	82
	0.2	0.10	18.1	15	143	81
	0.2	0.20	18.2	15	142	79
	0.2	0.50	18.6	15	139	76
	0.2	0.50	18.6	15	139	76
	0.2	1.00	20.1	13	133	71
Taubert's Model	–	–	–	45	441	300
HMMS	–	–	–	44	441	303

Table 8 : Maximum variation in Selling Price, Work – force, Sales and Production, predicted by the VPM for various values of m_t and K = 0,2, using Taubert's data in monopoly and oligopoly markets and for the following set of coefficients :

'a' = 2000, P_c = 101.20, d_t = 0.08, e_t = 32.00

Market Situation	Coefficient Values		Selling Price	Work – force	Sales	Production
	K	m				
Monopoly	0.0	0.0000	19.0	16	150	86
Oligopoly	0.2	0.1250	18.1	15	143	80
	0.2	0.1350	18.2	14	143	80
	0.2	0.1215	18.6	14	139	76
	0.2	1.0000	20.1	13	133	71
Taubert's Maximum Variation	–	–	–	45	441	300
HMMS Maximum Variation	–	–	–	44	441	301

Table 9 : Maximum variations in Selling Price, Work – force, Sales and Production, predicted by the VPM for various values of m_t and K, using Taubert's data, in monopoly and oligopoly markets, for the following set of data :
'a' = 2000, P_o = 101.20, d_t = 0.08, e_t = 32.00

Market Situation	Coefficient Values K	Coefficient Values m_t	Selling Price	Work – forsce	Sales	Production
Monopoly	0.0.	0.000	19.0	16	150	86
Oligopoly	0.10	0.135	18.1	15	143	81
	0.17	0.135	18.1	14	143	81
	0.20	0.135	18.2	14	143	80
	0.25	0.135	18.2	14	142	80
	0.40	0.135	18.3	14	141	78
Taubert's Model Results	–	–	–	45	441	300
HMMS	–	–	–	44	441	301