LEAST-COST FEED RATIONS FOR SHEEP BY LINEAR PROGRAMMING*

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Mathematically, there are two main procedures to examine potential adjustments for improving the present economic, efficiency of the sheep as a single enterprise: (a) by marginal analysis of the productivity of resources used, and (b) by least-cost feed rations. This paper concerns the second one. In fact, an attempt is made here to specify the optimum combination of various foodstuffs for sheep by using the linear programming technique, which minimises the cost of the ration.

The data have been derived from a representative sample of 70 sheep farms in the Epirus region of Greece, in three areas of that region: the plain, semi-mountainous and mountainous, with 23, 24, 23 farms respectively.

Introduction

It has been found (Zioganas, 1981) that feed costs represent nearly 30% of the total costs of production of sheep. Therefore, least-cost feed rations, i.e. the optimum combination of various food stuffs, which are available in the area, are of major importance from the economic as well as biological and nutritive point of view. There are various methods by which this question can be dealt with, but linear programming is considered to be the most successful and exact method (Waugh, 1951).

The linear programming model here is to minimise:

$Z = \sum_{j=1}^{n} c_j x_j$, subject to the constraints:

$b_i \geq$ or $=$ or $\leq \sum_{j=1}^{n} a_{ij} x_j \; (i = 1, 2, \ldots, m)$,

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where Z = feeding costs for a certain level of production,
Cj = market price per unit (drs/kg) of foodstuff j,
Xj = no. of units (kg) of foodstuff j,
bj = constraints, i.e. maximum or equal or minimum amount of nutrient components i required (i = dry matter, digestible protein, starch equivalent),
ay = amount of nutritive components i contained in one unit of foodstuff j.

Very often some variability exists in the various nutritive components of foodstuffs (Rahman and Bender, 1971). But here adequate knowledge of nutritive components of foodstuffs, stability in their quality and standard physiological attributes of sheep are assumed. When changes of prices occur, the least-cost ration should be recomputed, since the optimum combination of feeds is likely to be altered(Dent and Casey, 1967, p. 10).

In the gerion of Epirus, sheep are grazing on the pastures for about seven months of the year, and are fed on hay and concentrates for about five months. It is thus difficult to calculate a feeding ration which will partly cover the needs for maintenance and production (mainly milk), since the proportion of the needs covered by grazing is not known. To overcome this problem, the annual needs per ewe in nutritive components are calculated. These calculations are based on the daily requirements of one ewe of 50 kg l.w. producing one litre of milk containing 6% fat, as shown in Table 1. Next the rations of foodstuffs consumed by the sheep are expressed in nutritive components, as shown in Tables 2, 3, 4 (the amounts are based on the survey data). The difference between the total needs and those supplied by these rations are considered to be nutrients covered by grazing. Finally, Table 5 contains the relative figures expressed per ewe per annum. For the present analysis these figures are assumed to be constant (Kitsopanidis et al, 1980).

TABLE 1 Daily requirements in nutritive components of 1 ewe of 50 kg l.w. producing 1 litre of milk with 6% fat.

Coverage of needs for	Dry matter (kg)	Starch equivalent (units)	Digestible protein (kg)
Maintenance (50 kg l.w.)	1.0 - 1.5	400	0.050
Milk production (1 litre, 6% fat)	0.5 - 0.6	375	0.080
Total	1.5 - 2.1	775	0.130

Source: P. Kalaisakis: «Applied Animal Nutrition», Athens, 1965.

TABLE 2 Average annual per ewe nutritive components supplied by complementary feeding rations in the plain area.

Foodstuffs	Amount (kg)	Nutritive components		
		Dry matter (kg)	Starch equivalent (units)	Digestible protein (kg)
Maize	48.23	41.96	37619	3.09
Barley	16.71	14.37	11513	1.25
Forage hay	15.58	13.71	4908	0.76
Lucerne hay	110.40	97.15	35770	14.90
Total	190.92	167.19	89810	20.00

TABLE 3 Average annual per ewe nutritive components supplied by complementary feeding rations in the semi-mountainous area.

Foodstuffs	Amount (kg)	Nutritive components		
		Dry matter (kg)	Starch equivalent (units)	Digestible protein (kg)
Maize	44.70	38.89	34866	2.86
Barley	20.76	17.85	14304	1.56
Forage hay	22.14	19.48	6974	1.08
Lucerne hay	101.64	89.44	32931	13.72
Total	189.24	165.66	89075	19.22

TABLE 4 Average annual per ewe nutritive components supplied by complementary feeding rations in the mountainous area.

Foodstuffs	Amount (kg)	Nutritive components		
		Dry matter (kg)	Starch equivalent (units)	Digestible protein (kg)
Maize	33.11	28.81	25826	2.12
Barley	32.22	27.71	22200	2.42
Forage hay	26.12	22.99	8228	1.28
Lucerne hay	93.51	82.29	30297	12.62
Total	184.96	161.80	86551	18.44

TABLE 5 Average annual per ewe[1] requirements of nutritive components and proportions covered by complementary feeding rations and by grazing.

Area	Requirements	Dry matter		Starch equivalent		Digestible protein	
		Kg	%	units	%	Kg	%
Plain	Total[2]	522.25	100.00	191000	100.00	27.85	100.00
	Feeding rations[3]	167.19	32.01	89810	47.02	20.00	71.81
	Grazing	355.06	67.99	101190	52.98	7.85	28.19
Semi-mountainous	Total[2]	515.10	100.00	186125	100.00	26.81	100.00
	Feeding rations[3]	165.66	32.16	89075	47.86	19.22	71.69
	Grazing	394.44	67.84	97050	52.14	7.59	28.31
Mountainous	Total[2]	506.85	100.00	180500	100.00	25.61	100.00
	Feeding rations[3]	161.80	31.92	86551	47.95	18.44	72.00
	Grazing	345.05	68.08	93949	52.05	7.17	28.00

(1) Average liveweight:-50 kg.
(2) On the basis of Table 1, taking into acount that milk yield per ewe is:-
(i) plain area:- 92 litres (milked) + 28 litres (taken by lambs) = 120 litres in total;
(ii) semi-mountainous area:- 79 litres (milked) + 28 litres (taken by lambs) = 107 litres in total;
(iii) mountainous area:- 64 litres (milked) + 28 litres (taken by lambs) = 92 litres in total.
(3) These figures are taken from Tables 2, 3, 4.

The linear programming matrix

Table 6 presents the least-cost ration matrix for one ewe per year in the plain area. The sheep are in fact fed this ration of hay and concentrates for about 150 days a year, in winter. The matrices for the other two areas are the same except for minor changes in the levels of some of the constraints (See Appendix: Explanations of least-cost ration matrix).

The activities include ten foodstuffs available for use in the area. The prices are the average purchase prices found from the analysis of the sample of farms for the year 1979.

Those constraints which refer to the requirements per ewe (for 150 days) are based on the complementary ration (Table 5). For the other constraints explanations are given in Appendix (Explanations of least-cost ration matrix).

The minerals and vitamins required by sheep were not included in the matrix, partly because insufficient data were available and partly because they are usually supplied as 1% of the total ration and thus their cost is almost negligible compared with the total cost of the whole ration.

The nutritive components px(dry matter, digestible protein, starch equivalent) contained per kg in each of the foodstuffs (often referred to as the technical coefficients) have been taken from available nutrient tables (Kalaisakis, 1965).

Comparative results of present and optimum rations

The results are presented in Table 7 for the three areas separately. Both 1979 and 1980 prices were applied, all the other information in the matrix being kept the same for both years. In each area the optimum ration comprised exactly the same combination of foodstuffs in both cases (i.e. 1979 and 1980) despite the fairly substantial changes in relative prices between the two years.

The major changes proposed by the optimum rations are: barley is not selected; bran B is introduced at the maximum level allowed; forage hay is significantly increased; and maize and lucerne hay are substantially decreased. As can be seen in Table 7, the present ration does not contain any bran B, whereas the optimum ration contains the maximum possible amount. In fact sheep farmers in general have not used bran B for feeding sheep for some years now. This is because they are supplied maize and barley readily by the State, without any problems, whereas sometimes it is difficult to buy bran B or other concentrates from the open market.

As a result of these changes the cost of the optimum ration is reduced by 5.86%, 5.24% and 4.52% in the plain, semimountainous and mountainous areas respectively, for the year 1979. The corresponding figures for 1980 are 4.90%, 5.15 and 6.27%.

TABLE 6 Least-cost ration matrix in the plain area for 1 ewe (1979 prices)

		1	2	3	4	5	6	7	8	9	10
	Activities:	MAIZE	BARLEY	BRANA	BRANB	SOYABEAN	COTCAKE	SUGCAKE	LUCHAY	FORHAY	STRAW
		KG	KG	KG	KG	KG	KG	KG	KG	KG	KG
	Net revenues[1]:	5.52	5.52	4.00	4.30	14.00	6.70	5.00	4.50	3.00	2.50
	Constraints										
1. DMMAX	GR 195055⩾	870	860	862	881	865	889	901	880	880	899
2. DMMAIBAR	GR 97950⩾	870	860	0	0	0	0	0	0	0	0
3. DMBRABRD	GR 79350⩾	0	0	862	881	0	0	0	0	0	0
4. DMSOYCOT	GR 69300⩾	0	0	0	0	865	889	0	0	0	0
5. DMSUGCAK	GR 33750⩾	0	0	0	0	0	0	901	0	0	0
6 DMLUCFOR	*GR* 122600⩾	0	0	0	0	0	0	0	880	880	0
7 DMSTRAW	GR 101100⩾	0	0	0	0	0	0	0	0	0	899
8. STAREQUI	UNIT 89810=	780	689	364	455	695	429	505	324	315	116
9. DIGPROT	GR 20000=	64	75	88	110	427	172	43	135	49	3
10. DMMIN	GR 167190⩽	870	860	862	881	865	889	901	880	880	899
11. DMLUFOST	GR 75000⩽	0	0	0	0	0	0	0	880	880	899

[1] i.e. *prices* in drs per Kg; drs=drachmas, where: 1 drachma is equivalent to approximately 0.83 pence, or £1 is equivalent to approximately 120 drachmas.

TABLE 7 Comparative results of present and optimum rations

Foodstuffs and cost	Homogeneous area					
	Plain		Semi-mountainous		Mountainous	
	Present ration	Optimum ration	Present ration	Optimum ration	Present ration	Optimum ration
A. **Foodstuffs (kg)**						
Maize (grain)	48.23	27.63	44.70	26.78	33.11	23.62
Barley (grain)	16.71	–	20.76	–	32.22	–
Forage hay	15.58	36.99	22.14	45.43	26.12	52.15
Lucerne hay	110.40	48.24	101.64	39.79	93.51	33.08
Bran B	–	90.07	–	90.07	–	90.07
B. **Cost (drs/ewe)**						
(a) 1979 prices	922	868	897	850	862	823
(b) 1980 prices	1101	1047	1088	1032	1068	1001

Stability of the optimum rations

Table 8 shows the price range of each foodstuff over which the amount selected remains constant in the optimum ration, provided the prices of the other foodstuffs do not change. These results are exactly identical for the three areas both for 1979 and 1980 prices.

Table 9 shows to what level the price of each foodstuff not selected would need to be reduced in order to be selected in the optimum ration, provided the prices of all the other foodstuffs remained the same. These results were also identical in the three areas, both for 1979 and 1980 prices.

TABLE 8 Price range (drs/kg) over which each foodstuff stays in optimum plan at selected level.

Foodstuffs	Year 1979			Year 1980		
	Lower limit	Present	Upper limit	Lower limit	Present	Upper limit
Maize	5.27	5.52	5.90	7.23	7.50	7.70
Bran B	0	4.30	4.44	0	5.00	5.15
Forage hay	2.70	3.00	3.17	3.43	4.00	4.18
Lucerne hay	4.33	4.50	5.16	4.82	5.00	6.00

TABLE 9 Prices (drs/kg) of foodstuffs not in optimum plan to be selected.

Foodstuffs	Year 1979		Year 1980	
	Present price	Price needed for selection	Present price	Price needed for selection
Barley	5.52	5.19	7.50	6.82
Soya beans	14.00	11.15	15.00	10.65
Cottonseed cake	6.70	5.34	7.80	5.59
Sugar beet cake	5.00	3.60	5.00	4.87
Bran A	4.00	3.41	4.80	3.97
Straw	2.50	1.10	2.50	1.78

Conclusions

The linear programming method adequately solves the problem of specifying the minimum-cost feed rations for sheep.

The major changes proposed by the optimum rations are: on the one hand, barley is not selected and maize and lucerne hay are substantially decreased; on the other hand, bran B is introduced at the maximum level allowed and forage hay is significantly increased.

As a result of these changes the cost of the optimum ration is reduced by 5.4% on the average for all areas of Epirus.

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APPENDIX

Explanations of least-cost ration matrix

(i) List of activities (foodstuffs)

Column no.	Identification	Description
1.	MAIZE	Maize (grain)
2.	BARLEY	Barley (grain)
3.	BRANA	Bran A (Standard quality)
4.	BRANB	Bran B (higher quality)
5.	SOYABEAN	Soya beans
6.	COTCAKE	Cottonseed cake
7.	SUGCAKE	Sugar beet cake
8.	LUCHAY	Lucerne hay
9.	FORHAY	Forage hay
10.	STRAW	Straw (from cereals)

(ii) List of constraints

Row No.	Identification	Description
1.	DMMAX	Dry matter at maximum
2.	DMMAIBAR	Dry matter of maize and/or barley
3.	DNNBRABRD	dry matter of Bran A and/or Bran B
4.	DMSOUCOT	Dry matter of Soya bean and/or cottonseed cake
5.	DMSUGCAK	Dry matter of Sugar beet cake
6.	DMLUCFOR	Dry matter of Lucerne hay and/or forage hay
7.	DMSTRAW	Dry matter of straw
8.	STAREQUI	Starch equivalent
9.	DIGPROT	Digestible protein
10.	DMMIN	Dry matter at minimum
11.	DMLUFOST	Dry matter of Lucerne hay and/or Forage hay and/or Straw

(iii) Determination of levels of constraints

1. **DMMAX GR 195055** >: 167190 (rowlO) χ 2.1 (Table l)-=-1.8 (same Table).
2. **DMMAIBAR GR 97950** Daily maximum maize and/or barley 0.75 kg; on the

basis of maize which has more dry matter than barley, the daily maximum dry matter allowed is 0.75 Kg χ 870 gr/kg of maize = 653 gr; then 653 gr χ 150 days (average) = 97950 gr.

3. **DMBRABRB GR 79350** >: Daily maximum bran A and/or bran B 0.6 kg; on the basis of bran B with the most dry matter the daily maximum dry matter is 0.6 kg χ 881 gr/kg of bran B = 529 gr; then 529 gr χ 150 days = 79350 gr.

4. **DMSOYCOT GR 69300** >: Daily maximum soya beans and/or cottonseed cake 0.5 kg; on the basis of cottonseed cake with the most dry matter the daily maximum dry matter is 0.52 kg χ 889 gr/kg of cottonseed cake = 462 gr; then 462 gr χ 150 days = 69300 gr.

5. **DMSUGCAK GR 33750** >: Daily maximum sugar beet cake 0.25 kg; 0.25 kg χ 901 gr (dry matter) = 225 gr; then 225 gr χ 150 days - 33750 gr.

6. **DMLUCFOR GR 122600** >: Daily maximum lucerne hay and/or forage hay 1.5 kg; 1.5 kg χ 880 gr (dry matter, for both foodstuffs is the same) = 1,320 gr; then 1,320 gr χ 167190 gr (dry matter, in row 10)÷ 1,800 gr (average daily total requirements in dry matter) = 122600 gr.

7. **DMSTRAW GR 101100** >: Daily maximum straw 0.75 kg; 0.75 kg χ 899 gr (dry matter) = 674 gr; then 674 gr χ 150 days = 101100 gr.

8. **STAREQUI UNIT 89810** =: The starch equivalent is taken from Table 5.

9. **DIGPROT GR 20000** =: The digestible protein is also taken from Table 5.

10. **DMMIN GR 167190** <: As above.

11. **DMLUFOST GR 75000** < : Daily minimum lucerne hay and/or forage hay and/or straw = 500 gr in dry matter; then 500 gr χ 150 days = 75000 gr

(iv) Changes in levels of constraints for the other areas

The matrices for the semi-mountainous and mountainous area are almost identical to the matrix for the plain area. The only differences are in some of the constraints, because of different milk yields, as follows:

Row No.	Semi-mountainous	Mountainous
1	193270	188767
6	121480	118650
8	89075	86551
9	19220	18440
10	165660	161800

(v) Prices for the year 1980

Foodstuff	drs/kg
Maize	7.50
Barley	7.50
Bran A	4.80
Bran B	5.00
Soya beans	15.00
Cottonseed cake	7.80
Sugar beet cake	5.00
Lucerne hay	5.00
Forage hay	4.00
Straw	2.50