AN EMPIRICAL COMPARISON OF THE CAPM AND THE APT IN THE FOREIGN EXCHANGE MARKETS*

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I. PERSPECTIVES

The majority of research effort on asset valuation and market efficiency during the past two decades has been based on single - period asset pricing models that are founded on the early contributions of Markowitz [1952, 1959], and Tobin [1958]. Known generally by the acronym (CAMP), these one - factor models were first developed by Treynor [1961], Sharpe [1944], Lintner [1965], and Mossin [1966] and later extended through such variants as the «zero beta» of «two-factor CAMP» by Black, Jensen, and Scholes [1972], the «three factor CAMP» by Merton [1973], and the «four-factor CAMP» by Fama and MacBeth [1973[.

In its basic form, the one-factor CAMP assumes that assets are priced in a well integrated market, and specifies the expected return on an investment as a linear function of the risk free rate and the expected return on the market portfolio. More specifically, the model assumes the following analytical structure :

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$$E(\tilde{R}_{jt}) = R_f + \beta_j[E(\tilde{R}_{mt}) - R_f], \qquad (1)$$

where

R_{jt} = return on asset j at time t ;

R_f = return on risk-free asset f;

$R_{M,t}$ = return on the «market» portfolio ;

$\beta j = (\sigma_{JM}/\sigma^2_M)$ = the ratio of the covariance of asset j with the market over the variance of the market, also known as «beta» coefficient; and

E, ~ denote the expectation operator and random variable, respectively.

There has been a significant and consistent lack of succeess, however, in attempts to validate empirically expression (1) that led early researchers to efforts or improving the testing methodology and enriching the analytical content of the model. In early attempts to reconcile the theoretical implications and the empirical findings of the CAPM, Blume [1971], Vasicek [1973], and others, first suggested ways to adjust the empirical beta coefficients to their analytical counterparts[2]. Other efforts were directed toward the enrichment of the model through additional factors so as to account for such empirical anomalies as «size - effects», «turn-of-the-year effects», and «earnings/price rations effects»[3]. Nevertheless, some researchers seem to have come to the conclusion that the CAPM may not describe adequately the structure of asset returns. Hence, the need for a substitute to the CAPM that would overcome the previously outlined limitations, as well as answer the basic question of CAPM - testability raised by Roll [1977].

More recently, Ross [1976, 1977], pursued a different direction and developed an Arbitrage Pricing Theory (APT) that does not depend on knowledge of the market portfolio, and, hence, on the mean - variance efficiency of such portfolio, which is the main conclusion of the (CAPM). The APT allows for any number of risk pre-

1. See Jensen [1972] or Philippatos [1979] for a thorough review of the empirical investigations of the CAPM.
2. There are at least three well known beta - adjustment procedures - namely, the Blume [1971], the Vasicek [1973], and the Merrill Lynch. For a detailed review and comparison of these procedures, see Hawawini and Vora [1982].
3. See Schwert [1983] and the other papers in the same issue for the latest studies on what are euphemistically called «empirical regularities».

mia to influence expected returns and is regarded by the current literature as an alternative to the CAPM. Its return structure, equivalent to (1), is given by (2)·

Like the CAPM, researchers set out to test the empirical validity of (2). These tests, confined to their limitations, cover a wide spectrum of hypotheses. For example, some investigated the relevant factors to be included in the final form of (2)

$$E(\tilde{R}_{jt}) = R_f + b_{j1}[E(\tilde{R}^1_{Mt}) - R_f] + b_{j2}[E(\tilde{R}^2_{Mt}) - R_f] + \ldots b_{jm}[E(\tilde{R}^m_{Mt}) - R_f], \qquad (2)$$

where

b_{jm} = beta of asset j associated with factor m ; and

$E(\tilde{R}^m_{Mt})$ = expected return on factor m.

(Cehr [1975], Roll and Ross [1980]), while others tested the uniqueness of both the risk-free and excess factor returns across assets at a given point in time (Hughes [1982] and Brown and Weinstein [1983]). The results, although not heralded so, were not in general agreement. Gehr [1975] found that two or possibly three factors explained a large portion of variation in returns, but only one of the factors was significant in the pricing relationship. Roll and Ross [1980], on the other hand, reported five significant factors. Brown and Weinstein [1983] presented evidence conflicting with the five-factor model suggested by Roll and Ross, while Hughes [1982] provided evidence that there exists a unique risk-free rate across securities, in line with expression (2). Additional tests by Fogler, John, and Tipton [1981], Oldfield and Rogalski [1981], and Reinganum [1981], have attempted to identify the return generating process, given in expression (2), through empirical methods. Also, in a related research, Sharpe [1982] reported 8 systematic sector influences (Basic Industries, Capital Goods, Construction, Consumer Goods, Energy, Finance, Transportation, and Utilities)[4].

The purpose of this paper is to dwell on some serious weaknesses that characterize most of the empirical studies that have used latent variable techniques, such as factor analysis, to identify and measure the risc premia in the APT. The main conclusion of the paper is that a decomposition of the covariance structure of the

4. For some theoretical work on the APT the reader is referred to Ross [1976, 1977], Shanken [1982], Elton, Gruber, and Rentzler [1983], and Solnik [1983].

variables under consideration into m components, via factor analysis, may not necessarily imply an m-factor asset pricing model. That is, such decomposition simply partitions the over-all systematic risk of an asset into m additive components without contribution anything to the pricing relationship of the asset. Corroborating empirical verification is obtained by applying our methodology to the foreign exchange market. The empirical evidence can be summarized as follows : (1) very little explanatory power is gained beyond the first factor ; (2) the APT-type betas are not significantly different from the CAPM - type betas ; (3) the single - index model yields adequate results, although it appears that at least a two - factor model is necessary to approximate the covariance structure of currency returns. Needless to say, aside from the specific emplirical findings, we believe that the main conclusion of this paper is sufficiently general to hold for any market with a finite number of factors.

The paper is organized as follows. Section Π reviews the Arbitrage Pricing Theory and points out the weaknesses associated with its empirical tests. Section III presents the orthogonal factor model which constitutes the cornerstone for most empirical papers dealing with APT. Section IV is an application of the methodology described in Section III, to the foreign exchange market, and Section V summarizes the main conclusions.

Π. THE ARBITRAGE PRICING THEORY AND ITS TESTS

Following Ross [1976, 1977], the Arbitrage Pricing Theory assumes the existence of the following three conditions.

$$\sum_i x_i E_i = 0 \tag{3}$$

$$\sum_i x_i b_{ij} = 0, \quad \forall j, \ j = 1,2, \ldots m \tag{4}$$

$$\sum_i x_i = 0, \tag{5}$$

where

X_i = additions or withdrawals from the original portofolio weights ;

E_i = expected return on asset i ; and

by = sensitivity of asset i's returns to the movements of the common factor j.

Whereas derivation of expression (1) requires very restrictive assumptions (quadratic utility and/or multivariate normality of returns, for example), conditions (3) through (5) exploit the «law of large numbers» for the diversification of the unsystematic risk inherent in every asset's returns. Assuming complete diversification of the asset specific risk, the algebraic consequence of conditions (3) through (5) is given by expression (6), which is equivalent to expression (2),

$$E_i - E_0 = (E^1 - E_0)b_{i1} + \ldots + (E^m - E_0)b_{im}, \qquad (6)$$

where a subscripted E denotes expected asset returns (with 0 being the riskless asset) while a superscripted E denotes expected factor returns.

In contrasting expressions (1) and (6), one recognizes the ability of the APT to accommodate several sources of systematic risk. Although this may be viewed as an advantage over the CAPM, the fact that the m factors that constitute the sources of systematic risk cannot be observed poses an offsetting disadvantage for the APT[5]. Most of the empirical tests of APT employ factor analysis to validate expression (6). However, as discussed later in Section III, factor analysis attempts to reproduce the covariance structure of the original variables with as fewer common factors as possible.

Although this statistical procedure yields uncorrected common factors, which is desirable, the fact that it utilizes the relationship of the individual asset returns to construct these common factors is not, in general, any different from constructing a market portfolio from the individual asset returns. To evaluate this comparison let us reexamine expression (6) more closely. It is obvious that this expression ought to hold for individual assets as well as for fully diversified portfolios, such as the market portfolio. Thus, regardless of the fact that the market portfolio may not be directly observable, we may rewrite expression (6) as follows.

5. For a criticism of the APT on theoretical grounds, see Shanken [1982].

$$E^M - Eo = (El - Eo)\ b_{Ml} + \ldots + (1» - Eo) b_{Mm} \tag{6}$$

where M refers to the market portfolio.

Expression (6'), above, states that an m common factor model is needed to price the market portfolio. However, it also suggests that, for as long as the ex. pected returns of the common factors are combinations of the expected returns of the individual assets, the argument of asset pricing made earlier may be reversed. If this be the case, the market portfolio may be utilized to price the m com. mon factors and consequently the η individual assets - thus reducing the usefulness of these factors to negligible value. That is, the market portfolio may be directly utilized to price asset returns. In fact, all that is needed in this case is to apply factor analysis on the market portfolio and any one asset. This procedure may be sufficient to price any asset in the market under consideration.

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Since neither the market portfolio nor the various common factors can be observed, the causality of asset pricing is a matter of conjecture. Let us assume that the market portfolio may utilized to price asset returns adequately[6]. More specifically, let

$$Ei - Eo = \beta_{im}[E^M - Eo], \qquad j = 1, 2, \ldots, m, \tag{7}$$

where βJM is the sensitivity of factor j's returns to the movements of the market portfolio M. Expression (7) states that the market portfolio may serve as a pricing mechanism for the m common factors, which is exactly the opposite argument made earlier in expression (6')· Since expression (7) is not the result of a model based on factor analysis, the variance of the market portfolio is not restricted to unit value[7]. Note, however, that the factors in expressions (6) and (6') imply a unit variance. Indeed, the betas of these expressions are the factor loadings of the orthogonal factor model discussed later in Section III.

Replacing PJM with its definition $\frac{\sigma_{jm}}{\sigma_m^2}$, and substituting (7) into (6), we obtain

6. For a similar approach to this issue, see Sharpe [1977, 1982].

7. Expression (7) is similar to Sharpe's [1982] expression (9).

$$E_i - E_0 = \frac{\sigma_{1m}}{\sigma_m^2} b_{i1} [E^M - E_0] + \dots + \frac{\sigma_{mM}}{\sigma_m^2} b_{im} [E^M - E_0] \quad \textbf{(8)}$$

or

$$E_i - E_0 = [\frac{\sigma_{1m}}{\sigma_m^2} b_{i1} + \dots + \frac{\sigma_{mM}}{\sigma_m^2} b_{im}] [E^M - E_0] \quad \textbf{(8)}$$

Now, define $\frac{\sigma_{jm}}{\sigma_m^2}$ aswj; then (8') becomes

$$Ei - Eo = [Wib_{il}' + \dots + w_m b_{im}] [E^M - E_0] \quad (9)$$

or

$$E_1 - E_0 = \sum_j^m w_j b_{ij} \ E[^M - E_0] \quad (9')$$

or

$$Ei - E_0 = bi [E^M - Eo], \quad (10)$$

where $b_i = \sum_j^m w_j b_{ij}$

Expression (10) resembles expression (1), which is derived from an equilibrium model of asset prices (CAPM) under a set of very restrictive assumptions. This expression establishes our claim that a decoomposition of the covariance structure of the variables under consideration into m components, via factor analysis, may not necessarily imply an m factor asset pricing model if the beta in expression (10) is not significantly different from the beta in expression (1). That is, nothing can be gained by decomposing the systematic risk of an asset into m components, since we can always aggregate it back to its initial level.

To recapitulare, although the APT may be theoretically sound, empirically it suffers from the same criticism that the CAPM is subjected to. That is, it is not clear that although the covariance structure of the original variables can be reproduced by an m-factor model, it also ought to yield an m-factor asset pricing model. The traditional tests of the APT via factor analysis have adopted the above argument and consequently yielded conflicting results.

The next section reviews the orthogonal factor model that has been employed in most empirical tests of the APT.

III. THE ORTHOGONAL FACTOR MODEL

A. The Covariance Structure of the Orthogonal Factor Model

The orthogonal factor model with m (m<n) common factors may be expressed in matrix notation as

$$\underset{(nx1)}{r} = \underset{(nx1)}{\bar{r}} + \underset{(nxm)}{B} \underset{(nx1)}{f} + \underset{(mx1)}{u}, \qquad (11)$$

where

$\bar{r}$ = the observable random vector of asset returns ;

r = the mean vector corresponding to r ;

B= the matrix of factor loadings ;

f = the random vector of the unobservable common factors ; and

u = the random vector of the unobservable specific factors.

It is assumed that

$E(f)=0 \qquad E(ff')=I$

$E(u) = 0 \qquad E(uu') = U$, where U is a diagonal matrix.

$E(uf')=0$

The basic difference between the multivariate regression model and the orthogonal factor model, as expressed in (11), is that while in the former the independent variables can be observed, in latter the independent variables are generally unobservable.

The covariance structure implied by the orthogonal factor model is

$$1.\ Cov(r) = \Sigma = BB' + U \qquad (12)$$

or

$$Var(r_i) = \sum_{j=1}^{m} b^2_{ij} + u_i$$

$$Cov(r_i, r_j) = \sum_{k=}^{m} b_{ik} b_{jk}$$

$$2.\ Cov\ (r,f) = B \qquad (13)$$

or

$$Cov\ (r_i, f_j) = b_{ij}$$

The variance of the i^{th} asset's returns is decomposed into two components called «communality» and «unique» or specific variance, respectively [8].

$$Var(r_i) = \underbrace{\sum_{j=1}^{m} b^2_{ij}}_{\text{ith communality}} \qquad \underbrace{+u_i}_{\text{unique variance}}$$

8. This decomposition is analogous to the partitioning of the total risk into systematic and unsystematic within the CAPM framework.

B. Estimating the Covariance Structure of the Orthogonal Factor Model

Estimating the covariance structure of the orthogonal factor model, as expressed in (12), essentially amounts to reproducing the covariance structure of the original variables, V, with as fewer common factors as possible 9. Generally speaking, the more Σ deviates from a diagonal matrix (i.e., the higher the covariability between the security returns), the fewer the common factors necessary to reproduce Σ will be. This simplification results in considerable computational efficiency, especially when the population is sufficiently large as in the case of common stocks. Thus, the adequacy of the common factors retained in the orthogonal factor model deserves special attention. In the event that we specify fewer common factors than those needed to price the asset universe adequately, mispricing of some assets, will occur.

A common procedure for testing

$$H_0 : \underset{(nxn)}{\Sigma} = \underset{(nxm)}{B} \quad \underset{(mxn)}{B'} + \underset{(nxn)}{U}$$

versus

H_1 : Σ= any other positive definite matrix,

has been suggested by Bartlett [1954] and employed by recent researchers (Roll and Ross [1980] and Brown and Weinstein [1983]. Essentially, it assumes a normally distributed population and employs a chi-square test. The problem with this test, however, is that it will continue to reject H_0 for a large number of factors that may not provide significant additional insight to the estimated covariance structure. Instead, we suggest the following alternative approximation.

Step 1. Specify an initian number of factors (sufficiently small, i.e., **m** = 3) and estimate (11).

9. In the special case of η = m the factor analysis representation of (12) becomes exact and specific variances disappear. That is,

$$\underset{(nxn)}{\Sigma} = \underset{(nxn)}{B} \quad \underset{(nxn)}{B'} + \underset{(nxn)}{0} = BB'$$

Step. 2. Construct an efficient frontier utilizing the estimates obtained in Step 1. This amounts to employing a multi-index model to form an efficient frontier, a well known procedure in the literature.

Step 3. Retain an additional factor and estimate (11).

Step 4. Repeat Step 2.

Step 5. Contrast the new efficient frontier with its predecessor. If there is significant improvement, go to Step 3, otherwise terminate the iteration.

This procedure is equivalent to approximating the full covariance efficient frontier suggested by Markowitz, with an adequate multi-index model[10].

Let us now discuss the estimation of the covariance structure of the orthogonal factor model via principal components analysis[11]. To view the relationship between the principal components and the orthogonal factor model, we can decompose the factor loadings matrix B in (3) as follows

$$\underset{(n\times m)}{B} = \underset{(n\times n)}{V^{1/2}} \underset{(n\times m)}{L'}, \qquad (14)$$

where L' is the transpose of the loadings obtained from the principal components analysis, and $V^{1/2}$ is a diagonal matrix whose diagonal is the square root of the diagonal of the eigenvalue matrix, V. From (14) we observe that

$$L = B'V^{-1/2}, \qquad (15)$$

which derives from the principal components scheme of the form

10. The marginal contribution of each additional factor is positive, by construction.

11. For alternative estimation procedures the reader is referred to Johnson and Wichern [1982, pp. 407-423].

$$P = L r , \qquad (16)$$

where ρ denotes the vector of principal components (portfolio). Algebraically, the principal components are particular linear uncorrected combinations of the original variables. For example, the jth principal component it the linear combination $l_f r$ such that

$$\max_{l_j} \quad Var \quad (l_j r) \qquad (17)$$

$$\text{s.t.} \quad \text{(i)} \ l'_j l_j = 1, \text{ and}$$

$$\text{(ii)} \ cov\ (l_j r, l_j r) = 0 \qquad Ai < j$$

For our purpose it is important to keep in mind that we are interested in the linear uncorrelated combinations of the original and not the transformed variables. That is, we are interested in the eigenvector-eigenvalue pairs derived from the covariance and not the correlation matrix of the variables under investigation. A principal component vector constructed according to (17) will have as a first element the linear combination with maximum variance. Each subsequent element will capture successively lower proportion of the total variance. The contribution to the total variance of the j^{th} common factor is given by its corresponding eigenvalue, V_j.

IV. APPLICATION OF THE APT TO THE FOREIGN EXCHANGE MARKET

A direct test of our hypothesis is to contrast the betas of the CAPM with those suggested by expression (10). If the two sets of betas are approximately equal, then we can safely accept the hypothesis that approximating the covariance structure of the original variables by an m - factor model may not necessarily imply an m - factor asset pricing model. If, however, the two sets of betas differ significantly, then we may reject the hypothesis of a single - factor asset pricing model. We proceed to investigate empirically this issue by applying the CAPM and the APT to a data base from foreign exchange markets.

The foreign exchange market we will be dealing with in this section consists of 14 assets, that is, the exchange rates of the 14 industrial countries (Austria, Belgium, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Norway, Sweden, Switzerland, U.K., and U.S.A.), covering the period July 1974 through December 1981. These exchange rates are expressed in terms of the U.S. dollar, with the exception of the U.S. dollar which is expressed in terms of the special drawing right (SDR)[12].

Let us define the month - end exchange rate of country i at time t, $\tilde{X}_{it}$, expressed in units of country i's currency per U.S. dollar. Then, the continuously compounded return on currency i, at time t $\tilde{R}_{it}$, is given below

$$\tilde{R}_{it} = \ln(X_{it-1}/\tilde{X}_{it}), \tag{18}$$

where denotes a random variable.

Assuming that the fluctuations of returns (Rit) are generated by a «common factor,» the returns from any country's currency may be given by the following single - index model :

$$\tilde{R}_{it} = \alpha_i + \beta_i \tilde{R}_{Mt} + \tilde{\varepsilon}_{it}, \tag{19}$$

where

$\tilde{R}_{Mt}$ = continuously compounded return on the index, in this case the SDR, at time t, expressed in U.S. dollars per SDR ;

a_i, β_i = parameters to be estimated ; and

$\tilde{\varepsilon}_{it}$ = the t^{th} error term.

Table 1 shows the currency betas according to expression (19). Alternatively, we may estimate the currency betas via APT as noted in Section Π. The necessary

12. The SDR is a «basket» of currencies created by the IMF in 1969 to augment international liquidity. Although in 1974 it was a complified by keeping only the five «major» currencies with participation weights of 42 percent for the U.S. dollar, 19 for the Deutsche Mark, and 13 each for the Japanese Yen, the French Franc, and the British Pound. These weights broadly reflect the relative importance of the five major currencies in international trade.

nputs for the calculation of betas according to expression (10) are the eigenvalues and eigenvectors, derived from the variance covariance matrix of exchange rate returns. Tables 2 and 3 show the eigenvalues and eigenvectors, respectively[13]. Table 4 shows the betas according to expression (10), assuming various factor-pricing schemes.

TABLE 1

CURRENCY BETAS ACCORDING TO EXPRESSION (19)
PERIOD : JULY 1974 – DECEMBER 1981

Country (i)	β_i	$t(\beta_i)$	R^2
Austria	1.980	25.38	.880
Belgium	1.995	27.31	.894
Canada	0.238	2.60	.072
Denmark	1.868	22.18	.848
France	1.768	21.19	.836
Germany	1.992	25.74	.883
Italy	1.338	10.37	.550
Japan	1.357	7.93	.417
Netherlands	2.012	28.20	.900
Norway	1.529	18.16	.789
Sweden	1.444	12.90	.654
Switzerland	2.158	14.92	.717
U.K.	1.413	9.69	.516
U.S.A.	–1.000	–	–

In contrasting the betas of Tables 1 and 4, the following points are in order. First, although a multi - factor model was utilized to approximate the covariance structure of the currency returns, very little is gained beyond the first factor. This is also evident from the data of Table 2, where it can be seen that the first factor

13. The eigenvalues and eigenvectors of Tables 2 and 3 were obtained via the method of principal components analysis. That is, in terms of our notation of Section II, Table 2 shows the diagonal elements of the eigenvalue matrix V, while Table 3 shows the elements of the factor loadings matrix L.

explains more than 76 percent of the total variability in currency returns. Although the explanatory power of the first factor in this market is substantially higher than its equivalent in any other market, we still expect the first factor to be highly correlated with the market portfolio in question. This correlation was 0,98 in this market, while subsequent factors exihbited much lower correlation coefficients with the SDR. For example, the correlation of the second factor and the SDR was 0,14 and less than. 0,06 for the remaining factors.

Second, the APT - type betas are not significantly different from the CAMP - type betas. The minor discrepancies may be due to the fact that we utilized the method of principal components to estimate the factor loadings. An alternative estimation method may be the maximum likelihood technique. Of course, in such a

TABLE 2

FACTOR EIGENVALUES AND PROPORTION OF VARIANCE EXPLAINED BY EACH FACTOR – PERIOD : 7/1974 – 12/1981

Factor	Eigenvalue*	Proportion of Variance Explained
1.	0.009701	0.764110
2.	0.000912	0.071856
3.	0.000551	0.043416
4.	0.000485	0.038223
5.	0.000317	0.024991
6.	0.000239	0.018826
7.	0.000172	0.013559
8.	0.000128	0.010096
9.	0.000082	0.006469
0.	0.000063	0.004963
11.	0.000023	0.001835
112.	0.000014	0.001100
13.	0.000006	0.000445
14.	0.000001	0.000110

* The eigenvalues were obtained via principal components analysis of the variance - covariance matrix of the currency returns.

TABLE 3. EIGENVECTORS OBTAINED VIA PRINCIPAL COMPONENT ANALYSIS OF THE VARIANCE–CO-VARIANCE MATRIX OF THE CURRENCY RETURNS – PERIOD : 7/1974–12/1981

Country \ Factor	1	2	3	4	5	6	7
U.K.	0.198566	0.507631	0.700946	0.048291	-.426488	-.044634	-.090325
Austria	0.327562	-.172716	-.047140	0.045187	-.051843	0.122370	-.049583
Belgium	0.327008	-.130002	-.009959	0.120978	-.028662	0.245510	-.144010
Denmark	0.309686	-.112919	-.080971	0.152115	0.043001	0.094307	-.017528
France	0.282983	-.009685	0.105585	-.125442	0.268569	0.145516	-.127927
Germany	0.328580	-.162629	-.052154	0.082977	-.081419	0.164181	-.039493
Italy	0.212393	-.002054	0.338155	-.598666	0.574269	-.042328	0.179637
Netherlands	0.328735	-.122836	0.029809	0.075482	-.081866	0.248463	-.083952
Norway	0.249049	-.041717	0.025998	0.304603	0.148415	-.123928	0.015131
Sweden	0.233100	0.024608	0.042154	0.445242	0.274104	-.699766	0.156947
Switzerland	0.362332	-.076520	-.320208	-.510741	-.504174	-.435855	0.147970
Canada	0.031186	0.017945	0.045880	0.138291	-.098576	0.290198	0.928958
Japan	0.190116	0.791998	-.512048	-.005101	0.190402	0.145527	-.015254
U.S.A.	-.156368	-.073549	-.037505	-.010106	-.013530	-.042614	-.033782

Country \ Factor	8	9	10	11	12	13	14
U.K.	-.009956	0.002591	0.055883	0.088030	-.001694	0.011319	0.077928
Austria	-.158724	-.125955	-.283900	0.460936	0.192207	0.683562	-.012954
Belgium	-.142419	-.012805	0.022874	-.267900	-.814815	0.130485	0.076837
Denmark	-.013811	-.038609	0.881350	0.223115	0.141070	-.042344	-.025783
France	0.833909	-.267162	-.098335	-.008571	0.012950	-.030308	0.075263
Germany	-.198135	-.086516	-.312689	0.400123	0.020833	-.711722	0.096127
Italy	-.319443	0.109215	0.040586	0.009189	-.017636	-.012801	0.045945
Netherlands	-.185312	-.058512	-.077786	-.690860	0.517977	-.012819	0.072170
Norway	0.196705	0.866463	-.108530	0.023260	0.052126	0.016605	0.019950
Sweden	-.087129	-.358269	-.070413	-.106811	-.035985	-.007079	0.044100
Switzerland	0.129102	0.082066	0.029780	-.079407	-.050261	-.005307	0.016312
Canada	0.108833	-.041752	-.020636	-.020482	-.045960	0.028387	0.056818
Japan	-.091278	0.001485	-.039855	0.008301	0.017182	0.014909	0.078757
U.S.A.	-.007725	0.025830	0.064781	0.044171	0.032723	0.067965	0.976179

TABLE 4

Currency betas according to expression (10) assuming 1, 2, 3, 4, 5 and 14 factor asset pricing schemes - period 7/1974 - 12/1981

Beta* / Country	b^1_1	b^2_i	b^3_i	b^4_i	b^5_i	b^{14}_i
U. K.	1 2272668023	1.3662320572	1.4250514745	1.4260019423	1.4187308437	1.4163790837
Austria	2 0245458351	1 9772643984	1.9733086767	1.9741980513	1.9733141916	1.9741644488
Belgium	2 0211217554	1.9855333830	1.9846976801	1.9870787801	1.9865901281	1.9908681367
Denmark	1 9140605488	1 8831486903	1.8763540627	1.8793480040	1.8800811184	1.8989724291
France	1 7490186714	1 7463673785	1.7552274735	1.7527585126	1.7573372855	1.7602566180
Germany	2 0308377360	1.9863176408	1.9819411726	1.9835743335	1.9821862391	1.9821395482
Italy	1 3127266397	1.3121643521	1.3405404039	1.3287574052	1.3385479891	1.3414114340
Netherlands	2.0317957367	1.9981690747	2.0006704771	2.0021561207	2.0007604055	2.0047723227
Norway	1.5392845192	1.5278643862	1.5300459911	1.5360412150	1.5385715092	1.5406470811
Sweden	1.4407093441	1.4474458456	1.4509831704	1.4597464640	1.4644196018	1.4340680321
Switzerland	2.2394470101	2.2184994688	2.1916294276	2.1815769767	2.1729814269	2.1597721496
Canada	0.1927497280	0.1976622166	0.2015122063	0.2042340623	0.2025534621	0.2362213420
Japan	1.1750403160	1.3918517479	1.3488835806	1.3487831823	1.3520293034	1.3570604514
U.S.A.	-0.9664557645	-0.9865899871	-0.9897371942	-0.9899361014	-0.9901667714	-0.9872189713

* The supperscript indicates the number of factors assumed necessary to reproduce the variance-covariance matrix of the original variables.

case the assumption of the normality of the common factors and the specific factors is required[14]. Finally, although it is evident from Tables 2 and 3 that at least a two-factor model is needed to approximate the covariance structure of currency returns, the single-index model given by expression (19) seems to yield adequate results. That is, the SDR may be considered as an adequate proxy for the currency market portfolio. An interesting future research project may be to contrast the two methods of calculating betas before and after January 1, 1981, the date of the simplification of the SDR from 16 currencies to five. It is conceivable that such a simplification may not have been warranted, due to possible reduction in market representation.

V. CONCLUSIONS

This paper has demonstrated that the application of factor analysis to test empirically the Arbitrage Pricing Theory, may not be a valid procedure. More specifically, decomposition of the covariance structure of the asset returns into m components, via factor analysis, may not necessarily imply an m-factor asset pricing model. We applied our methodology to the foreign exchange market and verified the above hypothesis. However, we expect that the main conclusion of this research will hold for any market with a finite number of factors. From the empirical section of the paper we can also draw an additional inference ; that is, the SDR may be considered as an adequate proxy for the currency market portfolio in pricing national currencies.

14. For a full description or the method of the maximum likelihood and a comparison with the principal components method, the reader is referred to Johnson and Wichern [1982, pp. 415-420].

REFERENCES

Bartlett, M. S., 1954, «A Note on Multiplying Factors for Various Chi - Squared Approximations». *Journal of the Royal Statistical Society* (B), 16, pp. 296-298.

Black, Fischer, Michael C. Jensen, and Myron Scholes, 1972, «The Capital Asset Pricing Model : Some Empirical Tests.» In Michael C. Jensen, éd., *Studies in the Theory of Capital Markets* (New York : Praeger Publishers), pp. 79 - 121.

Blume, Marshall, E., 1971, «On the Assessment of Risk». *Journal of Finance* (March), pp. 1-10.

Brown, Stephen J. and Mark I. Weinstein, 1983, «A New Approach to Testing Asset Pricing Models : The Bilinear Paradigm». *Journal of Finance* (June), pp. 711 -743.

Connor, Gregory, 1982, «Asset Pricing in Factor Economies». Unpublished Doctoral Dissertation, Yale University.

Dybvig, Philip H. and Stephen A. Ross, 1985, «Yes, the APT is (bestable». *Journal of Finance* (September), pp. 1173-1188.

Elton, Edwin, Martin Gruber, and Joel Rentzler, 198ύ, «The Arbitrage Pricing Model and Returns on Assets Under Uncertain Inflation». *Journal of Finance* (May), pp. 525-537.

Fama, E. F. and J. D. MacBeth, 1973, «Risk, Return, and Equilibrium : Empirical Tests». *Journal of Political Economy*, 81 (May - June), pp. 607 - 636.

Fogler, Russel H., 1982, «Common Sense on CAPM, APT, and Correlated Residuals». *Journal of Portfolio Management* (Summer), pp. 20-28.

Fogler, Russell H., Kose éohn, and éames cbipton, 1981, «cbhree Factors, Interest Rate Differentials and Stock Groups». *Journal of Finance* (May), pp. 323 - 335.

Gehr, Adam Jr., 19τ5, «Some Tests of the Arbitrage Pricing Theory». *Journal of Midwest Finance Association*, pp. 91- 105.

Hawawini, Gabriel A. and Ashok Vora, 1982, «Investment Horizon, Diversification, and the Efficiency of Alternative Beta Forecasts». *Journal of Financial Research* (Spring), pp. 1 - 15.

Huberman, Gur, 1982, «A Simple Approach to Arbitrage Pricing Theory». *Journal of Economic Theory*, 28, pp. 183-191.

Hughes, Patricia, 1982, «A Test of the Arbitrage Pricing Theory». Unpublished manuscript : University of British Columbia.

Jensen, Michael C, 1972, «Capital Markets: cbheory and Evidence». *The Bell éournal* of *Economics and Manadement Science* (Autumn), pp. ύ5ύ-ύ98.

Johnson, Richard A. and Dean W. Wichern, 1982, *Applied Multivariate Statistical Analysis* (Englewood Cliffs, N.J. : Prentice-Hall, (nc).

Lintner, John, 1965, «The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets». *Review of Economics and Statistics* (February), pp. 13-37.

Markowitz, Harry M., 1952, «Portfolio Selection». *Journal of Finance* (March), pp. 77-91.

Markowitz, Harry M., 1959, *Portfolio Selection: Efficient Diversification of Investments*, (New York : John Wiley et Sons, Inc.).

Merton, Robert C, 1973 «An Intertemporal Capital Asset Pricing Model.» *Econometria* 41 (September) pp. 867 - 887.

Mossin, Jan, 1966, «Equilibrium in a Capital Asset Market». *Econometrica* (October), pp. 768-783.

Oldfield, George S., Jr. and Richard J. Rogalski, 1981, «Treasury Bill Factors and Common Stock Returns». *Journal of Finance* (May), pp. 337-350.

Philippatos, George C, 1979, «Mean-Variance Portfolio Selection Strategies». In James L. Bicksler, éd., *Handbook of Financial Economics* (North Holland Publishing Co.), pp. 309-337.

Reinganum, Marc R., 1981, «The Arbitrage Pricing Theory : Some Empirical Results». *Journal of Finance* (May), pp. 313-321.

Roll, Richard, 1977, «A Critique of the Asset Pricing Theory's Tests». *Journal of Financial Economics* 4 (March), pp. 129- 176.

Roll, Richard, 1977, and Stephen A. Ross, 1980, «An Empirical Investigation of the Arbitrage Pricing Theory». *Journal of Finance* (December), pp. 1073- 1103.

Ross, Stephen A., 1976, «The Arbitrage Theory of Capital Asset Pricing». *Journal of Economic Theory* (December), pp. 341 - 360.

Ross, Stephen A., 1977, «Return, Risk, and Arbitrage». Iα Irwin Friend and James L. Bicksler, eds., *Risk and Return in Finance*, I, (Cambridge, Mass. : Ballinger) pp. 189-218.

Schwert, G., William, 1983, «Size and Stock Returns, and Other Empirical Regularities». *Journal of Financial Economics* 12 (June), pp. 3- 12.

Shanken, Jay, 1982, «The Arbitrage Pricing Theory : Is it Testable ?» *Journal of Finance* (December), pp. 1129 - 1140.

Sharpe, William F., 1964, «Capital Asset Prices : A Theory of Market Equilibrium Under Conditions of Risk.» *Journal of Finance* (September), pp. 425 - 442.

Sharpe, William F., 1977, «The Capital Asset Pricing Model : A' «Multi-Beta» Interpretation»- In Haim Levy and Marshall Sarnat, eds., *Financial Decision Making Under Uncertainty* (New York : Academic Press, Inc), pp. 127- 135.

Sharpe, William F., 1981, «Factor Models, The Arbitrage Pricing Theory and Capital Asset Pricing Models». Unpublished manuscript : Stanford University.

Sharpe, William F., 1982, «Factors in New York Stock Exchange Security Returns, 1931 - 1979». *Journal of Portfolio Management* (Summer), pp. 5-19.

Solnik, Bruno, 1983, «International Arbitrage Pricing Theory». *Journal of Finance* (May), pp. 49 - 457.

Tobin, James, 1958, «Liquidity Preference as Behavior towards Risk.» *Review of Economic Studies* (February), pp. 65-85.

Treynor, Jack L., 1961, «Toward a Theory of Market Value of Risk Assets». Unpublished manuscript.

Vasicek, Oldrich A., 1973, «A Note Using Cross - Sectional Information in Bayesian Estimation of Security Beta». *Journal of Finance* (December), pp. 1233- 1239.