

THE MISSING PARTS OF THE WALRASIAN LAW OF GENERAL EQUILIBRIUM *

By
Dr ANGHEL N. RUGINA
Prof. Emeritus, Northeastern University

INTRODUCTION :

Is there anything new to be said about the Walrasian System?

Ever since the works of Paul Samuelson,¹ Kenneth J. Arrow and Gerard Debreu,² Lionel McKenzie³ and others including forerunners like Sir John Hicks⁴, Nicholas Kaldor,⁵ Ragnar Frisch⁶ and A. Wald,⁷ in what is called now the

A word of thanks is due to my friend and colleague, Dr. Edward R. Willett from the College of Business at Northeastern University, who kindly read the first draft of this paper and helped to improve its style and content. This paper was presented at the International Atlantic Economic Conference in Freeport-Lucaya, Bahamas, February 10-15, 1980.

1. Paul Samuelson, *The Foundations of Economic Analysis* Harvard University Press, Cambridge (1947) Fourth Printing 1955.
2. Kenneth Arrow & Gerard Debreu, *Existence of Equilibrium for a Competitive Economy* *Econometrica* 22 (1954)
3. Lionel McKenzie, *On the Existence of General Equilibrium for a Competitive Market* *Econometrica* 27 (1959)
4. John R. Hicks *Value and Capital*
An Inquiry into some Fundamental Principles of Economic Theory (1939) Second edition (1946) reprinted in 1957. Oxford At the Clarendon Press
5. Nicholas Kaldor, *A Classificatory Note on the Determinateness of Equilibrium* *Review of Economic Studies*, vol. I No. 2, Febr. 1934.
6. Ragnar Frisch, *On the Notion of Equilibrium and Disequilibrium*, *The Review of Economic Studies* vol. III No 2 Febr. 1936
7. Abraham Wald, *Über Einige Gleichungssysteme der Mathematischen Ökonomie* *Zeitschrift für Nationalökonomie* No. 7.1936

«Formalist Revolution in Economics»⁸ revived Leon Walras' law of general equilibrium, the Walrasian model has usually been equated simply with a system of pure competition. The greatest credit for the revival of Walras' work probably should be given to Joseph Schumpeter who evaluated Walras as the greatest of all economists and his performance as a sort of «Magna Charta» for the exact science of economics⁹.

No other author, to my knowledge, has raised the question that pure competition as a force cannot work efficiently in a vacuum (an abstract model based on formal assumptions not related in any way to reality) to attain an equilibrium position and, by itself, maintain equilibrium prices. The same problem exists for natural sciences whenever, for instance, the force of gravitation is assumed alone without an adequate space-time framework.

All those economists who assume the Walrasian model to be equal to a system of pure competition and nothing else, are being trapped into thinking that a force as such can ever act in an orderly manner in a vacuum. In both nature and society, a force, however, can act orderly and create a stable equilibrium (stability-from-within), if it is correlated with, or embedded in, a suitable framework (in nature given ; in human societies constructed) which has a very strong natural parameter in the form of a constant (axis, magnitude, institution, value) consistent with the force in question. This may be called the Universal Law of the Natural Parameter which holds together systems under conditions of stable equilibrium. A keen observer can envision how the Universal Law of the Natural Parameter may be the key for the methodological unification of all sciences to).

Leon Walras was intuitively aware of this universal law even though he did not formulate it precisely. But when he developed his law of general equilibrium,

8. Benjamin Ward, *What's Wrong with Economics?*
Basic Books, Inc. New York. 1972

9. Joseph Schumpeter, *History of Economic Analysis*
Edit, by Elizabeth Boody - Schumpeter
Oxford University Press, N.Y. (1954) 2nd print. 1955 p. 968

10. Anghel N. Rugina, *The Problem of Value-Judgments in Science and a Positive Solution.*
Max Weber and Ludwig Wittgenstein Revisited.
Paper presented at the International Atlantic Economic Conference in Salzburg, Austria, on
May 14, 1979.

he specifically indicated as prerequisites not only perfectly free markets (pure competition) but also the requirement that all prices be expressed in terms of the «numeraire» (or a monetary unit = a certain quantity of a selected standard commodity) which is a constant magnitude serving as the axis of the system. This is the natural parameter of the numeraire (NaPa).

Lacking an explicit definition of the law of the natural parameter, Walras left out the equivalent of the space - time relationship. He omitted an adequate institutional framework where the force of competition can act from 0 (zero)-inertia—to 100 per cent intensity. Specifically he did not include in his model government, modern banks and other financial institutions, organized stock-, exchange-and commodity markets. His excuse was that they do not belong to pure theory but rather to applied theory or «Economie Politique Appliquée.» This is a poor and inconsistent excuse. First, as already mentioned, a given force like competition cannot work efficiently unless it is in a suitable environment consistent with the law of the natural parameter of the numeraire (NaPaNu) and other conditions of stable equilibrium. Second, what real meaning has «applied theory» on modern banks, for instance, if we do not have a pure theory on banks, under both equilibrium and disequilibrium conditions ? Consequently, if we speak of «applied science» in a particular field, we must first have a pure theory for that field. Walras ; assumption that institutions are outside of pure science is, therefore, untenable.

Even within that part of theory which Walras completed there are portions, such as the theory of profit, which require reexamination and correction.

The purpose of this paper is to undertake the investigation necessary to uncover the missing parts and to prove that they can be integrated with the original formulation of the law of general equilibrium. In this way it is hoped to show the path to follow to achieve a truly general law of stable equilibrium in pure economics.

There was earlier criticism of the internal structure of Walras' model but more of mathematical details, such as the use of constant coefficients of production or the lack of dynamic elements in the system. Such criticism cannot be answered effectively until we know precisely how far or when the model in question is static and when or how far it is dynamic.

This paper is unique in that we shall examine the Walrasian system with the help of a new research program characterized by a simultaneous equilibrium vs. disequilibrium approach in order to see conceptually (leaving

the mathematical translation as an extra accomplishment) what important pieces are missing or do not fit this celebrated law and its underlying model. Then an attempt will be made to reformulate the Walrasian law so that it may be called truly general.

The guiding thought is that the application of the new research program discloses a methodological similarity between social and natural sciences. This is consistent with Walras' philosophy of science. It was his ambition to discover that economic theory was constructed on the same foundation as the theory of physical sciences. For this purpose he thought that the use of mathematics was indispensable. After he finished the presentation of his system, he wrote :

«Very few of us are capable of reading Newton's *Philosophiae Naturalis Principia Mathematica* or Laplace's *Mécanique Céleste* ; and yet, on the word of competent scientists, we all accept the current description of the universe of astronomical phenomena based on the principle of universal gravitation. Why should the description of the universe of economic phenomena based on the principle of free competition not be accepted in the same way? There is no reason why the proof of the system, once established, may not be taken for granted, nor why the assertions involved may not be used in the study of questions of applied practical economics.»¹¹).

The questions which has intrigued this author is that 100 years having passed since Walras (like Newton) formulated his unique law to explain the universe of economic phenomena, why has this law not yet been accepted, even though during this time so much work has been done by so many illustrious economists on the same subject? Something must be wrong and indeed is. The methodology used by Walras and those who followed him, critics included, is incomplete. It will become clear when the new research program is applied that the Walrasian law is like a table with one or two legs : it can never stand by itself, even though, as far as he went, Walras' system with some corrections is consistent with conditions of stable equilibrium. An important lesson must be learned by the economics profession : an economist's reasoning should be conducted basically in economic concepts

11. Leon Walras, *Elements of Pure Economics*
Or the Theory of Social Wealth

Transi, by William Jaffe and published by the Amer. Economic Assoc, and the Royal Economic Society. Richard D. Irwin. Inc. Homewood, Illinois, 1954 p. 428

(which have not only form but also content) exactly as a mathematician's reasoning must develop along the line of mathematical concepts. Otherwise the distinction between the various fields of knowledge is lost and we also are lost. This does not mean that cooperation between mathematics and economics or other sciences is not desirable. It simply expresses the fact that primary concepts in economics and other social sciences are non-mathematical in character and therefore it depends upon the nature of the problem whether mathematical treatment is necessary or possible.

It may well be that, «in the last analysis», as Schumpeter remarked, «Walras' system is perhaps nothing but a huge research program»¹²) which has delivered an unfinished product deserving further study and completion.

I. A NEW RESEARCH PROGRAM AND AN ORIENTATION TABLE FOR ECONOMICS

The new research program stems from the Walrasian system taken as a limiting case - the regime of pure competition with all its adequate milieu - and runs through all possible mixed systems defined in terms of imperfect competition and or imperfect monopoly or monopolistic competition of various degrees, until it reaches the other limiting case - the regime of pure monopoly, again with all its suitable milieu.

It consists of few interrelated theorems which lead to the development of an Orientation Table for Economics and all other social sciences. With the help of this table we can clearly identify the unique location and character of the Walrasian system vis-a-vis all other possible systems. The same table shows how complicated the relationship is between static and dynamic analysis, outside of the area referred to as «comparative statics». It indicates further in what respect the Walrasian system is static and in what respect it is dynamic.

No less important for Walrasian economics is the clarification of the issue of how far a position of unstable equilibrium is to a position of stable equilibrium. The table answers this question. The murky subject of minor (weak and strong) vs. major (weak and strong) disequilibria (two oceans of disequilibria moving in opposite directions) is clarified by consulting the table. In fact, through the table

12. Joseph Schumpeter, op. cit. p. 1026

the entire science of economics appears in a new light because the table provides a logical frame for any theoretical work (which has some truth) indicating where and how far it is valid.

In addition, the new research program can be extended not only to the other branches of social science but also to natural sciences. In this way a path is revealed for the methodological unification of all sciences.

A. Here are the component parts :

1. Axiom 1: The Universal Hypothesis of Duality

It is assumed that the physical universe in which we live, as well as human societies, are composed of stable (equilibrium) and unstable (disequilibrium) elements, forces, behavior and values.

Without this hypothesis we cannot completely explain and understand stability and disturbances in nature, or progress, crises and revolutions in human societies. It is so close to reality, a sort of self-evident truth, that we can consider it an axiom which needs no further proof.

2. Corollary: The Simultaneous Equilibrium vs. Disequilibrium Approach

Any concept and any explanation (theory) consequently can be envisioned at least in two versions in micro-analysis : one in a framework which satisfies the conditions of stable equilibrium and another in a framework where the conditions of equilibrium are negated (disequilibrium).

3. Axiom 2: The Universal Law of the Natural Parameter

Theorem : Any system composed of individual parts, both in the physical universe and human societies, in order to reach and maintain a position of stable equilibrium (stability - from - within) must have a very strong (at the limit 100 per cent) natural parameter which, in conjunction with a suitable force and consistent framework, holds the whole system together.

The natural parameter appears as a real, more or less perfect axis, magnitude, institution or value which changes not at all or very little over a long span of time. The planet earth has a definite axis which does not change daily and deviations from

this axis are nominal and finite. For the physical universe at the macro-level, the SUN is the natural parameter and the suitable force is the law of gravitation. It was Newton who discovered and formulated the law of gravitation, but he did not make use of the concept of the natural parameter which actually determines the framework (space-time relationship) where a force may act, partially or to the maximum. Thus he did not see that the universal hypothesis of duality changes the picture of the real world by introducing exceptions to the law of gravitation as well as to the other known classical laws. Einstein noticed these exceptions and developed the concept and theory of relativity which, in reality, does not negate but complements Newton's theory.

For a national economy, it was Leon Walras (1834- 1910) who formulated the law of general equilibrium and proved theoretically that stable equilibrium was possible only when two basic conditions are satisfied :

- (1) the assumption of a chain of free markets where monopoly forces are reduced to zero at the limit (pure competition), and
- (2) that all prices without exception are expressed only in terms of the numeraire or 100 percent - backed currency.

Even though Walras did not explicitly name the concept of the natural parameter, his numeraire nevertheless is a constant magnitude (an axis at the center of the system) which, in conjunction with a suitable force (competition), helps to attain and maintain a position of stable equilibrium (With minimal deviations exactly as in the physical universe the constant position of the SUN and the axis of the earth together with the force of gravitation assure stability according to Newton's law.

4. Corollary 1 of the NaPa: The Law of Consistency

The law of consistency says : No force in any system - in nature or human societies - can act in such a way to produce stable equilibrium (or stability - from -within) without the existence of a suitable milieu or an adequate framework consistent with the NaPa and the force in question.

The extraordinary beauty and miracle of the physical universe in which we live, the mother nature that we sometimes abuse, lies in the fact that a very strong natural parameter (as a guess, 90 percent or above) and a pure force (gravitation) were established in the act of creation and have not changed significantly over millions of years. There are changes in nature, some of them violent, which show

that the proportion between stable and unstable elements and forces is not fixed. Nevertheless, the physical universe recovers quickly whatever is lost in terms of matter or energy during violent changes or turbulations. Consequently, the overall framework possesses consistency between the natural parameter, the major equilibrium force (gravitation) and the rest of the universe.

When we go to the economic and social universe of the Walrasian system, it is this point of consistency in the institutional framework which is missing. Indeed, he identified clearly the natural parameter of the numeraire and the suitable force of pure competition and showed the manner in which they work, but he did so without being aware that there was a third basic element—a consistent institutional framework—which is indispensable for the attainment and maintenance of stable general equilibrium. The institutional framework here is considered as a theoretical problem to be dealt with in ideal types (Weber). Walras did not have the tools necessary to include this topic in his law of general equilibrium. Therefore he placed it outside of pure theory in the domain of «applied theory» and did not realize the consequences.

5. Corollary 2 of the NaPa: The Compensatory Law or the Natural Law of Full Employment

The Compensatory Law says : The aggregate volume of investment, income and employment in a system of stable equilibrium can never shrink by itself but rather will either increase or remain the same, depending upon the existing conditions and wishes of the people.

Indeed, in a system of stable equilibrium with a 100 percent numeraire - currency and all forms of credit - money also of the numeraire type, that is 100 percent-backed, there is a compensatory flow of real investment, income and employment, running back and forth from the non - monetary to the monetary sector and vice - versa, which thus secures the realization and maintenance of full employment.^{12a)} The dream of Lord Keynes never became a total reality because his model of reasoning (based on conditions of disequilibrium) lacked this compensatory law and the other conditions of stable equilibrium.

12a. Anghel N. Rugina, American Capitalism at a Crossroads!
Where Do We Go From Here ?
Exposition Press, N.Y. 1976 p. 58-59

The compensatory law certainly also exists in nature, and thus the entropy law has limited application, specifically only to unstable elements. The Lavoisier law of the conservation of matter is likewise correct, but according to the universal hypothesis of duality, not in its absolute form but rather restricted to the stable elements in nature.

*
* *

With the help of the new research program we can identify theoretically the institutional framework which is suitable to the Walrasian system and which then has to be included in the law of general equilibrium.

B. Application to Economics

1. Two Bipolar Forces in Economic Life

Using the equilibrium vs. disequilibrium approach, instead of treating each separately as in the past, we can easily align the two well-known bipolar forces which can govern the economic life of a country : Pure competition as an equilibrium force and pure monopoly as a disequilibrium force.

Both competition and monopoly in real life may take varied forms but all are derived theoretically from their pure form.

2. The General Possibility Theorem

Given the dual nature of economic reality in its pure form, there is an unlimited number of possible combinations or systems which, for study purposes, can be reduced to seven basic models.

3. An Orientation Table of Economics

A systematic application of the general possibility theorem leads to the development of a methodological map of all possible systems which can be reduced to seven basic models. A sample of an Orientation Table for Economics follows :

Co = pure competition	ECO == economic system	Nu = numeraire
Mo = pure monopoly	MON = monetary system	anti—Nu=anti-numeraire

Model No 1 :

ECO 100 Co+ MON 100 Nu+a perfect institutional framework. This is the Walrasian system at the limit, a 100% stable economic system. It may be called the «Economics of Certainty, «and this is what Walras had assumed theoretically before considering the real world. This may be identified also as a «Flexible Static System» open to change and usable in «Comparative Statics».

Model No 2 :

ECO 95 Co+ 5 Mo +MON 95 Nu+5 anti-Nu+an adequate institutional framework.

This incorporates a less perfect version of the Walrasian system but of course is more realistic in view of the universal hypothesis of duality. In a rather crude form it approximates the economic model in classical economics as envisioned by Adam Smith with the theory of the natural price and the «invisible hand». In practice this is not a second stage but rather a first best possible because it can be improved probably up to the limit of ECO 98+MON 98. This is the last possibility in the application of the Walrasian system to the real world. One may identify this model as a «System of Normal Dynamics». With such a strong natural parameter (95 - 98 %), monopoly cannot develop and survive (as Adam Smith intuitively saw). Business fluctuations would be reduced to simple and finite adjustments with no possibility for the business cycle phenomenon, as known in modern times.

Here we enter the territory of weak minor disequilibria. Further down we are faced with models representing «Systems of Abnormal Dynamics» and we are in the domain of «Economics of Relativity».

Model No. 3 :

ECO 65 Co+ 35 Mo+MON 65 Nu+35 anti-Nu+a similar institutional framework.

This is a mixed economy type A where equilibrium forces prevail but the natural parameter of the numeraire is not strong enough to impede the development of monopoly and cumulative fluctuations which characterize the phenomenon of the business cycle.

England, during the second half of the 19th century, at certain dates probably reached this stage. Otherwise modern capitalism in terms of stability and social equity never passed the line of 65 Nu.

Here is the area of strong minor disequilibria.

Model No. 4 :

ECO 50 Co+ 50 Mo+MON 50 Nu+50 anti-Nu+a similar institutional framework.

This is a mixed economy type B where equilibrium and disequilibrium forces are of equal intensity. Consequently this represents the model of an organically static system. It looks like equilibrium but it is not a stable one. It is an unstable equilibrium and perhaps better called «stable disequilibrium». Keynes called it «equilibrium with unemployment» and Ricardo envisioned it as a stationary state of the capitalist system in stagnation. This is an «Inflexible, Frozen Static System» which is not open to change.

Here is the area of weak major disequilibria.

Model No. 5 :

ECO 35 Co+ 65 Mo+MON 35 Nu+65 anti - Nu+a similar institutional framework.

This is a mixed economy type C where disequilibrium forces prevail. Such a weak system with a thin natural parameter is exposed to wild fluctuations in all directions and the business cycle phenomenon becomes unmanageable in the sense that contradictions among the principal goals (f.i., price stability and full employment) appear as inevitable and incurable. This model may approximate the capi-

talistic system during the great depression of the 1930's. This is when Keynes noticed the inconsistency between domestic stability with full employment and the balance of payments in equilibrium. Further along the situation deteriorates to the point where inflation and unemployment may increase at the same time (Stagflation). At that point any government policies are doomed to failure.

This is the territory of strong major disequilibria.

Model No. 6 :

ECO 5 Co+95 Mo+MON 5 Nu+95 anti-Nu+a similar institutional framework.

The mixed economy type C under the repeated storms of major disequilibria collapsed and is replaced by a government planned and controlled economy where the means of production are either in collective ownership (Socialism-Communism) or directly controlled by the government through organized monopolies (Fascism).

Model No. 7 :

ECO 100 Mo+MON 100 anti-Nu+an adequate institutional framework.

This is the limiting case of pure monopoly, not owned by private business but by the government. The State monopolist has absolute powers. This is a theoretical model of 100 per cent disequilibrium hidden by the absolute powers of the State. The solution to any problem here is indeterminate and therefore this may be called the «Economics of Complete Uncertainty» in the full sense of the term. The universal hypothesis of duality negates the possibility of having such a system realized at the limit. Experiments with communism and fascism in this century confirms this conclusion.

II. THE WALRASIAN LAW OF GENERAL EQUILIBRIUM AND ITS MISSING PARTS

The theory of general equilibrium in Walras is composed of four consecutive parts (according to Schumpeter) like four stories in a building. These four parts are :

1. The law of the establishment and variation of equilibrium prices for commodities in general exchanged in the market or simply the theory of the exchange of products ;
2. the same law as «1», for services of the factors of production ;
3. the theory of capital formation and the prices of capital goods, and
4. the theory of money.

1. The **Theory of the Exchange of Products**

The first law is described by Walras thusly :

«Given several commodities, which are exchanged for one another through the medium of numeraire, for the market to be in a state of equilibrium or for the price of each and every commodity in terms of the numeraire to be stationary, it is necessary and sufficient that at these prices the effective demand for each commodity equal its effective offer. When this equality is absent, the attainment of equilibrium prices requires a rise in the prices of those commodities the effective demand for which is greater than the effective offer, and a fall in the prices of those commodities the effective offer of which is greater than the effective demand»¹³⁾

The formation of equilibrium prices is explained by a process of groping (« tâtonnement ») where buying and selling is first practiced nominally with tickets at different changeable prices until finally an equilibrium price is reached as described above.

13. Leon Walras, op. cit. p. 172

In the market there will be only one price at which consumers will attain «the greatest possible satisfaction of their wants». If the exchanges take place «in a market ruled by free competition» and «if the prices are cried in terms of a numeraire,» concludes Walras, «the condition of general equilibrium is fulfilled ipso facto. Otherwise, arbitrage transactions are required for the attainment of general equilibrium.¹⁴).

It is this statement which in my view is incomplete. It should read :

- (1) Given a chain of open markets ruled by free competition alone ;
- (2) given that all prices were expressed in terms of a numeraire, and
- (3) given a consistent institutional framework for the entire economy, then the basic conditions of general equilibrium are fulfilled ipso facto. The other characteristics are derived from the state of equilibrium.

2. The Theory of Production

From the theory of exchange Walras went to the analysis of the process of production where he considered the services of the classical three factors : Land, Labor and Capital. Maintaining the same basic assumptions he arrived at similar results expressed in the following manner :

«Production in a market ruled by free competition is an operation by which services can be combined and converted into products of such a nature and the in such quantities as will give the greatest possible satisfaction of wants within limits of the double condition, that each service and each product have only one price in the market, namely the price at which the quantity supplied equals the quantity demanded, and the selling price of the products be equal to the services employed in making them.»¹⁵.

Briefly, according to Walras the state of general equilibrium is characterized by the following conditions :

14. Leon Walras, bid p. 173-174

15. » » ibid p. 255

- (1) A system of free markets where monopoly forces are reduced to zero at the limit or pure competition ;
- (2) All prices for products and services are expressed only in terms of a numeral-
- (3) The effective demand is equal to the effective supply ;
- (4) There is the greatest possible satisfaction of consumers' wants or maximum of utility ;
- (5) For each product and service there is only one price in the market ; and
- (6) The selling price for each product is equal to the cost of the services used for its production.

To this list we should add:

- (7) A consistent (equilibrium) institutional framework to support the realization of the state of general equilibrium.

3. The Theory of Capital Formation and Capital Goods

As to the analysis of the market for capital goods, Walras reached similar results as in the other markets, of course under the same assumptions, He wrote :

«Then for the market for capital goods to be in equilibrium, or for the prices of all new capital goods in terms of numeraire to be stationary, it is necessary and sufficient : (1) that at selling prices equal to the ratio of net incomes to the current rate of net income, the effective demand for these new capital goods be equal in terms of numeraire to their effective supply ; and (2) that the selling prices and the costs of production of the new capital goods be equal.»¹⁶⁾

The theory of capital formation in the Walrasian system has to be restricted only to voluntary savings, or in Walras' words : «the positive difference between the excess of income over consumption and the amount necessary to cover the depreciation and insurance of capital goods proper.»¹⁷⁾ In view of condition (7) this

16. Leon Walras, *ibid* p. 294

17. » » *ibid* p. 274

has to be included specifically as a requirement for the state of equilibrium :

(8) Capital formation = voluntary savings.

It has serious implications related to the modern banking system and the organization of credit, of which Walras was not quite aware. The state of equilibrium therefore requires further that :

(9) Banks should not be permitted to monetize credit in the form of bank notes, bank deposits or in any other form because this would introduce an anti-numeraire or disequilibrium element alien to the system.

(10) Credit used in the economy must be real in the sense that it is always fully covered by a commodity or by an equal sum of money.

Therefore it is limited to the volume of real income in the economy. In other words, real credit is not monetized.

Walras does not touch upon the subject of reforming the modern banking system. «The capitalist», he wrote, «accumulates his savings in money and lends this money to the entrepreneur who, at the expiration of the loan, repays the money. This operation is known as credit.»¹⁸⁾ The concept of real credit is evident here. This quotation presents, however, another requirement for the state of equilibrium :

(11) The volume of new investment = the amount of voluntary savings, which is consistent with conditions (8), (9) and (10).

Another important issue which Walras did not completely clarify is the requirement that :

12) The money market (short term loans) must be clearly separated from the capital market (long term loans).

Walras did not explicitly separate the two markets, even though from his text it is clear that the first deals with the liquidity - function and the second with the investment - function. In the desired cash balance, he included both «cash or savings.»¹⁹⁾ He also did not specifically require that in a system of stable equilibrium all forms of money must be of the numeraire type or 100 percent backed. Only under such a requirement is another condition of the state of equilibrium fulfilled :

18. Leon Walras, *ibid* p. 270

19. » » *ibid* p. 321

(13) Money (numeraire) capital = real capital (in form of capital goods).

Lacking (12) and (13), Walras was faced with problems in the theory of the rate of interest. He wrote :

«Thus the rate of interest, which is the ratio of net profit to the price of securities, manifests itself, to be sure, in the market for numeraire capital, that is to say in the banking system, though actually it is determined in the capital goods market, that is to say in the stock exchange, as a rate of net income which is the common ratio of the net price of services to the price of landed capital, personal capital as well as capital proper. It is clearly seen now that the key to the whole theory of capital is to be found in thus eliminating capital loans in the form of numeraire so that attention is directed exclusively to the lending of capital in kind.»²⁰)

Using (12) and (13) there is no need to introduce a non-realistic assumption that capital loans were eliminated (when actually they were not!). In a modern economy like that with which Walras was concerned, we cannot use the assumption of «lending capital in kind» since that belongs to a primitive economy.

If requirements (12) and (13) are fulfilled, there is no problem in constructing the curves for effective demand for liquidity and the supply of immediately available cash balances in order to determine the equilibrium rate of interest in the money market. It is also possible to draw the curves for the effective demand for new investment (production of capital goods) and the available supply of capital (voluntary savings) in order to determine the equilibrium rate of interest in the capital markets.

Even though Walras did not use these additional specifications, nevertheless since his system was constructed to be 100 percent consistent, he managed to complete his analysis of capital formation and the production of capital goods, with results that could be integrated with his previous conclusions in the theory of exchange and production of consumer goods. In his own words :

«Capital formation in a market ruled by free competition is an operation by which the excess of income over consumption can be transformed into such types and quantities of new capital goods proper as are best suited to yield the greatest possible satisfaction of wants both to the individual creators of savings and to the whole body of consumers of the services of the new capi-

20. Leon Walras, op. cit. p. 290

tal goods, within limits defined by the condition that the depreciation and insurance of capital goods proper be covered at the expense of consumers of the capital - service and not at the expense of the owners of the capital goods.»²¹⁾

From this quotation we can see that the investment in the production of new capital goods is equal to the «excess of income over consumption» or voluntary savings and that there is no other way to finance new investments. This satisfies all the requirements from (8) to (13).

4. The Theory of Entrepreneurship and Normal Profits

When discussing the production of consumer and capital goods, we cannot ignore the services of the entrepreneur (or manager) designated by Walras as the «fourth person, entirely distinct» from the landowner, worker and capitalist (saver) «whose role is to lease land from the landowner, hire personal faculties from the labourer, and borrow capital from the capitalist, in order to combine the three productive services in agriculture, industry or trade.»²²⁾ In continuation he wrote: «From the scientific point of view, we must keep these roles separate and avoid both the error of the English economists who identify the entrepreneur with the capitalist and the error of a certain number of French economists who look upon the entrepreneur as a worker charged with the special task of managing the firm»²³⁾

There is no more explicit way to identify entrepreneurship as a fourth factor of production. And if there is a market for the services of the other factors (land, labor and capital), then to this we must also add a market for managerial or entrepreneurial services. The assumption that in a system of stable equilibrium profit = zero is not only confusing but is unrealistic and logically and economically unjustified.

The responsibility for this widespread confusion, in fact a negation of a legitimate normal income (profit) for the entrepreneur, rests entirely with Walras who wrote :

«Thus, in a state of equilibrium in production, entrepreneurs make neither

21. Leon Walras, op. cit. p. 305

22. » » op. cit. p. 222

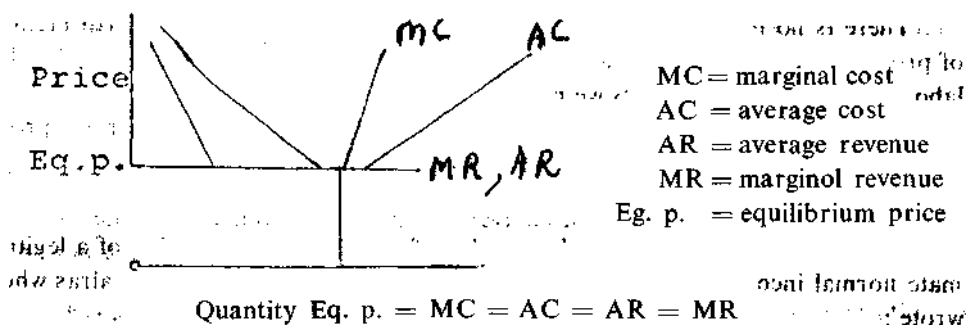
23. » » ibid cit, p. 224

In fairness and also as a requirement of the economic process in a modern economy, the entrepreneur deserves to have his own legitimate income, determined by the same principle of free competition, which can be properly named «normal profit» in the same way as we have normal (equilibrium) wages, rent and interest. Thus a new condition for the state of equilibrium can be stated :

- (14) In a system of stable equilibrium, prices = the lowest cost of production when a normal profit for the entrepreneur is included.

The normal rate of profit, under equilibrium conditions, is equal to the marginal profit nor loss ('les entrepreneurs ne font ni bénéfice ni perte). They make their living not as entrepreneurs, but as landowners, labourers or capitalists in their own or other business.»²⁴⁾ utility or the marginal productivity of the factor, exactly as in the case of the other three factors. Consequently, the condition that profit = zero, under equilibrium conditions must be dropped.

Among the living economists, to my knowledge, Mrs. Joan Robinson has defined normal profits very adequately as «that level of profits at which there is no tendency for new firms to enter the trade, or for old firms to disappear out of it.»²⁵⁾ These are equilibrium profits. For an individual firm in equilibrium she added further : «The total receipts of the firm are then exactly equal to the total costs including normal profits.»²⁶⁾ She even provided a diagram of a firm under equilibrium conditions, which is different from diagrams in other textbooks.²⁷⁾



In other textbooks the Eq.p. line is above the least cost (the bottom of AC curve)

24. Leon Walras, op. cit. p. 225

25. Joan Robinson, The Economics of Imperfect Competition (1933) MacMillan & Co. Ltd London, 7th reprint. 1948 p. 92

26. » » op. cit. p. 94

27. » » op. cit. p. 96

because profits are not included and this is a sort of anomaly in the study of general stable equilibrium. Very definitely, the theory of a normal profit is an important missing factor in the Walrasian system of general equilibrium.

There is another subject which also may easily create confusion in the theory of profits under conditions of equilibrium. It is true that Walras did not define entrepreneurship as did Schumpeter in terms of innovations²⁸) explicitly, but he does allow new capital formation and growth when population is increasing. The entrepreneur could not do justice to such a goal unless he uses new, more efficient methods of production or a new combination of available resources and this is what «innovation» in modern business means.

In brief, whenever there is a deficit in the available supply (f.i., in the case of increasing population), even in a system of stable equilibrium, the effective demand will appear relatively higher and therefore a rise in prices is inevitable. In this way the entrepreneur receives a signal that output has to be expanded and an immediate reward in the form of a small percentage of a «profit - differential» is provided by the existing market conditions. This profit - differential is a legitimate income which, however, does not remain with the entrepreneur since under equilibrium conditions the available resources are already committed. Consequently, to have more capital for an expansion of output he has to pay a higher rate of interest ; to hire more labor he has to offer higher wages ; and to use more raw materials he has to bid a higher price for raw materials (or a higher rent). All these changes lead to a new economic rearrangement where the profit - differential is shared by the other factors in the form of interest, wages and rent.

It is this profit - differential which Walras had in mind when he wrote :
«... free competition consists, on the one hand, in allowing entrepreneurs to expand output in case of profits and to restrict output in case of loss ; and on the other hand, in allowing land-owners, workers and capitalists, as well as entrepreneurs, freedom to buy or sell services and products by bidding against one another.»²⁹)

We must therefore add another condition for the state of equilibrium :

(15) In a system of stable equilibrium in action (in real life) a small percentage

28. Joseph Schumpeter, *Theory der Wirtschaftlichen Entwicklung* (1912) English Transl. *Theory of Economic Development* 1934

29. Leon Walras, *op. cit.* p. 255

of a profit-, wages-, rent- and interest-differential must be allowed as an incentive and reward to produce more whenever necessary.

It is true that when production and prices are in a stationary state, the profit-differential is and must be = zero but not the normal (equilibrium) profit. This must be positive and at the same level of marginal utility (rareté) and productivity as exists for the other factors used. When, for some reason, the expansion of output has been unsuccessful, then the profit - differential becomes negative and represents a decline in the normal profit.

The profit - differential must not be confused with what is called in literature «pure profit» as a compensation for additional risk which cannot be calculated (Frank H. Knight)³⁰) as is the case in the capitalist system when a significant disequilibrium prevails. In the Walrasian system uncertainty and risk are reduced to a minimum (in fact to zero at the limit), and therefore there is no legitimate reason for the existence of «pure profit».

5. The Theory of Natural vs. Artificial (Pure) Speculation

One other subject on which Walras is not quite clear is the issue of speculation. The process of groping («tâtonnement») by which he explained the development of equilibrium prices in practice is based on arbitrage or speculation. In his «Etudes d'Economie Politique Appliquée» he has a whole chapter on «La Bourse,» where speculation and arbitrage are described in much detail but only from the institutional point of view.³¹) No proper theoretical analysis is provided since he thought that this subject belonged to applied and not to pure economics.

Walras did not see—as Sir Nicholas Kaldor later did—³²) a clear distinction between two different kinds of transactions on the organized stock -, exchange - and commodity - markets :

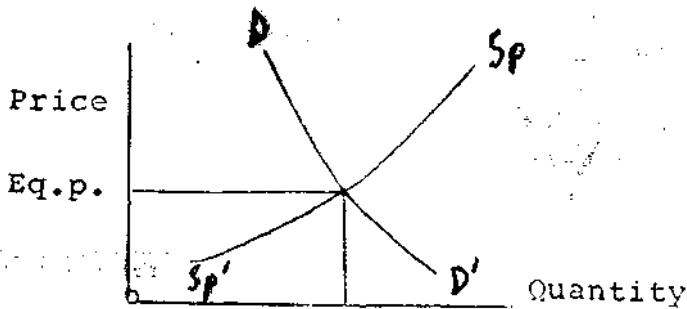
30. Frank H. Knight, Risk, Uncertainty and Profit (1921)
The London School of Economics and Political Science. Series of Reprints No. 16
Eighth Impression 1957 p. 46-47
31. Leon Walras, Etudes d'Economie Politique Appliquée
Théorie de la Production de la Richesse Sociale Lausanne, F. Rouge et Cie, Second edition, 1936 pp. 401 - 445
32. Nicholas Kaldor, Speculation and Economic Stability
The Review of Economic Studies vol. VII, No. 1, Oct. 1939

- (a) Natural, real transactions where the object of business is an actual transfer of ownership (stock, foreign exchange or commodities), and
- (b) artificial, nominal transactions where the object of business is a wager that future prices of the items in question will rise (bullish speculators) or decline (bearish speculators).

A very important argument can be raised as to the consequences of the two different types of transactions. Of course, in both there is an element of speculation but in the first case the speculation is limited and covered whereas in the second case it is unlimited and not fully covered.

In the first type of transaction, speculation is natural (real) in the sense that the seller has possession (direct or by order) of the respective item and the buyer acquires ownership of the full value of the item by use of his own savings or borrowed funds. Both are therefore 100 percent covered and the speculation in question (the purpose of gaining a normal profit) is limited to the available supply of the item (on the seller's side) and to the existing supply of funds or circulation - capital (on the buyer's side).

We can easily construct the curves representing the effective demand and available supply for this type of normal or natural speculation. These are the normal Marshallian curves which indicate the equilibrium price for the respective item.



With the second type of transaction the situation is entirely different. The speculation is artificial (nominal) or pure in the sense that it is unlimited and very little, and sometimes not at all, covered. The seller does not own the items in which he wants to speculate and the buyer is not interested in actually acquiring those items. The seller is willing to sell now at a certain price since he hopes that on liquidation day when he purchases to offset his sale the price will be lower and, in this way, he will reap a profit without ever seeing the object which was traded fictitiously. The buyer is not interested in acquiring the item in question. He is willing to buy now at a certain price with the hope that on the liquidation day when

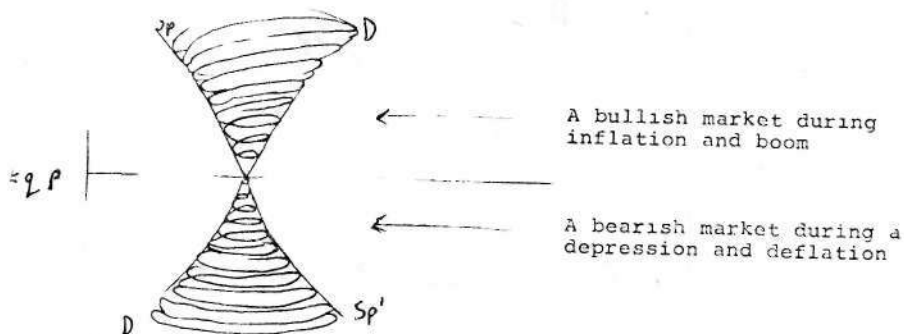
he sells to offset his purchase, the price will be higher and thus he will be the one to reap the profit, again without ever seeing the item traded fictitiously. The purchaser is the bullish speculator and the seller is the bearish.

The result:

If the bulls are stronger than the bears, then the prices for the respective items will go up and vice versa when the bears are stronger, regardless of real conditions in the economy. In real life we see this often ; this is when professionals in the market cannot explain why the market moved in a certain direction.

In a bullish market the prices go up and up because the speculative demand increases due to the premium of an extra profit. In this case the demand curve for pure speculation has a positive slope and is forward rising. This is a typical disequilibrium demand curve which we find also during a prolonged inflation and a boom. The supply curve of pure speculation has a negative slope and is backward bending. Indeed, in a bullish market the expectation of an extra profit for a bear diminishes as prices increase, and he may decide meanwhile to change positions. This is a typical disequilibrium supply curve of pure speculation which again we can trace during an inflation.

The opposite happens in a bearish market or during a depression with deflation. We can express the pure speculation through the following diagram :



Pure speculation, therefore, as distinguished from normal, natural speculation, is harmful to the economy as a whole because it produces cumulative price fluctuations in conjunction with other elements of disequilibrium. During the recent grains embargo for sales to the Soviet Union announced by President Carter, farmers requested the Commodity Futures Trading Commission to halt grain trading as being harmful, and the Commission sided with the farmers, but only for two days. The Chairman of the Commission had only this to say : «The market users wanted the market closed. Only the traders and the exchanges wanted the market to stay

open ...» as if this was the whole story. (The Boston Globe, January 11, 1980)

The science of modern economics has not yet recognized the curves necessary to express «pure speculation» even though Marshall, who coined the term «normal» demand and supply curves, in a footnote pointed out that nominal transactions done by speculators «to corner the market» can not be included in that concept.³³⁾

Walras did not see the distinction and the consequences of the existence of the two entirely different types of transactions in a modern economy. He reasoned as if all speculations were natural in the sense of being limited to the «excesses (in numeraire) of income over consumption» which are savings. He saw the role of the speculator as one whose «business is to classify capital»,³⁴⁾ which again falls in the same category. The conclusion is another condition for a state of general equilibrium :

- (16) In a system of stable equilibrium, transactions on the organized stock-, exchange-and commodity-markets have to be restricted to normal, natural or fully covered operations. Nominal transactions or pure speculations must be prohibited by law, since they represent alien or disequilibrium elements in the system.

6. The Theory of Money

Walras developed his law of general equilibrium under the assumption of what may be called a «numeraire - economy» but, according to his own statements the numeraire was not yet declared money. Only in Lesson 29 is money introduced in the form of numeraire-currency (full value gold or silver coins), following the same economic process as for all other commodities. He did not face any difficulty because the equations of circulation and money were consistent with the law of general equilibrium presented before. All that was still needed was one paragraph saying that the law of general equilibrium requires that all forms of money in circulation must be of the numeraire - currency type or 100 percent covered and freely convertible. Then a chapter on the nature and functioning of numeraire-currency within a domestic economy and on foreign exchange markets would have been sufficient to complete his theory of general equilibrium. Only one issue would have still presented a problem — the business of government, the public sector of the economy.

Walras, unfortunately, fell into a trap. In Lesson 30 he leaves the domain

33. Alfred Marshall, Principles of Economics (1890)
8th edition. The Macmillan Co. N. Y. 1952

34. Leon Walras, Elements, p. 310

of pure theory and desires also to be a «realist», or, in his own words, «to pass from the theoretical solution which is reached in the market.» For this purpose he introduced money which «is neither a commodity nor anything that can serve as the numeraire,»³⁵⁾ in other words, inconvertible paper money (or anti - numeraire) without realizing that by doing this he departed from the proper domain of his system of stable equilibrium (Models No. 1 and No. 2 on our Orientation Table).

When he further developed his equations he reached the conclusion that «the equation of monetary circulation, when money is not a commodity, comes very close, in reality, to falling outside the system of equations of (general) economic equilibrium.»³⁶⁾ Instead of being alarmed by this result and attempting to discover the error, he continued to apply pure mathematical reasoning and develop a money market and a theory of desired cash-balances with conventional «raretés» (marginal utilities) on the questionable belief that what counts is the «service of money» and not its background.

Walras was wrong conceptually in reaching such a conclusion even though mathematically one may not be able to find the error since the equations in question can have mathematical (in form) but not economic (in content) consistency. Indeed this is the case in the argument raised here. Inconvertible paper money as well as not fully covered bank notes and bank deposits, both in their nature and functioning, represent an anti - numeraire or a disequilibrium type of money.³⁷⁾ The service of money is entirely different (good or bad) depending whether money is numeraire or anti - numeraire. Numeraire - currency in any form has inherent stability in a system of free markets, whereas any form of anti-numeraire-currency has inherent instability in the same system. This is what Walras did not clearly perceive. If he had, he would have then rejected the introduction of inconvertible paper money or any form of credit money as being inconsistent with a system of general equilibrium.

Walras minimized the problem but Schumpeter noticed that there were difficulties in passing from «numeraire - economy» to «money-economy» (where credit-money and paper money was mixed with numeraire - currency) and that «the question of stability (and of the presence of a tendency in the system to realize the equilibrium values of its elements) is now much more difficult to answer than it was before.»³⁸⁾ But even Schumpeter did not put his finger on the real cause

35. Leon Walras, op. cit. p. 325

36. » » op. cit. p. 327

37. Anghel Rugina, American Capitalism, p. 206, 216, 228 and 261

38. Joseph Schumpeter, op. cit. p. 1025

for the difficulties, namely the introduction of anti-numeraire forms of money in the Walrasian system. Walras, however, immediately sensed the true reason for those difficulties, which he did not face mathematically but only intuitively. He clearly pointed out that a service which is «an immaterial thing can not be used as money. The only thing that can be used is a final product or a raw material. In fact, it seems that nature has conspired to bestow all the attributes of money, homogeneity, great scarcity, divisibility and immutability upon two precious metals, gold and silver, which are final products and raw materials at one and the same time.»³⁹⁾

What is inconvertible paper money or credit - money, in the form of not fully covered bank notes and bank deposits, but an immaterial thing based on political power or the sheer confidence of the public? A few pages later Walras even more emphatically stressed the qualities of unnumeraire - currency. «What is most remarkable in the case of a commodity which serves both as money and as numeraire, is the manner in which all prices rise and fall in terms of A (numeraire) in response to an increase or decrease in the rareté or value of this commodity in its monetary use when there is a decrease or increase in its quantity.»⁴⁰⁾

After all of these quotations the ever recurring question is : Why did Walras not state explicitly that in a system of general equilibrium there is no place, either in theory or in practice, for any anti - numeraire form of money in circulation? The most plausible explanation for this failure by such a great thinker as Walras appears to be the fact that first, he did not have the law of the natural parameter or the numeraire and its corollary, and second, he was not fully aware of the distinction between numeraire (equilibrium) and anti-numeraire (disequilibrium) forms of money and even less aware of the consequence of this distinction.

It is true that Walras later in his life changed some of his monetary views but he did not change the two deficiencies just mentioned. In a proposal presented at the International Monetary Conference for the continuation of the Latin Monetary Union (1884), for instance, he abandoned his previous favorable attitude to bimetallism and instead recommended a system of gold-money and silver as a regulating bullion.⁴¹⁾ In a subsequent study : *Théorie de la Monnaie* (1886), Walras reiterated his proposal of gold-money with silver as regulating bullion and stressed «the prohibition of any issue of bank notes in which I recapitulate today the theory of circulation.»⁴²⁾

39. Leon Walras, *Elements*, p. 329

40. » » *op. cit.* p. 333

41. » » *Etudes d'Economie Politique Appliquée* p. 19

42. » » *op. cit.* p. 73

In conclusion the following statement must be added explicitly to the previous list :

In conclusion the following statement must be added explicitly to the previous list :

- (17) In a system of general equilibrium all forms of money in circulation must be of the numeraire-currency type. At the same time all forms of anti - numeraire money must be prohibited by law.

7. The Theory of Foreign Exchange Markets

Foreign exchange rates appear as a ratio of foreign money expressed in terms of domestic currency or vice - versa depending upon the custom of quotation (continental vs. British technique). Much credit is used in international transactions, but as long as this credit is fully backed by a commodity, a service like shipping or a certain amount of numeraire - currency, the credit in question is real and non-monetized. Therefore such fully covered credit is limited and consistent with the law of general equilibrium. Bills of exchange are issued and traded along with these transactions. As long as such bills of exchange are issued and traded under the above conditions, freely convertible in numeraire and not monetized by banks, they do not constitute a problem because they represent an equilibrium type of circulation media.

We can therefore identify another condition of the state of general equilibrium :

- (18) Given (a) that foreign exchange is fully backed and freely convertible in numeraire - currency or any numeraire type of money ;
- (b) that it is traded freely in open markets where monopoly is excluded and pure competition prevails ; and
- (c) that numeraire - currency itself is freely imported and exported, then the state of equilibrium will be marked by the fact that the effective demand for foreign exchange is equal to the available supply of foreign exchange. This is the equilibrium exchange rate and is at par with the official rate.

Consequently, the normal Marshallian demand and supply curves will also be applicable to the foreign exchange markets.

Should the state of equilibrium be disturbed then, according to Walras, arbitrage (natural speculation) in bills of exchange will restore it. Of course, in addition to this, the well - known mechanism of foreign exchange itself, the gold or numeraire-

re - import and export points, the rate of interest and the level of prices will complete the adjustment process.

Under conditions of stable equilibrium, a large number of international transactions are settled through the use of bills of exchange. Therefore only a small quantity of gold or silver (as numeraire) need move from one country to another to reestablish equilibrium whenever it is disturbed. Walras is absolutely right when he concludes :

«Thus the world market for bills of exchange serves as a vast clearing house where the transactions of all countries are liquidated by the mere payment of differences. And this result is obtained purely and simply by the automatic operation of the mechanism of free competition. The law of supply and demand regulates all these exchanges of commodities just as the law of Universal gravitation regulates the movements of all celestial bodies. Thus the system of the economic universe reveals itself, at last, in all its grandeur and complexity : a system at once vast and simple, which, for sheer beauty, resembles the astronomic universe.»⁴³).

8. The Theory of the Business of Government (Public Sector)

The classical tradition, as inherited from Adam Smith and other thinkers, was that government is a consumptive institution and therefore the analytical apparatus of the principle of free competition cannot be applied to an explanation of how public income (taxation) and expenditures are determined or should be determined. It seems that Walras shared this classical tradition, even though not explicitly. In any case he left the public sector out of his law of general equilibrium. There may have another stronger reason, namely, in his plan for the nationalization of land with the government as an administrator of rent where he assumed land to carry a surplus value in all progressive societies. Land was supposed to be purchased by the government and then rented to individuals and corporations.⁴⁴)

In this way, he thought, two principal problems could be resolved. First, by creating conditions of equality in the social milieu, the individual may develop his unequal God-given talents and desires so that the social ideal of justice would be accomplished. Second, when he recommended that the government be the admi-

43. Leon Walras, *Elements*, p. 374

44. » » *Etudes d'Economie Sociale*
Théorie de la Repartition de la Richesse Sociale 2-nd edition, 1936.
Lausanne, F. Rouge & Cie p. vi

nistrator of land and receive all rents, he believed that this should be the unique source of public income and all taxation could be abolished. Walras especially opposed a personal income tax. In view of the negative results under the collectivization of agriculture in different socialist countries in this century, Walras could have a second thought on this issue even though, to be fair, his proposal is different from the socialist experiment. The point here is that government is not a good administrator, and by nationalizing land—a principal factor of production—more problems may be created than resolved.

Let us go back and see how he justifies his position from the theoretical point of view of pure economics :

«Our proof implies a fundamental distinction between individual wants i.e private utility which the individual is capable of estimating, and social wants or public utility which is estimated in an entirely different way. Therefore; the principle of free competition, which is applicable to the production of things for private demand., is not applicable to the production of things where public interest is involved.»⁴⁵)

This is a widespread view even today, a hundred years after Walras expressed these thoughts. A critical examination of the issue may prove that there is more truth in the opposite view of an organic interdependence between the public and the private sectors even though each one may retain its own particularities. Indeed, more analytical work in the study of government and government finance, by the application of the new research program of equilibrium vs. disequilibrium approach, could provide support for a new view that government, under conditions of stable equilibrium is actually a fifth factor whose services are indispensable for a complete and successful economic process in a modern economy as are the other four factors, each with its own peculiarities.

Let us examine this issue more carefully. First, there is an effective demand for and an effective supply of public services for which a price is to be paid—taxation. Consequently there is a market for public services whose production and distribution requires the cooperation of the other four factors. Second, there is free competition (assumed in democratic countries) among the political parties which want to be in charge and administer public services exactly as there is competition among the entrepreneurs who want to be in charge of and manage private corporations. Third, the resources are limited in both the private and public sector and therefore in both we are faced with the problem of making a choice among

45. Leon Walras, *Elements*, p. 257

different uses of the available resources (again competition). Fourth, we strive toward the same goal of maximum utility : individual utility in the private sector and social utility in the public sector. Fifth, we want the highest possible degree of efficiency in the use of available resources in both sectors, that is to produce and distribute services (private and public) at a price or taxation equal to the lowest cost of production.

If you consider all of these things, it appears that the principle of competition and other rules of efficiency are not out of order in the public sector as Walras assumed. Consequently, since no modern economy can exist or function normally without the existence of an adequate form of government, we may conclude that under conditions of stable equilibrium government must be considered as a fifth factor of production with a legitimate right to have its own revenue measured and evaluated by the same principle of marginal utility (marginal productivity) as for the other four factors.

There is a problem of social equity which must be resolved in both theory and practice. It is the quest for «just taxation.» The same problem also exists for the private sector-namely, the requirement for «just prices» of both finished products and services. The solution therefore is not to separate the private from the public sector and leave the latter outside of pure science, as Walras wanted, but rather to investigate rigorously under what precise conditions prices charged by private business and prices (taxes) charged by government are truly equilibrium prices and taxes. In other words, we need a theory and a model of an equilibrium form of government and government finances, which then can be attached to the scheme of the Walrasian law of general equilibrium making it complete and truly general.

We cannot pursue this important subject further here, but it must be remembered that a number of economists during the last part of the 19th century and the first two decades of this century have diligently worked on this issue and reached valuable results. Unfortunately, these results were later abandoned due to the strong influence of Keynes's and Marx's works which run in the opposite direction-the study of disequilibrium conditions. Pantaleoni (1857-1924) was probably the first who, as far back as 1883, attempted to prove that the laws of value, specifically the criterion of marginal utility used by Walras and Jevons, also had application to the evaluation of public expenditures.⁴⁶⁾ An entire Italian

46. M. Pantaleoni, *Contributo alla Teoria del Riparto delle Spese Pubbliche* (1883) quoted in Antonio de Viti de Marco, *Principi di Economia Finanziaria* (1833) Prefazione di Luigi Einaudi. Edizioni Scientifiche Einaudi, 1953 p. 18.

school which dealt with the same subject of public finance based on the same principle developed at the turn of the century. In particular, Antonio de Viti de Marco (1858-1943) and Luigi Einaudi distinguished themselves in the same attempt to prove that the laws of value in pure economics are applicable also in the domain of public finance. Erik Lindahl⁴⁷ and Knut Wicksell⁴⁸) from the Scandinavian school as well as Emil Sax⁴⁹) from the Vienna School have made important contributions to the understanding of the problem of «just taxation» and the optimum level of public expenditures.

We can therefore conclude that in a system of stable equilibrium, government can be considered as the fifth factor of production under the following conditions :

- (19) No public expenditure can be undertaken without a prior adequate public income derived from taxes or borrowing. In other words, first income then expenditure. This rule can be linked to the private sector under the same equilibrium conditions— first capital formation (voluntary savings) then investment.

This may be called the principle of budgetary stability.

- (20) No taxation should be imposed upon the people without a prior direct consultation and the consent of the majority through a referendum on a regional basis (like Switzerland).

This may be called the principle of social equity and democracy in public finance.

- (21) The state of general equilibrium can be expressed then by the formula :

$$\begin{array}{l} \text{The marginal utility of \$ 1.00} \\ \text{spent for public services} \end{array} = \begin{array}{l} \text{the marginal utility of \$ 1.00} \\ \text{spent for private goods.} \end{array}$$

It means that at the limit both private and public goods are equally productive in the sense of equally valuable, as they should be in a free, just and stable society and economy. This would correspond to conditions with an equilibrium level of taxation and an optimum level of public expenditures combined with an equilibrium level of prices for private goods. Then people would not object to paying taxes.

47. Erik Lindahl, DieGerechtigkeit der Besteuerung.
Eine Analyse der Steuerprinzipien auf Grundlage der Grenznutzentheorie. 1919

48. Knut Wicksell, Finanztheoretische Untersuchungen. 1896

49. Emile Sax, Grundlegung der Theoretischen Staatswirtschaft 1887

The principle of maximum utility and efficiency will then govern both sectors and not the least important, the ethical principle of justice of equity will be fulfilled.

Walras ever came close to the realization that if the proof was given that one and the same principle of marginal utility and/or marginal productivity determined the production of both private and public goods and also the distribution of all major sources of income (rent, wages, interest, profit and taxes), then this ipso facto would achieve justice of equity or of equitable shares. Of course, this refers to a model of ideal conditions. If we want to have such conditions in the future, this requires resolving the problem of social inequities inherited from the past through a reasonable social reform before we introduce conditions of stable equilibrium.

Unfortunately Walras had a very narrow concept of science equated with pure theory and nothing else. Consequently ethics for him was a subject outside of science. After he succeeded in putting the theory of exchange, production and capital formation under the same roof, proving to be «a great master of universal interdependence.» as Schumpeter called him,⁵⁰) he wrote :

«Are these conditions of maximum utility just? That is for the ethical theory of the distribution of social wealth to say ; only then can the economic theory of the production of social wealth boldly proceed to work out in detail the application of the principle of free competition to agriculture, industry, commerce, banking and speculation.»⁵¹)

Even though he frequently uses the term «justice of equity» and mentions the trinity, «Truth, Utility and Equity,» as making a «perfect social science,»⁵²) nevertheless in application he splits the concept into «justice of equity» for the private sector and «justice of equality» for the public sector, without realizing that in this way a logical inconsistency is inevitably created in the system. He did this with the honest belief that a reconciliation (accommodation) between absolute individualism (Liberalism of the laissez-faire type) and absolute collectivism (Socialism-Communism) was possible by following this road. But such a solution is a doctrinaire compromise, very debatable in science — then and perhaps even more today. Schumpeter, who otherwise gives Walras full credit for his unique performance in pure economics, on this issue remarked on «his questionable philosophies about social justice, his land - nationalization scheme and other things

50. Joseph Schumpeter, op. cit. p. 1025

51. Leon Walras, Elements, p. 306

52. » » Etudes d'Economie Sociale, p. 31

that have nothing to do with his superb achievement in pure theory. They have nothing to do with superb achievement in pure theory. They have cost him the goodwill of many a competent critic, and must, I imagine, try the patience of many of his readers.»⁵³).

In short, there seems to be enough evidence of the possibility of uniting the private and the public sectors under the common roof of pure economics. It is not byway of submitting the private sector unconditionally to the power of government but rather in recognizing an organic interdependence among all parts of a modern society and economy. The task of science then is to provide a consistent link which will unite the two sectors without creating problems of dependence, all of which is possible in a generalequilibrium scheme. Walras did not have the adequate methodological tools to discover the link and therefore he left the two sectors separate.

The new research program can provide such a consistent link by identifying the stable forces, elements, behavior and values vs. the unstable ones in human societies, economy and government. In this way a new condition emerges :

(22) It appears both logical and realistic that in a law of general stable equilibrium, we must include only stable elements, forces, behavior and values. Otherwise a qualification is needed and the solution to a given problem is no longer determinate in the complete sense of the term.

9. The Concept of «Pure» Competition Often Misinterpreted

The concept of pure competition often is confusing, especially when the prefix «perfect» is added. Walras used both terms interchangeably as if there were no difference between them. Later, however, other economists insisted that the two terms must be differentiated. Edward H. Chamberlin, for instance, interpreted pure competition as requiring (a) a relatively large number of buyers and sellers ; (b) a perfectly homogeneous product ; and (c) not being alloyed with monopoly elements. As to the meaning of «perfect» competition, he added more requirements, like the «absence of friction» and «perfect knowledge of the future and the consequent absence of uncertainty.»⁵⁴) All these requirements were interpreted, not only by Chamberlin but by many other economists, as leading to the conclusion

53. Joseph Schumpeter, op. cit. p. 827-28

54. Edward Chamberlin, The Theory of Monopolistic Competition (1933) A Re - Orientation of the Théorie of Value. Harvard University Press, Cambridge, 1962 8th ed.p. 6, 16

that pure competition actually is impossible, an abstraction that never existed and never could exist.

Unfortunately there is a misinterpretation in the above presentation which requires clarification. In the first place, competition in theory is, of course, an «abstraction from reality» like any other concept but, as Boulding stressed, «without these abstractions ... we cannot hope to understand reality.»⁵⁵)

The law of NaPaNu and its corollary can help clarify this argument. In general when a force like competition is isolated analytically from opposing forces, it is always pure by definition ; this is what Walras had in mind. «Perfect», on the other hand, does not relate to competition per se but rather to the institutional framework where competition is supposed to work, a framework which indeed must be perfectly consistent with NaPaNu and the force in question. Only under such a framework as delineated in Model No. 1 on our Orientation Table can competition be pure and perfect in theory, and in Model No. 2 in theory and practice.

In conjunction with the same issue, the problem of natural monopolies is often raised as proof against the possibility of having pure competition in practice. The answer to this is simple. In a system of general stable equilibrium, natural monopolies must and can be neutralized completely by a law of social and economic justice requiring such monopolies, as well as other businesses, to sell their product or service in question at an equilibrium price, equal to the lowest cost of production, wherein a normal rate of profit and a small percentage for incentive to innovation and growth are included.⁵⁶) Summing up, pure competition in action does not need to destroy itself through mergers and monopolies if we provide an adequate institutional framework.

It is a pity that during the 19th century when the doctrine of liberalism was put into practice (without a suitable milieu having been prepared) and again during this century, competition was given free reign to act wildly in an institutional framework full of inherited inconsistencies and contradictions. Moreover, this state of affairs has been defended and perpetuated under the false image of an archaic concept of economic freedom which has frustrated any sense of social equity, and under the illusion that the law of supply and demand can by itself successfully resolve problems created by hidden contradictions in the system.

55. Kenneth Boulding, *Beyond Economics*
Essays on Society, Religion and Ethics
University of Michigan Press, 1968 p. 98

56. Anghel Rugina, *American Capitalism*, p. 10, 90

From this we can draw another condition for a state of general equilibrium:

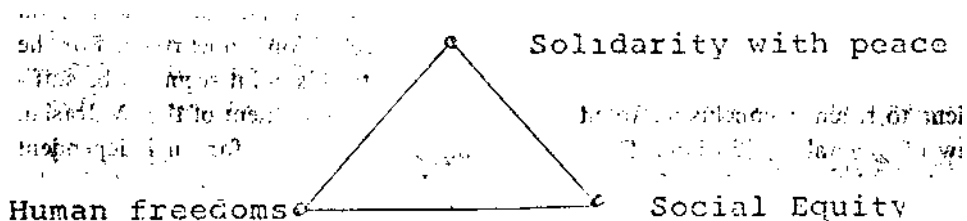
- (23) Competition in action is, as Walras noticed, «a self-driven and self-regulating mechanism»⁵⁷) throughout the whole economic system, but he failed, to add : only when it is anchored into an institutional framework consistent with NaPaNu. Furthermore, all other opposing forces must be eliminated or neutralized to the point where they are no longer an impediment in the normal functioning of the system.

Finally, the concept of competition has another dimension that escaped Walras because of his narrow, one may say puritanic, view of science which he equated to pure theory and nothing else. In a broader perspective, that is looking at society as a whole, competition appears as one aspect of a larger principle, namely, that of human freedoms (social, economic, political, cultural) which runs through all social science, economics included. Competition is indeed a branch of the tree of human freedoms. The central weakness in the economic thinking of the 18th and 19th century was the fact that the thinkers of that time (influenced by the French and American Revolutions) concentrated on only one factor — human freedoms as the basis of a free, democratic society and economy, and gravely neglected the other two principal pillars—social equity and human solidarity with peace. Thus the liberal experiment of that time ended with half success : progress and failure.

The 20th century thinking tormented by two world wars of destruction, by the Great Depression of the 1930's and the Russian Revolution, shifted toward more social justice, not of equity but more of equality, thus introducing new inconsistent elements in the already weakened institutional framework, all at the expense of individual freedoms. Following this road, the result cannot be called a success because we are approaching a crisis of large dimensions characterized by an open conflict between the ideal of human freedoms and that of social equity. We are no more satisfied with present conditions than we would be to return to the past. To turn the wheels of history in the right direction, we need to learn the right lesson from past as well as from our present experiences.

The new research program provides a methodological tool with which to judge and interpret competition as well as any other present and past institution, practice, behavior or value in human societies, with a view toward the ideal conditions of stable general equilibrium. This instrument can be shown as a triangle expressing the interdependence of three basic principles which form a logical entity :

57. Leon Walras, *Elements*, p. 305



If competition is a part or an aspect of human freedoms, then certainly according to this standard it has a natural limit in respecting the other two basic principles. Thus a new condition for a state of general equilibrium arises :

(24) Competition in theory and practice is naturally limited by two other cardinal principles which together harmoniously unite society, economy and government : Social Equity and Solidarity with Peace. This is the true image of fair, workable, balanced (equilibrium) competition. Equilibrium prices, as defined earlier in this paper, satisfy this requirement.

All three principles are interrelated and consistent with each other. Thus they form a logical entity. Each one determines simultaneously a natural limit to the other two. To be able to enjoy the fruits of a new, better social order of tomorrow constructed in accordance with conditions of general stable equilibrium, we need to learn how to live and think in terms of an equilibrium (balanced) concept of competition, human freedoms in general, social equity, human solidarity, and peace. Whatever does not satisfy all three basic principles of the triangle, contains characteristics and problems of disequilibrium.

From these observations we can determine a new condition for a state of general equilibrium, which is purely methodological :

(25) A theory of general equilibrium in a particular field like economics must be structured in such away as to remain open to an organic link with similar theories in related fields. This is particularly true for all branches of the social sciences.

Of course, we have to understand that a general theory can be constructed only within the limits of a given system and not for all possible systems, the latter case belonging to the Impossibility Theorem.⁵⁸⁾

58. Anghel Rugina, American Capitalism, p. 36-37, 226-228

This line of investigation into the basic patterns of a state of stable general equilibrium could be continued and probably new conditions unearthed. For the purpose of this paper, however, what has been done to this point seems to be sufficient to reach a conclusion about the nature and improvement of the Walrasian law of general equilibrium. There is one important question for an independent study, which was omitted for lack of space : «Is the Walrasian system static, dynamic, or both?»

III. THE WALRASIAN LAW COMPLETED AND REFORMULATED

1. Review of what Walras Overlooked

In reviewing the previous discussion of the missing parts and correction of the Walrasian law, we can identify the following major points :

1. Walras correctly saw the necessity of the «numeraire» as a constant magnitude (natural parameter) of a system of stable equilibrium but he did not perceive nor formulate the law of the NaPaNu.
2. Walras did not see, either theoretically or practically, that a force like pure competition cannot act in a vacuum so as to produce and maintain a position of stable equilibrium. He was not aware that fundamentally his system required an adequate institutional framework consistent with the NaPaNu and the force of competition. In other words, he missed formulating the law of consistency.
3. Walras overlooked the compensatory law of real investment, income and employment between the monetary and non-monetary sectors in a system of stable equilibrium. This law is indispensable in order to see how stability-from-within is maintained in practice and how the goal of full employment without government intervention can be achieved.
4. Walras did not consider the universal hypothesis of duality in the physical and socio-economic universe. Thus he was not sufficiently aware of the difference between what may be called «ideal reality» (real life as manifested under conditions of stable equilibrium) and «actual reality,» which in his time was a mixture of equilibrium and disequilibrium conditions, perhaps not as bad as today but still disequilibrium. This goes back to the fact that he did not envision clearly the difference between nature and human societies, specifically that the natural parameter and the adequate framework in the physical universe

are given, whereas in human society, economy and government both have to be discovered, constructed with, and maintained through the best possible means. Walras underestimated the complexity of problems posed by the real world in order for his system of stable equilibrium to be realized in practice.

5. Walras was not aware of the important distinction between numeraire as an equilibrium and anti - numeraire as a disequilibrium form of money. He thus neglected to point out explicitly that a state of general equilibrium requires a 100 percent numeraire-currency system and all anti - numeraire forms of money should be banished by law.

6. Walras did not have an adequate nor complete view regarding the relationship between money-and capital-and capital-markets. Even though he distinguished the liquidity - function vs. the investment - function, nevertheless he did not complete the reasoning leading to the formulation of two sets of interest rates, namely, one for the money-market (short term loans) and the other for the capital market (long term loans). He mingled savings with cash balances.

7. Walras failed to develop an adequate theory of entrepreneurship and normal profits, even though he described the function in question very well. He did not see the difference between normal profits and profit-differentials, the latter having the task to innovate and increase output whenever necessary. His assumption that under conditions of equilibrium, profits equal zero is not only confusing but also not justified once the function of entrepreneurship has been identified as different from the task of the other factors.

8. Walras was not aware that the greatest impediment for the realization of a system of stable equilibrium in his time was modern banking, which then as today represented an immense factory of monetized credit, i.e., of manufacturing and pouring into circulation anti-numeraire or a disequilibrium form of money.

9. Walras also was unaware that in an organized stock-, exchange-, and commodity market two entirely different kinds of transactions take place daily. Natural transactions represent real buying and selling where speculation is limited and always fully covered. This is consistent with conditions of stable equilibrium. The other kind of transactions is just nominal and consists of a wager that the future prices may go up (bullish speculators) or down (bearish speculators). In this case speculation is unlimited and not fully covered. This, however, is inconsistent with equilibrium conditions. Walras considered that all transactions were natural and therefore no problem of disequilibrium existed on the organized stock-, exchange- and commodity markets («La Bourse» in French).

10. Walras did not have an adequate solution to unite the theory of the private sector with the theory of the public sector even though logically this was required by a truly general law of equilibrium. He kept the two sectors separate so that his law of general equilibrium actually pertains only to the private sector of the economy.
11. As a result of 10, in a way Walras was forced to split the concept of social justice in two-justice of equity for the private sector and justice of equality for the public sector. He did not realize that by so doing an anomaly was created in his system of stable equilibrium.
12. Walras did not perceive that competition in itself was just an economic aspect of the larger principle of human freedoms (social, economic, political) which, in conjunction with two other basic principles (social equity and human solidarity with peace) can unite all social sciences by creating a consistent social universe where the Walrasian system with new additions and corrections can be perfectly integrated.

The application of the new research program was instrumental in unravelling the missing parts and realizing that the law of general equilibrium, both in theory and action, is much more complicated than envisioned by Walras. Nonetheless, when next we will put all the pieces together, it will become evident that the constructed whole fits in very well with the foundation provided by Walras. That is why Walras' contribution is so great and Schumpeter was so right in his evaluation.

2. The Walrasian Law Completed and Reformulated

The Walrasian law in its complete form follows. Given :

- (1) a system of free markets ruled by competition alone ;
- (2) that all prices for products and services of the factors and implicitly all incomes including foreign exchange are expressed in terms of a numeraire- currency which serves as the natural parameter of the system ; and
- (3) that the law of consistency and the compensatory law are satisfied by a joint complex of an institutional framework especially tailored to support (1) and (2) to the maximum ;

then we have fulfilled the first line of conditions for a state of general equilibrium which is characterized by :

- (4) The effective demand is equal to the effective supply ;
- (5) For each product and service there is one price in the market ;
- (6) The greatest possible satisfaction of consumers' wants or maximum utility to both consumers and producers is achieved ; and
- (7) The selling price for each product and service is equal to the marginal utility, marginal cost, marginal revenue, average cost.

The complex institutional framework capable of supporting the realization and maintenance of stable equilibrium at the point of full employment, requires a second line of conditions as follows :

- (8) Capital formation = voluntary savings, which means there is no «forced saving» in the system due to the inflationary effect or other means ; in other words, the prevailing rule is : First Saving then Investment ;
- (9) Banks are converted into intermediary institutions between «savers» and «investors» and by law are prohibited to monetize credit in the form of bank notes, bank deposits or any other form ; all other financial institutions (private or public) must conduct their business under this same rule.
- (10) Credit in the economy is free and real but not monetized ; its natural limit is determined by the aggregate volume of real income ;
- (11) The aggregate of new investments = the aggregate of voluntary savings;
- (12) The money market is separate from the capital markets and each one has its own interest rate, one for liquidity, the other for investment ;
- (13) Money (numeraire) capital = real capital (in form of capital goods expressed in terms of numeraire) ; in general, any form of money- income (rent, wages, interest, profit and taxes) = real income ;
- (14) The function of entrepreneurship is recognized as a fourth factor of production with its own income-normal profit, which is to be added to the other cost elements.

The equilibrium price therefore— the least average cost where normal Sprofits are included ;

- (15) A small percentage of a profit - differential is allowed over and above the

normal profit as an incentive for innovation and as a reward for producing more, whenever necessary ;

- (16) Traders in the organized stock-, exchange - and commodity markets are required by law to limit their business to real transactions, that is, outstanding buying and selling always fully covered. Nominal transactions or pure speculation (formal betting on future prices) is prohibited by law as being harmful to society and economy as a whole ;
- (17) The Central Bank of the country is entrusted to create and put into circulation only numeraire - type of currency ; all forms of anti - numeraire money issued by the Central Bank or government are prohibited by law ;
- (18) The business on the foreign exchange market follows similar patterns :
 - foreign exchange is fully backed and freely convertible in a numeraire - currency ;
 - foreign exchange is traded in open markets where pure competition prevails and monopoly is excluded ,
 - free import and export of numeraire - currency : then
 - the effective demand for foreign exchange will tend to be = the effective supply of foreign exchange, and
 - the foreign exchange rate will be = the official parity ;
- (19) Government business and its finances are conducted according to the rule : First income and then expenditure. It means that no public expenditure can be undertaken without a prior adequate public income derived from taxes or borrowing. This is the principle of budgetary stability.
- (20) No taxation is imposed upon the people without a direct consultation and the consent of the majority through a referendum on a regional basis. This is the principle of social equity and democracy in taxation.
- (21) The state of equilibrium in public finance is reached when :
$$\frac{\text{The marginal utility of \$ 1.00 spent in the private sector}}{\text{The marginal utility of \$ 1.00 spent in the public sector}} = \frac{\text{The marginal utility of \$ 1.00 spent in the private sector}}{\text{The marginal utility of \$ 1.00 spent in the public sector}} ;$$
- (22) A careful inspection of all major institutions and practices in the system confirms that all are consistent with the three basic principles of the triangle : Human Freedoms, Social Justice of Equity, and Human Solidarity with Peace ;

- (23) All inconsistent elements and practices are eliminated or neutralized to the point where they are no longer an impediment in the normal functioning of the system ;
- (24) In a law of social and economic justice it is made clear that fair competition as a part of human freedoms has natural limits with respect for the other two basic principles of social equity and human solidarity with peace.
- (25) Finally, a law of general equilibrium in a given study like economics must have, from the methodological point of view, a kind of built-in open bridge that, when necessary, can be linked to a similar theory in related fields.

When all these 18 requirements of a consistent institutional framework plus the initial seven are fulfilled, we can say that it is possible to attain and maintain a state of general stable equilibrium, both in theory and practice. Without such an explicit assumption-a vast complex of institutional machinery - the Walrasian system and the law of general equilibrium will remain an imaginary, unfinished product of the mind, a vision without a chance ever to be realized in practice.

Above all, the time of Walras is not over but coming. There seems to be a cyclical movement in the history of modern economic thought. The 17th and the first half of the 18th centuries were dominated by thinking in terms of disequilibrium conditions, at the centre being the mercantilist doctrine. During the second half of the 18th and the 19th centuries a shift took place in the direction of thinking in terms of equilibrium conditions and this was the classical school. In the first half of this century, attention appears to have been concentrated more on conditions of disequilibrium. Indeed the main contribution of Lord Keynes and his disciples was to establish a more perfect study of «Economics of Disequilibrium» or «Economics of Uncertainty & Relativity», even though this goal was always clearly stated.

We now stand on the threshold of a new era in economic thinking and hear of «New Times and New Economists». The position of the new economists, all in the 30's and early 40's, so far is critical of prevailing economics and hints at possible new roads of analysis, without having brought out yet a specifically defined method of analysis. But according to the above mentioned cyclical movement in the history of modern economic thought, next in line is a phase of thinking in terms of a model of more perfect equilibrium conditions, which we desperately need now more than ever. This may be called the «Economics of Stable Equilibrium» or the «Economics of Certainty», which this author has advanced for more than twenty five years. And that is also what Walras stood for all his life.