THE STATE - VARIABLE FORM OF AN ECONOMETRIC MODEL OF THE GREEK ECONOMY

By
COSTAS RIGAS
Piraeus School of Industrial Studies

1. INTRODUCTION

The theory of optimal control has been applied to many economic problems by a number of investigators [1].

The applications to the economic stabilization has been considered by Chow [2], Sengupta[3] and Theil[4].

The long-term economic development problem has been treated by Dobell-Ho [5], Fox - Sengupta - Thorbecke [6], Kendrick - Taylor [7], Wall - Westcott[8] and

1, Costa A. Riga (In Greek) «The theory of optimal Control in Economics», Spoudai, Vol KZ, 3 p. 707-730,1977.

2. Chow, G. «On The optimal Control of Linear Economic systems», IBM Research Report. Poughkeepsie, New York, 1969.

3. Sengupta, J. K. «Optimal Stabilization Policy with a a Quadratic Criterion Function», Rev. Econ. Stud. January, 1970.

4. Theil, H., «Optimal Decision Rules for Government and Industry», Amsterdam, the Netherlands, North - Holland, 1964.

5. Dobell, A.R., Ho, Y.C. «Optimal Investment Policy : An Example of a Control problem in Economic Theory», IEEE Trans, Autom, Control, AC - 12, pp. 4-14 1967.

6. Fox, K., Sengupta, K., Thorbecke, E., «The theory of Quantitative Economic Policy». Amsterdam, the Netherlands : North - Holland, 1966.

7. Kendrick, D. Taylor L. «Numerical Solution of Nonlinear planning Models», Econometrica, 1970.

8. Wall, K. D. Wescott J. H. «Policy Optimization Studies with a simple control Model of the U.K. Economy», procedings of the 6th IFAC Congress, Boston, Mass, USA., 1975.

Shell [9]. Pindyck has applied optimal discrete - time systems concepts to the short-term economic stabilization policy problem[19].

Jamshidi and Ozgumer have been applied the economic model of Pindyck for the long - term stabilization problem [11].

In this paper we specify the state - variable form of an econometric model of the Greek Economy.

2. THE DISCRETE-TIME OPTIMAL CONTROL MODEL

The optimal control problem would be to regular the economic state Xi and policy vector u_i as close to the nominal value $\hat{X}i$ and Ui respectively. For this we introduce the following cost functional,

$$(2.1)\quad J = \frac{1}{2} \Sigma \left[(x_i - \hat{x}_i)' \quad Q(x_i - \hat{x}_i) + (u_i - \hat{u}_i)' \quad R(u_i ;- \hat{u}_i) \right]$$

where Q and R are nxn positive semi - definite and rxr positive definite weighting matrices respectively[12,13].

9. Shell, K., «Essays on the Theory of optimal Economic Growth», Cambridge, Mass., ed. MIT Press, 1967.

10. Pindyck, R.S. {«An application of the Linear Quadratic Tracking Problem to Economic Stabilization policy», IEEE Trans. Autom. Control, AC - 17, No 3, pp. 287-300, 1972.

11. Jamshidi M. Ozguner U. «An optimal adaptive policy for long-term Economic Stabilization Problem» Models and Decision Making in National Economies J. M.L. Janssen, L.F. Pan, A. Straszak (eds.), North - Holland Publishing Company, 1979.

12. Robert S. Pindyck, Optimal planning for Economic Stabilization, North - Holland American Elsevier, 1973.

13. John Pitcford and Stephen J. Turnovsky Applications of Control Theory to Economic Analysis, North - Holland, 1977.

The state - variable form of an econometric model has the following discrete - time structural form :

(2.2) $x_{t+1} = Axt + But + Cz_t$

(2.3) $\chi_0 = \xi$

where x_t is the n-vector of the endogenous variables. The u_t is the r-vector of the control (or policy). The z_t is the s - vector of exogenous variables.

The A,B,C are properly - sized time - invariant real matrices.

The optimal control problem would be simply to find a set of policy sequence $\{u_i^*, \ i = 0, 1, 2, \ ...\}$ such that the cost functional (2.1) is minimized subject to the constraints :

(2.4) $\chi_0^* = \xi$

(2.5) $x^* = Ax_i + Bu_i + Cz_i$

The solution of the above optimal control problem is straight forward by the application of the Pontryagin's minimum principle in the discrete form.

The solutionis the following :

(2.6) $u_i^* = -(R+B'\ K_{i+1}B)^{-1} \quad B'K_{i+1}(I+A)X_i^*$

$(R+B'K_{i+1}B)^{-1}\ B'K_{i+1}BR^{-1}B'g_{i+1} -$

$R^{-1}B'g_{i+1}.\ (R+B'K_{i+1}B)^{-1}B'K_{i+1}(B\hat{u}_i+Cz_i)+\hat{u}_i$

where

(2.7) $K_i = Q + (I+A)'\ (K_{i+1} - Ki_{+1}B(R+B'K_{i+1}+B)^{-1}\ B'K_{i+1})(I+A)$

$$(2.8) \quad K_T = Q$$

$$(2.9) \quad g_i = -(I+A)'(K_{i+1} - K_{i+1}B(R+B'K_{i+1}B)^{-1}$$

$$B'K_{i+1})BR^{-1}B'g_{i+1} + (I+A)' \; g_{i+1} + (I+A)'K_{i+1}$$

$$-K_{i+1}B(R+B'K_{i+1}B)^{-1}B'K_{i+1}(\hat{B}u_i + Cz_i) - Q\hat{x}_i$$

$$(2.10) \quad g_T = P^*_T - K_T x^*_T - Q\hat{x}_T$$

$$(2.11) \quad P^*_T = Q(x^*_T - \hat{x}_T)$$

The (2.10), (2.11) are the boundary conditions.

3. AN ECONOMETRIC MODEL OF THE GREEK ECONOMY

:

The econometric model of Greece employed in this paper is a version of a model developed by Zonzilos - Brisimis for the period 1959-75(14). In this paper parameter values for the equations were estimated using official Greek data for the period 1959-1984.

The equations and variables which make up the model are presented in tables 3.1 and 3.2.

(14) SPOUDAI Vol. KH, 1 p. 95-115,1978 (in Greek)

Table 3.1.

Econometric model of Greece (1959 – 1984)

1. Consumption

$$C_t = 3{,}5024 + 0{,}4148 Y_t^d + 0{,}5383\, C_{t-1}$$
$$(1{,}6752) \quad (7{,}3751) \quad (9{,}2095)$$

$R^2 = 0{,}9985$ $\quad \bar{R}^2 = 0{,}9984$ $\quad$ D.W. = 1,9395

2. Investment

$$I_t = -0{,}8795 + 0{,}4535\,(Y_t - Y_{t-1}) + 2{,}6553\, R_t + 0{,}9105\, I_{t-1}$$
$$(-0{,}2402) \quad (5{,}3376) \quad (0{,}0382) \quad (13{,}8596)$$

$R^2 = 0{,}9607$ $\quad \bar{R}^2 = 0{,}9553$ $\quad$ D.W. = 2,5153

3. Stocks

$$H_t = -21{,}4892 - 0{,}1325(Y_t - Y_{t-1}) + 0{,}1415 Y_t - 0{,}1590\, K_{t-1}$$
$$(-4{,}1683) \quad (-1{,}1511) \quad (5{,}0687) \quad (-3{,}7641)$$

$R^2 = 0{,}6580$ $\quad \bar{R}^2 = 0{,}6114$ $\quad$ D.W. = 1,2265

4. Interest Rate

$$R_t = 0{,}0197 + 0{,}0002\, Y_t - 0{,}0002\, M_{t-1}$$
$$(2{,}1271) \quad (1{,}5130) \quad (-0{,}4308)$$

$R^2 = 0{,}6913$ $\quad \bar{R}^2 = 0{,}6645$ $\quad$ D.W. = 0,6455

5. Imports

$$\Theta_t = -15{,}6261 + 0{,}2350\, Y_t$$
$$(-6{,}6456) \quad (34{,}9931)$$

$R^2 = 0{,}9808$, $\quad \bar{R}^2 = 0{,}9800$, $\quad$ D.W = 1,1828

Identities

6. $Y_t = C_t + I_t + H_t + G_t + E_t - \Theta_t$

7. $Y_t^d = Y_t - T_t$

8. $K_t = H_t + K_{t-1}$

Table 3.2.

List of variables in the Greek econometric model

1. C = private consumption.
2. I = Gross private fixed asset formation.
3. G = Government Consumption plus government investment.
4. H = Changes in capital stocks.
5. E = Exports.
6. Θ = Imports.
7. Y= Gross National Product.
8. Yd = Personal disposable income.
9. T = Direct Taxes.
10. K = capital stocks.
11. M = Supply of Money
12. R = Interest rate.

The data have been received by 1) the National Accounts of Greece of the Ministry of National Economy and 2) by the monthly bulletins of Bank of Greece.

The values are at constant prices 1970 for the period 1959 - 1984.

The estimation was single equation OLS.

The usual mean - variance assumptions were made about the stochastic error term.

The R^2 is the corrected R^2 and the D.W. is the Durbin- Watson statistic.

4. THE STATE-VARIABLE FORM

The state - variable form of the econometric model of § 3 is the following :

$$(4.1) \qquad x_{t+1} = A_o X_{t+1} + A_1 X_t + B_1 ut$$

where

$$
x_t = \begin{vmatrix} G_t \\ I_t \\ H_t \\ R_t \\ \Theta_t \\ Y_t \\ Y_t{}^d \\ K_t \\ E_t \\ G_t \\ T_t \end{vmatrix}, \quad u_t = \begin{vmatrix} 1 \\ \\ \\ \\ \\ M_t \end{vmatrix}, \quad B_1 = \begin{vmatrix} 3{,}5024 & 0 \\ -0{,}8795 & 0 \\ -21{,}4892 & 0 \\ 0{,}0197 & -0{,}0002 \\ -15{,}6261 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{vmatrix}
$$

$$
A_0 = \begin{vmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0{,}4148 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2{,}6553 & 0 & 0 & 0{,}4535 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0{,}0090 & 0 & 0 & 0 & 0 & 00 \\
0 & 0 & 0 & 0 & 0 & 0{,}0002 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0{,}2350 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{vmatrix}
$$

$$
A_1 = \begin{vmatrix}
0{,}5383 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0{,}9205 & 0 & 0 & 0 & -0{,}4535 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & ,3125 & 0 & -0{,}1590 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{vmatrix}
$$

The A_0 is a 11x11 matrix, the A_1 is 11x11 and the B_1 is 11x2.

(4.1.) can also be expressed as :

(4.2) $x_{t+1} = (I - A_0)^{-1} A_1 x_t + (I - A_0)^{-1} B_1 u_t$

Substituting

(4.3) $A = (1 - A_0)^{-1} A_1$

(4.4.) $B = I - A_0)^{-1} B_1$

we obtain

(4.5) $X_{t+1} = Axt + But$

which is the state - variable form of the econometric model.

5. CONCLUSIONS

In this paper an econometric model of the Greek economy was presented at the state - variable form.

The model consisted of 11 state variables and one control (policy) variable (the money supply).

The next work is to find the optinal control variables, following the analysis of § 2.

A resercher could construct an other version of this model with more equations, including more variables (wages, unemployment etc) and more policy variables such as the government spending, a taxation mechanism etc.

REFERENCES

1. Chow, G, «On The optimal Control of Linear Economie systems», IBM Research Report. Poughkeepsie, New York, 1969.

2. Dobell, A.R., Ho, Y.C. «Optimal Investment Policy : An Example of a Control problem in Economic Theory», IEEE Trans, Autom. Control, AC- 12, ρ p. 4- 14. 1967.

3. Fox, K., Sengupta, K., Thorbecke, E., «The theory of Quantitative Economic Policy», Amsterdam, the Netherlands : North - Holland, 1966.

4. Jamshidi, M., Ozguner, U., «An optimal adaptive policy for long-term Economic Stabilizaticn Problem», Models and Decision Making in National Economies, J.MX. Janssen, L.F. Pan., A. Straszak (eds.), North - Holland Publishing Company, 1979.

5. Kendrick, D. Taylor, L., «Numerical Solution of Nonlinear planning Models», Econometrica, 1970.

6. Pindyck, R.S., «An application of the Linear Quadratic Tracking Problem to Economic Stabilization policy», IEEE Trans. Autom. Control, AC-17, No 3, pp. 287-300, 1972.

7. Robert S. Pindyck, «Optimal planning for Economic Stabilization, North- Holland», American Elsevier, 1973.

8. John Pitcford and Stephen J. Turnovsky, «Applications of Control Theory to Economic Analysis», North - Holland, 1977.

9. Costa A. Riga (In Greek), «The theory of optimal Control in Economics», SPOUDAI, Vol KZ, 3 p. 707-730,1977.

10. Sengupta, J.K. «Optimal Stabilization Policy with a a Quadratic Criterion Function», Rev. Econ. Stud. January, 1970.

11. Theil, H, «Optimal Decision Rules for Government and Industry», Amsterdam, the Netherlands, North-Holland, 1964.

12. Shell, K., «Essays on the Theory of optimal Economic Growth», Cambridge, Mass., ed., MIT Press, 1967.

13. Wall, K.D., Wescott, J.,H., «Policy Optimization Studies with a simple control Model of the U.K. Economy», procedings of the 6th, IFAC Congress, Boston, Mass, USA., 1975.