THE NET PRESENT VALUE METHOD UNDER CONDITIONS OF ANTICIPATED INFLATION

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ABSTRACT

This paper discusses the conventional formula for the discounted cash flow estimation of net present value (NPV) under conditions of inflation, and describes and analyses the required modification to the conventional formula.

Inflation will undoubtedly remain with us for a considerable period of time and therefore it would be advisable to allow for its effects when evaluating investment projects.

In order to keep our analysis as simple as possible we shall assume a riskless and taxless world dominated by perfect capital markets. Under conditions of constant prices the net present value (NPV) of a project X may be written as

$$NPV = \sum_{t=1}^{\pi} \frac{X_t}{(1+r)^t} - X_0 \qquad (1)$$

where X_t stands for the net cash flow in year t
r is the market discount rate and
Xo is the capital outlay for the year zero.

For a revenue producing project Xt is the difference between sales receipts and payments for operating costs. Let us assume that the physical amount to be sold every year (beginning with year 1) for η years is equal to Q and that labour hours expected to be used up every year for η years are equal to C. In the absence of inflation the selling price in any year (t) will be equal to P_0 the current year (year 0) market price of the product. Also, the wage rate per hour in year t will be equal to W_0 the current wage rate. The net present value equation may now be written as

$$NPV = \sum_{t=1}^{n} \frac{Q_t P_t - C_t W_t}{(1+r)^t} - X_0 \tag{2}$$

But since $P_t = P_0$ and $W_t = W_0$, equation (2) can be written as

$$NPV = \sum_{t=1}^{n} \frac{Q_t P_0 - C_t W_0}{(1+r)^t} - X_0 \tag{3}$$

According to equation (3) net present value is based on cash flows calculated by using current prices and costs.

Jet us now assume that inflation sets in. With rising selling prices and wage costs the project's future cash flows (both inflows and outflows) will be adjusted upwards. The resultant cash flows will be called nominal or money cash flows. By nominal cash flows in year t we will mean the actual (observable) cash flows calculated by using prices and wage rates prevailing in that year. Now the selling price in year t will be equal to

$$Po \quad (1+p)^t$$

and the wage rate in the same year will be equal to where (p') is the rate by which P_0 will incëease every year and w is the rate by which W_0 will increase every year. The expressions within brackets above are also known as specific selling price and wage cost indices. It follows therefore that to find our actual sales receipts in year t we must multiply the number of units sold by the selling price prevailing in that year, that is,

$$Po(1+P')^t$$

An analogous procedure will determine the actual cash operating costs in year t.

A similar adjustment must be made to the denominator of the present value equation. To understand the need for adjustment recall that an interest rate (discount rate) tells us how much better off we will be in terms of per cent per period if we invest a certain amount of money in the market. In the absence of inflation if the interest rate is equal to r per annum, then one pound invested today, t_0, for one year will grow to (1+r) received in t, has the same purchasing power as (1+r) received in t_0. When inflation sets in our idealised capital market, investors will try to maintain the purchasing power of their investments. In our case the appropriate adjustment will be achieved if and only if the t_x value of 1 invested in t under conditions of inflation has the same purchasing power as the t_4 value of 1 invested in t' under conditions of constant prices. We know that when prices are expected to remain constant over time investors want to be r (100) per cent (in physical terms) better off in t_x than they were in t_0. Uuder conditions of inflation this physical relationship will be maintained if our investors (lenders) receive in t_t a sum of money equal to $(1+r)$ plus $(1+r)\rho$ or $(1+r)(1+p)$, where ρ is the rate by which prices are expected to increase in the year. Thus the denominator of our net present value equation should now be $(1+r)^t\ (1+p)^t$.

Note that (1+p) may be thought of as the consumer price index for year 1 expressed as a ratio. A price index is a summary statistic which endeavours to measure changes in the cost of a basket of goods and services over time. Thus, if the consumer price index for year 1 is expected to be (1.1), with the present being our base year, the expression (1.1) will signify that a given basket of consumer goods will cost 10 per cent more in t_x than it cost in the base year. We can construct indices for labour costs, raw materials costs, selling prices, etc.

In this paper, future selling price and wage cost indices will be used in order to determine, respectively, the future nominal sales receipts and payments for operating costs. The consumer price index will be used for different purposes. To begin with we will use it in order to determine the nominal (actual) interest rate that will prevail in perfect capital markets when inflation is expected to occur at ρ per cent per year. Now we know that when inflation is expected to occur at ρ per cent, the t, value of 1 invested today will be equal to (1+r) (1+p). Let this expression be equal to (1+i)· Hence the following relationship will hold.

$$(1+r)(1+p)=(1+i) \tag{4}$$

where i is the nominal rate of interest which will prevail in our market when everybody expects inflation to occur at ρ per cent per annum. From (4) i can be expressed in terms of r and p, that is,

$$i = r+p+rp \tag{5}$$

If on the other hand ρ and i are known and we wish to determine the real rate of interest, then (4) should be solved for r, that is,

$$r = \frac{1 - \rho}{(1+P)} \tag{6}$$

Note that (6) is a very useful relationship which can be used to determine the real performance of any investment.

A consumer price index can also be used to determine the real value of a future cash flow. It should be noted that the consumer price index is used to determine real values or nominal values having the same purchasing power as a given present cashflow, in order to keep in line with one of the major assumptions made by financial analysts ; namely that investors strive to maximise the utility from their consumption over time. The real value of a (nominal) cash flow received in year t refers to the amount of base year pounds that could buy the same amount of physical goods in to the base year as the cashflow received in year r. It follows therefore that to calculate the real value of a future cash flow we must (a) specify our base year, and (b) construct the appropriate consumer price index for the future year. We then divide the future cash flow Xt by the price inhex for the same year, that is

$$\frac{Xt}{\quad} = Po \tag{7}$$

where P_0 indicates the present (base year) amount of money that has the same purchasing power as X received in year t.

We have therefore seen that under conditions of fully anticipated inflation both our future cash flows and discount rate must be adjusted upwards to allow for the effect of rising prices, cost, and a more expensive basket of consumer goods and services. We now have the necessary background knowledge to proceed with finding the net present value of our project under conditions of inflation. The first and obvious way would be to use (actual) or nominal future cash flows and an actual or nominal discount rate. Algebraically this can be written as :

$$NPV = \sum_{t=1}^{n} \frac{Q_t P_o \ (1+p')^t - C_t W_o \ (1+w)^t}{(1+i)^t} - X_0 \qquad (8)$$

where all terms have been defined before. Alternatively, one may wish to use real cash flows and a real interest rate., that is :

$$NPV = \sum_{t=i}^{n} \frac{[Q_t P_o(1+p')^t - C_t W_o(1+w)^t] + (1+p_t)^t}{(1+i)^t + (1+p)^t} - X_o \qquad (9)$$

It is immediately obvious that equations (8) and (9) yield identical results. Algebraically this is quite obvious since to obtain equation (9) we divide the numerator and denominator of the first part of the equation (8) by the same price index. Economically our results are not surprising since present value, however calculated, is at the same time both nominal and real. We can explain this as follows. By present value we mean the minimum sum of money the company will accept now, a nominal concept, in order to part with the project. Recall that by real present value we simply mean nominal present value divided by the consumer price index for year zero. But since the value of the price index for year zero is, by definition, equal to 1, then value, however calculated, must at the same time be both nominal and real. There is no need therefore to distinguish between real and nominal present value or net present value.

One may well wonder whether net present value based on cash flows calculated by using current prices and costs would yield the same result as that expected from either equation (8) or (9). The answer depends on whether inflation is uniform or differential. Uniform inflation is said to exist when prices of all goods and services and wages increase uniformly over time. On the other hand, when prices and wages increase at different rates over time, inflation is said to be differential. When inflation is uniform equation (8) reduces to

$$NPV = \sum_{t=1}^{n} \frac{(Q_t P_o - C_t W_o)(1+p)^t}{(1+r)^t (1+p)^t} - X_o$$

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$$NPV = \sum_{t=1}^{n} \frac{Q_t P_o - C_t W_o}{(1+r)^t} - X_o \tag{10}$$

Equation (10) follows from the fact that by definition, $p = w = p'$. Hence, given our assumptions if prices and wages throughout the economy are expected to change with uniformity over time, then the net present value calculations based on current prices and costs and a real discount rate (properly calculated) will yield the same results as net present value based on nomial parameters. However, when inflation is differential, that is, when $p \quad w \quad p'$, using current prices and costs would not yield the same result as a net present value calculation based on either nominal or real parameter.

A simple example may clarify the main issues discussed in this paper. Let us consider a project, the cash flows of which were calculated on the assumption of nil rates of cost and price inflation. In addition we shall assume a nil rate of inflation for all goods and services. The cashflows are shown in Table 1.

If in the absence of inflation the relevant discount rate shareholders require is equal to 5 per cent per annum, the net present value of the project will be equal to

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$$NPV = \frac{£5{,}200}{(1.05} + \frac{£6{,}600}{(1.05)^2} + \frac{£4{,}200}{(1.05)^3} - £12{,}000 = £2{,}566.$$

TABLE 1

NET CASH FLOWS WITH A NIL RATE OF INFLATION

	End of year			
	0	1	2	3
Cash receipts (£)	—	10,000	12,000	8,000
Cash payments (£)				
Capital outlay	(12,000)	—	—	—
Labour	—	(1,500)	(1,600)	(1,000)
Materials	—	(3,000)	(3,500)	(2,500)
Overheads	—	(300)	(300)	(300)
Total	(12,000)	(4,800)	(5,400)	(3,800)
Net cash flows	(12,000)	5,200	6,600	4,200

Let us now assume that uniform inflation sets in. Specifically let us assume that all prices in the economy are expected to increase at 7 per cent per annum. Now since by definition the investors' rate of inflation coincides with the company's rate of cost and price inflation ; the NPV will remain unchanged. In Table 2 the cash flows are inflated by 7 per cent per annum and the interest rate is increased from 5 per cent to 12.35 per cent, by application of equation (5) : $i — r+p+rp$.

Table 2 shows quite clearly why the net present value of the project will be the same irrespective of whether our cash flows are calculated by using (a) current prices and costs, or (b) prices and costs prevailing in years 1,2 and 3. We would have obtained the same result had we deflated our nominal cash flows and used a real discount rate.

By contrast, let us now turn to differential rates of inflation of company costs

TABLE 2

NET PRESENT VALUE WITH A 7 PER CENT RATE OF UNIFORM INFLATION

End of year	(a) Cash flow	(b) Discount factor	NPV in £ = (a)×(b)
0	(12,000)	1	(12,000)
1	5,200 (1.07)	$\frac{1}{(1.1235)} = \frac{1}{(1.05)} \times \frac{1}{(1.07)}$	= 4,952
2	6,600 $(1.07)^2$	$\frac{1}{(1.1235)^2} = \frac{1}{(1.05)^2} \times \frac{1}{(1.07)^2}$	= 5,986
3	4,200 $(1.07)^3$	$\frac{1}{(1.1235)^3} = \frac{1}{(1.05)^3} \times \frac{1}{(1.07)^3}$	= 3,628
Total			£2,566

and prices different from the shareholders' rate of inflation. Consider for instance, sales prices growing at 5 per cent per aunnum, labour costs at 8 per cent per annum, materials at 10 per cent per annum, overheads at 6 per cent per annum, and a 7 per cent per annum inflation for shareholders' personal expenditur. The nominal cash flows of the company are shown in Table 3.

In Table 4 we derive the NPV by discounting the nominal cash flow at the nominal discount rate, that is, 12.35 per cent.

Alternatively, the same result could have been obtained (subject to rounding errors) if we had discounted the real cashflows at the real rate of interest (discounted rate) as shown in Table 5.

TABLE 3

NOMINAL CASH FLOWS

Source/use of cash	Cash flow at pre-inflation prices and costs (£)		Inflation adjustment			Nominal cash flow
Sales (year 1)	10,000	×	(1.05)	=		10,500
Labour (year 1)	(1,500)	×	(1.08)	=	(1,620	
Materials (year 1)	(3,000)	×	(1.10)	=	(3,300)	
Overheads (year 1)	(300)	×	(1.06)	=	(318)	
Total outflow (year 1)						(5,238)
Net cash flow in nominal terms year (1)						5.262
Sales (year 2)	12,000	×	$(1.05)^2$	=		13,230
Labour (year 2)	(1,600)	×	$(1.08)^2$	=	(1,866)	
Materials (year 2)	(3,500)	×	$(1.10)^2$	=	(4,235)	
Overheads year 2)	(300)	×	$(1.06)^2$	=	(337)	
Total outflow (year 2)						(6,438)
Net cash flow in nominal terms (year 2)						6,792
Sales (year 3)	8,000	×	$(1.05)^3$	=		9,261
Labour (year 3)	(1,000)	×	$(1.08)^3$	=	(1,260)	
Materials (year 3)	(2,500)	×	$(1.10)^3$	=	(3,328)	
Overheads (year 3)	(300)	×	$(1.06)^3$	=	(357)	
Total outflow (year 3)						4,945
Net cahs flow in nominal terms (year 3)						4,316

TABLE 4
NET PRESENT VALUE ADJUSTING FOR INFLATION

Year	Nominal cash flow		Nominal discount factor at 12.35%		NPV in £
0	(12,000)	×	1	=	(12,000)
1	5,262	×	$\frac{1}{(1.1235)}$	=	4,684
2	6,792	×	$\frac{1}{(1.1235)^2}$	=	5,381
3	4,316	×	$\frac{1}{(1.1235)^3}$	=	3,043
Total					1,108

TABLE 4

NET PRESENT OALUE AOJUSTING FOR INFLATION : ALTERNATIVE METHOD

Year	Nominal cash flow	Discount factor for 7% rate of inflation		Real cash flow	Discount factor for 5% real rate of interest		NPV in £
0	(12,000 ×	1	=	(12,000) ×	1	=	(12,000)
1	5,262 ×	$\frac{1}{(1,07)}$	=	4,918 ×	$\frac{1}{(1.05)}$	=	4,684
2	6,792 ×	$\frac{1}{(1.07)^2}$	=	5,932 ×	$\frac{1}{(1.05)^2}$	=	5,381
3	4,316 ×	$\frac{1}{(1.07)^3}$	=	3,523 ×	$\frac{1}{(1.05)^3}$	=	3,043
Total							1,108

CONCLUSIONS

From our simplified analysis we can derive the following conclusions :

1. Consistency entails that both the numerator and denominator of the present value equation are cast either in nominal or real terms. One's results will be biased if the numerator is expressed in nominal terms whereas the denominator is expressed in real terms.

2. If the company or investor establish their discount rates by reference to market rates then, providing the market is reasonably perfect and free from government interventions, it is a matter of indifference whether one uses real or nominal parameters.

3. Since inflation is bound to be differential and market rates will embody an element for inflation, using current prices and costs to determine the project's net cash flows will yield inaccurate results.

4. If the investor or company believe that the market is significantly imperfect, optimal results can only be obtained by using a personal nominal or real discount rate.

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