STOCHASTIC CONTROL OF POPULATION DISTRIBUTIONS IN GREECE

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Summary

This paper is concerned with the stochastic control of population distributed on 9 geographic regions in Greece. Particularly, having the transition matrix between urban, semi-urban and rural pupulation and the corresponding growth operator matrix in each geographic region, we determine the maintainable and attainable region of the population distribution and the corresponding intervention factor to obtain a desired population distribution. These results of the analysis seems to be very helpful in the regional planning.

1. Introductory remarks

In order to apply an appropriate policy in the regional planning, the basic element of the investigation is the determination of the population distribution. By this way, the first problem in the application arises immediately with the desired or goal distribution. It is very questionable if the desired population distribution can be reached by an initial distribution. Of course, the solution of the problem slould depend on the transition matrix which reflects the high or low mobility of the population. On the other hand, in order to obtain a goal distribution, we have the opportunity to intervene an appropriate policy factor which denotes the number of people added or withdrawn in each subregion of the regarded system. From these reasons, it will be helpful to specify more clearly what is that we wish to control and the means which are available to exercise control. Thus, we shall aim to control a population distribution in a time-interval and in a set of subrogions, thereby altering **the** total number of people in each subregion of the population system and holding **the** non-diagonal elements of the tranition matrix out of the control.

Such considerations on the control policy we will apply in the 9 geographic regions in Greece. Thus, having a subdivision in urban, semi-urban and rural population, we estimate the transition probabilities between the regarded subdivision in **each** geographic region. As we mentioned before, the population mobility can be not under control, because the population is free to move everywhere in the region. On the other hand, it could be under control the number of people which are immigrated or emigrated abroad, because of the needed permission, and the number of people moving to or from other regions in the country, because of the migration conditions. Moreover, this concept can be extended in other factors, including for example birth control, depending every time on the aim of the regional planning policy. Thus, we have under control the diagonal elements of the growth operator matrix which is a sum of matrices, consisting of the transition matrix and the diagonal birth, death and migration matrices.

However, the control problem is treated in two aspects in the following analysis, which arise immediately from the above considerations. The steady-state control is used to refer to the problem of holding the grade structure at some specified values. This is the first problem, because the main and first aim in the regional planning policy is a preservetion of the observed population distribution. The more generalised case, sequential control refers to the problem of changing a given population distribution to some desired distribution by a sequence of adjustments to the control parameters. This is the second and more specified problem in the regional planning policy.

2. Steady-state control

We consider the population distribution $w(t)=(w_j(t), i=1,2,3)$ at the time point t in each geographic region, where exist a subdivision in urban (i=1), semi-urban (i=2) and rural (i=3) population. The basic equation of the population development can be written as follows:

$$w(t+1)=w(t)\ (P+B-D+N_1+N_2)=w(t)\ G \tag{2.1}$$

where B,D,N_1,N_2 are diagonal matrices, whose diagonal elements denote the crude birth, death, net migration rate abroad and net migration rate to other regions respectively and P the transition matrix.

Under these considerations, we can introduce the following decomposition of the growth operator matrix G, in order to determine the maintainable region of the population distribution. Thus, we have:

$$w(t+1)=w(t)\ R+w(t)\ Q, \tag{2.2}$$

where Q is a diagonal matrix with elements the diagonal entries of G and R is a (3x3)matrix with elements the non-diagonal entries of G.

A population distribution can be maintained if we can find a control parameter f such that $w(t)= w(t+1)=w$. Then we can rewrite the equation (2.2) as follows:

$$v=v\ R + v\ Q\ f, \tag{2.3}$$

where Q now a column with entries the diagonal elements of G and V are the relatives sizes:

$$v_i(t) = \frac{w_i(t)}{\bar{w}(t)}, \quad \bar{w}(t) = \sum_{i=1}^{n} w_i(t), \quad (n=3).$$

It is evident from the formulation of the equation (2.3) that the intervening factor f takes in account a policy not only in migration but also in births and deaths.

If the intervention factor f is the only set of parameters amenable to control, we have to determine an f satisfying the equation (2.3). A solution for f could be derived by direct computation from (2.3). Thus, we have:

$$f = v(I - R)/vQ. \tag{2.4}$$

It is easy to check that the elements of f may be not all positive, but they add up to one, if G is a stochastic matrix. If the elements of f are not positive the distribution is not maintainable.

A positive contribution of the introduced decomposition is the possibility of determining the set of population distributions which can be maintained. A simple characteristic of the maintainable region M, which follows directly from (2.3) is that it is the set of v for which v>vR. (Bartholomes, 1973).

The boundary of M may be found from (2.3). Hence

$$v = Q f d - R)^{-1}, \tag{2.5}$$

where the inverse always exists. The vector f may be written as:

$$f = \sum_{i=1}^{n} f_i e_i, \quad (n=3), \qquad (2.6$$

where ej is a vector with an one in the ith position and zeros elsewhere. Substituting in (2.5), then gives:

$$v = v Q \sum_{i=1}^{n} f_i \{e_i (I - R)^{-1}\}. \tag{2.7}$$

By post-multiplying both sides of (2.7) by a column vector of ones, vQ can be determined and by substituting in (2.7) the vector v becomes the form:

$$v = \sum_{i=1}^{n} \frac{f_i d_i}{\sum_{j=1}^{n} f_j d_j} \frac{1}{d_i} \{e_i (I - R)^{-1}\}, \tag{2.8}$$

where di is the sum of the elements of the ith row of $(I - R)^{-1}$.

The vector v has been presented as a convex combination of the points with coordinates d ei(1 — R) '. Thus, the maintainable region is a convex hull with this points as vertices. These vertices are easily computed by taking the rows of (I — R)~' in turn and scaling their elements, so that the row sums are one.

In a further application, we can introduce a growth factor a (t) given by:

$$a(t)=\frac{(\bar{w}(t+1)-\bar{w}(t))}{\bar{w}(t)} \quad . \tag{2.9}$$

Thus, the corresponding intervention factor may be determined by:

$$f= (v (I - R) + a v\}/(v Q + a) \tag{2.10}$$

and the arqument leading to the determination of the vertices of M goes through in this case, with obvious modification to give the vertices of M with coordinates proportional to:

$$ei(I(1+a)-R)^{1}. \tag{2.11}$$

It is evident from the above considerations that the maintainable region may be easily determined according to the introduced decomposition of the growth operator matrix, so that the inverse (I — R)~' always exists and has positive elements.

3. Sequential control

Let us now suppose that there is a goal population distribution g which must be attainable from at least one other distribution y in one step. Thus, the basic equation for the population development is:

$$g=yR+yQf, \tag{3.1}$$

where R and Q represent the introduced decomposers of the growth operator matrix G and f the intervention factor.

The first problem in the sequential control is to find the attainable region A. This may be determined directly from (3.1) having the necessary condition g yR.

The boundary of A may be found, more convenient, according to the introduced decomposition of G. Thus, the attainable region A is a convex hull with coordinates:

$$ejR+Qej,((ij=l,2,...,k), \tag{3.2}$$

where ej is a k-dimensional vector with one in the ith position and zeros elsewhere.

Of course not all points will be vertices of the attainable region, because some of them may be interior points. It is easy to check, for small k, the interior points and as a result of this, which points will be vertices of A.

The more interesting and difficult problem in the sequential control is the determination of control strategies. This means that we have to find a sequence of intervention factors which lead from an initial population distribution to a desired distribution. More explanatory, we define the problem in the following way. Let us consider the equivalent equation of the population development in the form:

$$w(t+1) = w(t)\ G + r(t), \tag{3.3}$$

where r (t) is a row vector which denotes the new intervention factor at the time t. The problem is to find a t' and a sequence of vectors (r(t)}, t=1,2,...,t', such that t' is the smallest t for which w(t)=g.

In our applications the target population distribution may not be something which has to be attained precisely but rather an indication of the limit structure of the declined population distribution. From this reason, we ignore any optimal condition about the time. Furthermore, in an other simplification, we consider the intervention factor independent of the time, so that the feasible goal population distribution g will satisfy, after t time-periods, the equation:

$$g = w(0)\ G^t \sum_{s=1}^{t-1} r\ G^s. \tag{3.4}$$

In a gradualy declining or stationary population, where the dominant characteristic root of G is less or equal to unit, it is easy to determine by simple algebraic calculations the unique r, which is given by:

$$r = g\ (I-G), \tag{3.5}$$

where the goal distribution is feasible after t= time-periods.

The constrain for infinite number of steps may appear to be impossible to enforce an infinite number of conditions. However, the theorem of Kemeny and Snell, (1962), provides an algorithm for establishing feasibility in a finite number of steps.

It is evident from the equation (3.5) that the elements of the intervention factor r may be not all positiv althought the goal distribution is feasible. Furthermore, they add up to zero, if G is a stochastic matrix.

4. Results of the applications

In order to apply the procedures of stochastic control, introduced in the foregoing paragraphs, the first element of the investigation is an estimation of the growth operator matrix in each geographic region. Thus, having the statistical data about interregional migration during the time-period 1966-1971, published by the Greek Statistical Service as results of the population and housing censuses in 1971, we estimate the growth operator matrix on the basis of distributional data in each geographic region, where the distributional data in 1971 were gathered during the census and in 1966 were estimated according to a stochastic projection of the population distribution in each geographic region (Tziafetas, 1982). This method (Rogers, 1968) may be used if data about birth, death and migration are not available or if they are not reliable. Thus, we illustrate in table (4.1) the growth operator matrix G for the 9 geographic regions in Greece with the intrinsic rate of growth, determined by the calculation of the dominant characteristic root of G.

TABLE 4.1

The growth operator matrix and the intrinsic rate of growth of the 9 geographic regions in Greece

	Growth operator matrix			Intrinsic rate of growth
Rest of Central Greece	1.04147	0.01168	0.00796	1.04650
	0.01394	0.96161	0.01253	
	0.03169	0.04284	0.91974	
Peloponessos	0.93295	0.00126	0.01321	0.94766
	0.01679	0.89035	0.01267	
	0.04073	0.01517	0.90181	
Ionian Islands	0.73770	0.00131	0.03540	0.91067
	0.0129	0.80886	0.01440	
	0.05687	0.03905	0.89239	
Epirus	0.97254	0.00289	0.01965	0.97821
	0.01532	0.75665	0.02452	
	0.02092	0.02160	0.89408	
Thessaly	0.95006	0.00622	0.00824	0.96621
	0.03926	0.81805	0.02243	
	0.07033	0.02723	0.91263	
Macedonia	1.02760	0.00413	0.00940	1.03321
	0.03933	0.85383	0.01680	
	0.05379	0.02144	0.91064	
Thrace	0.89527	0.00574	0.01930	0.95304
	0.01269	0.83697	0.00176	
	0.04490	0.02372	0.93166	
Aegean Islands	0.87907	0.00773	0.01694	0.93626
	0.01196	0.91957	0.01452	
	0.02838	0.01936	0.90006	
Crete	0.94665	0.00341	0.02788	0.97357
	0.02186	0.81312	0.02584	
	0.06780	0.03086	0.89246	

Because of a very fast changing structure of the population in Greece, the first problem on the regional planing policy is a preservation of the observed population distribution. It means that we have to determine in steady-state control the maintainable region of the population distribution which is a convex hull with vertices determined by (2.11). Thus, in table (4.2) we illustrate the maitainable region for the 9 geographic regions in Greece with the corresponding intervention factor, determined by (2.10), in order to preserve the observed population disiribution.

TABLE 4.2

ntainable region and the corresponding intervention factor f for a steady-state control in the 9 geographic regions in Greece

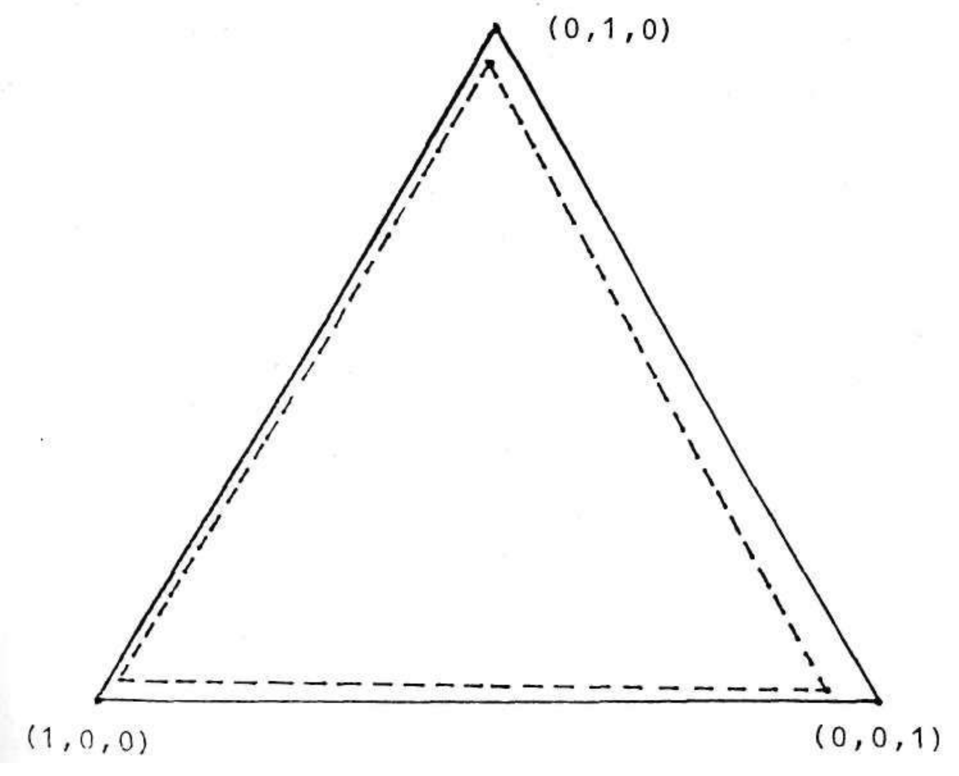
	Maintainable region			Intervention factor f'
Rest of Central Greece	0.98114	0.01126	0.00760	0.23107
	0.01334	0.97489	0.01177	0.24648
	0.02847	0.03851	0.93302	0.51378
Peloponessos	0.98472	0.00153	0.01375	0.25687
	0.01774	0.96906	0.01320	0:12619
	0.04084	0.01517	0.94399	0.67637
Ionian Islands	0.95967	0.00298	003735	0.12473
	0.01513	0.96896	0.01591	0.17777
	0.05703	0.03885	0.90412	0.74862
Epirus	0.97697	0.00332	0.01971	0.21505
	0.01555	0.96007	0.02438	0.06983
	0.02082	0.02121	0.95797	0.78604
Thessaly	0.98487	0.00658	0.00855	0.30726
	0.03973	0.93815	0.02212	0.16780
	0.06707	0.02599	0.90694	0.55523
Macedonia	0.98682	0.00413	0.00905	0.40982
	0.03688	0.94738	0.01574	0.15916
	0.04923	0.01952	0.93125	0.43469
Thrace	0.97378	0.00636	0.01987	0.25163
	0.01388	0.96383	0.02229	0.12286
	0.04424	0.02347	0.93229	0.64358
Aegean Islans	0.97384	0.00841	0.01775	0.25069
	0.01288	0.97183	0.01530	0.22031
	0.02908	0.01991	0.95101	0.60463
Crete	0.96789	0.00427	0.02783	0.28397
	0.02316	0.95094	0.02590	0.09688
	0.06383	0.02898	0.90719	0.56285

For example, in Peloponessos the intervention factor is (0.25687, 0.12619, 0.67637). Having in the year 1966 a total number of 1038.5 (xlOOO) people, distributed 284.6 in urban, 126.3 in semi-urban and 627.6 in rural regions, it is expected, according to the diagonal entries of the growth operator matrix and the in trinsic rate of growth, a number of 293.5 will be in urban, 122.3 in semi-urban and 571.2 in rural regions. According to the introduced intervention factor, from the in titial population should be, after a five years time-period, 269.6 people in urban, 119.7 in semi-urban and 594.7 in rural regions. It means that the urban and semi-urban population should be decreased by 23.9 and 2.6 people respectively and the rural population should be increased by 23.5 people, during the time-period 1966-1971, in order to preserve the observed population distribution in 1966.

Taking the coordinates of the vertices of the maintainable region in each one of the geographic regions, it is easy to have a geometric representation in a triangular diagram, as it was ploted in fiqure (4.1) for Peloponessos, and then to check if an observed population distribution is maintenable or not.

Figure 4.1

A geometric representation of the maintenable region in Peloponessos.



In order to solve the attainability problem in regional planning policy, we deter mined the attainable region A of the population distributions from which a goal dis tribution can be reached in one step. According to our considerations, the coordinates of the vertices of. the attainable region may be elected from the coordinates determined by (3.2). For example in rest of Central Greece the set of coordinates are: (1.04147, 0.01168, 0.00796), (1.05541, 0.0, 0.01253), (1.07316, 0.04284, 0.0), (0.0, 0.97329, 0.00796), (0.01394, 0.96161, 0.01253), (0.03169, 1.00445, 0.0), (0.0, 0.01168, 0.92770), (0.01394, 0.0, 0.93227), (0.03169, 0.04284, 0.91974).

Having, a geometric representation in atriangular diagram, scaling the coordinates so that they add up to one, it is easy to check that only the first point is an interior one. It means, that a point may be interior in a convex hull if his coordinates are greater than one.

In a further application of the stochastic control on population distributions, we estimated the intervention factor in each one of the 9 geographic regions in Greece, in order to obtain a goal distribution which provides an increase of 4% in urban regions and has the same number of people in semi-urban and rural regions, as the observed population in the year 1966.

As we see in table (4.1), 7 regions have an intrinsic rate of growth less than one, so that the goal distribution is feasible after a great number of steps, according to the equation (3.4). In this case we determine the intervention factor from (3.5). For example in Peloponessos, we estimated the intervention factor (-7.836, +3.954, +56.114) and a goal distribution (296.0, 126.3, 627.6). Thus, having the initial population distribution (284.6, 126.3, 627.6) we found, for some steps, the following distributions:

1. step: (285.6, 126.3, 627.5)
2. step: (286.0, 126.3, 627.6)
3. step: (286.7, 126.3, 627.6)

The population system investigated in the other 2 geographic regions of Greece is an intractable one to analyse within the above framework. It is an expanding population system, whose total population increases over time, as a result of an excess of births and immigration over deaths and emigration. However, this is the most commonly observed population system. Unfortunately, no analytic solution is immediately apparent. From this reason, we are obliged to apply an heuristic alogorithm which leads to a succesful result.

Having the fundamental equation (3.4) in an expanding population distribution exposed to an unchanging regime of growth, G, with an intrinsic rate of growth, $\lambda>1$, we etablish a generalisation of (3.4) by:

$$g = w(0)\, G^t + \sum_{s=0}^{t-1} r\, \lambda^s\, G^s.$$

Thus the suggested heuristic algorithum has the following steps:

1. Transform the growth operator into a stationary population counterpart by

reducing each diagonal element until the elements of each row in the operator sum to unit.

2. Find the intervention factor, r, for the stationary population system.

3. Increase the intervention factor, r, by the intrinsic rate of growth, λ, at each intervention, that new the intervention vector is λ- r.

Applying in the -2 expanding geographic regions the above algorithm, we estimate the intervention factor r for the corresponding stationary system as they have been illustrated in table (4.3).

TABLE 4.3

The intervention factor r and the goal distribution g in the 9 geographic regions in Greece

	Intervention factor r			Goal distribution g		
Rest of Central Greece	-14.250	-17.090	+31.340	245.8	255.3	489.7
Peloponessos	-7.836	+3.954	+56.114	296.0	126.3	627.6
Ionian Islands	+0.437	+1.780	+12.350	31.7	36.1	130.1
Epirus	-3.346	+3.334	+22.848	71.9	26.1	235.1
Thessaly	-17.420	+9.948	+25.730	227.0	114.1	345.2
Macedonia	-45.244	-3.361	+48.605	824.9	307.1	797.1
Thrace	+1.003	+1.761	+11.745	101.3	43.7	201.6
Aegean Islands	+5.615	+1.935	+21.093	112.9	93.6	243.8
Crete	-12.431	+0.265	+24.700	146.2	50.3	279.9

* For the expanding regions the intervention factor was estimated according to the corresponding stationary population system.

Literature

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