THE ARBITRAGE PRICING MODEL : SOME PROBLEMS IN ESTIMATION OF THE PORTFOLIO RETURNS GENERATING MODEL USING PORTFOLIOS OF LONDON STOCK EXCHANGE STOCKS

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The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965), and MoSsin (1966), has been the premier model of the financial literature for many years. Roil (1977), however, raised some legitimate questions regarding the testability and usefulness of the model. He forcefully argued that the CAPM may be valid, but it can not be tested unambiguously since the market portfolio is empirically unobservable.

Roll's work has attracted a great deal of attention and has led to interest in the empirical examination of an alternative model of asset pricing, namely the Arbitrage Pricing Model (APM), introduced by Ross (1976, 1977). The APM has received increased interest because it based upon a less restrictive set of assumptions than the CAPM, and does not involve any use of the market portfolio. There have been a number of empirical studies of the APM mainly for the U.S. capital market. These studies generally can be classified into two broap categories :

1) Those which attempted to verify empirically the model. Studies of this type were conducted by Gerh (1978), Roll and Ross (1980), Chen (1981), Reinganum (1981), Johnson (1981), Brown and Weinstein (1983), Chan and Beenstock (1984), and Lee and Wei (1984).

2) Those which focused on the assumptions required to transform the theoretical APM into a testable relationship. Studies of this area were offered by Gibbons (1981), Kryzanowski and To (1983), Dhryrnes, Friend, Gultekin (1984), Dhrymes, Friend, Gultekin, and Gultekin (1985), and Diacogiannis (1986a, 1986b).

Among the studies of the first category Gehr (1978), Reinganum (1981), and Jonhson (1981), utilized porfolios and they provided some evidence indicating that the CAP M can be verified empirically. Unfortunately these studies have not given attention to the problems encountered in testing the APM using a portfolio methodology. As a consequence their conclusions may be misleading and should be interpreted with caution.

The objective of this paper is to address the methodological problems asso ciated with the security return generating mode! of the Arbitrage Pricing Theory-(ÂPT) using portfolios of London Stock Exchange Stocks.

This paper has the following structure. The first section discusses the APM. The second describes the sample and the statistical methodology used. The next section presents the empirical results. Finally a brief summary of the paper is contained in the fourth section.

THE /ARBITRAGE PRICING THEORY

The basic assumption of the APM is the existence of unique return generating model that describes the t^{tb} ex-post return of any security under consideration in terms of a small number of connon factors. That is, the ex - post return of any security i is generated by the linear K - factor model :

$$\tilde{R}_{it} = E(\tilde{R}_{it}) + b_{i_1}\tilde{\delta}_{1t} + b_{i_2}\tilde{\delta}_{2t} + \dots + b_{ik}\tilde{\delta}_{kt} + \tilde{e}_{it} \qquad (1)$$

where $i = 1,2,\dots\dots N.$

$E(\tilde{R}_{it})$ = the single period expected return on security i.

$\tilde{\delta}_{it}$ = an unknown common factor, with $E(\tilde{\delta}_{rt}) = 0$ that affects the security returns during the period t, where $r = 1,2,\ldots, K$, and $K < N$.

b_{ir} = the sensitivity of the security i's returns to the fluctuations in the common factor $\tilde{\delta}_{rt}$. The coefficient δ_{ir} is called the r - factor coefficient.

$\tilde{e}_{it}$ = the security specific disturbance with $E(\tilde{\varepsilon}_{it}/\tilde{\delta}_{kt}) = 0$. The e's are commonly distributed. Also the security's e's are independent with any other security's e's and each disturbance has finite variance.

If $X_p = [x_{p1}, x_{p2},\ldots x_{pN}]$ is a (Nx1) vector of investment proportions, defining a portfolio p, then making use of equation (1) it follows that

$$\tilde{R}_{pt} = E(\tilde{R}_{pt}) + b_{p1}\tilde{\delta}_{1t} + b_{p2}\tilde{\delta}_{2t} + \ldots + b_{pK}\tilde{\delta}_{Kt} + K_t + \tilde{e}_{pt} \tag{2}$$

where

$\tilde{R}_{pt} = \sum_{i=1}^{N} \tilde{R}_{it}$ = the rate of return on porfolio p in period t.

$E(R_{pt}) = \sum_{i=1}^{N} E(\tilde{R}_{it})$ = the single period expected return on portfolio p.

$b_{pt} = \sum_{i=1}^{N} b_{it}$ = the sensitivity of porfolio p's returns to the fluctuations in the common factor $\tilde{\delta}_{rt}$

$\tilde{e}_{pt} = \sum_{i=1}^{N} \tilde{e}_{it}$ = the portfolio p's specific disturbance.

$$\sum_{i=1}^{N} x_{ip} = 1$$

Assuming that no arbitrage profits can be made and taking into account Ross's (1966, 1977) procedure, then the security return generating model represend by equation (2) implies ythe following approximately linear expected return - risk relationship :

$$r_p \approx r_Z + b_{p1}\,(r_{L1} - r_Z) + b_{p2}\,(r_{L2} - r_Z) + ... + b_{pK}\,(r_{LK} - r_Z) \qquad (3)$$

where

r_{L1} = the expected return on a portfolio with unit sensitivity to the K^{th} factor and zero sensitivity on the remaining K - 1 factors.

r_Z = the expected return on a portfolio that is otrhogonal with each portfolio LK, for each K.

I. DATA AND PORTFOLIO GROUPS

The London Share Price Database (LSPD) constitutes the source of data for this study. The LSPD monthly stock log - return file includes all securities that have traded on London Stock Exchange (LSE) since January of 1955. A firm was included in the sample if it satisfied the following criterion :

a) It is listed continuously on the LSE during a sample period beginning on January 1st 1972 and ending on December 31st 1983.

Given this objective 899 securities were selected with continuous data during the 12 year period. Such an objective may introduce a survival bias in the sense that it has only included firms in existence during 144 months, the sample is thus bias towards long - lasting firms and the results of this study have to be interpreted with this in mind.

Among those firms there were some whose securities had at least on month with no recorded trade. Including in the sample such firms will bias the estimates of second order moments which in turn will produce biased correlations (covariances) matrices. Therefore it was necessary to consider another criterion for the inclusion of the firm in the sample. Namely :

b) Over the entire sample period of 144 monthly observations securities having no recorded trade in more that there months must be excluded.

The decision to include securities having no recorded trade in two or three months over the entire sample period was made, because there were only a very small number of such securities and of these the months in which there was no recorded trade were normally mnot sequential. The second selection criterion reduced the sample of this study to 672 securities. The new sample may be biased by the deletion of the securities which had no recorded trade in more than three months.

From the total number of securities were randomly selected 6 portfolio groups of size 30, containing 5, 10, 15, *20*, 25 and 30 securities respectively. The selection of the portfolios comprised of 5, 10, 15, 20, and securities was made by using the method of no replacement, whereas the selection of the portfolios containing 30 securities was achieved by employing the technique of replacement. From each portfolio group 6 subgroups were formed comprised of 5, 10, 15, 20, 25, and 30 portfolios, respectively.

Π. PROBLEMS ASSOCIATED WITH THE SECURITY RETURNS GENERATING MODEL OF THE ARBITRAGE PRICING THEORY USING A PORTFOLIO METHODOLOGY

The theoretical validity of the APM requires the number of securities to be large enough so as to assure the application of the law of large numbers. As a consequence the model has to be tested by utilizing large samples of securities. Given limited computer processing capacity, the joint analysis of a large number of securities becomes impossible, and this in turn necessitates the division of the entire samble of securities into different groups. Since the theory behind the APM does not specify the nature of the relevant factors which have an impact on security returns, the most important problen of this methodology is the difficulty of assessing whether the same factors -generate the returns in each security group. Several different procedures have been adopted in order to overcome this problem. Among these is the grouping of the total number ofsecurities in the samble into portfolios and using the time series rate of returns of these portfolios to conclude- trie common factor structure. From this approach, however, a number of problems is emerged. These are investigated, explained and discussed in the following subsections.

Hi The suitability of the covariance correlation matrix for factor analysis

The APM describes an approximate linear relationship between expected returns on securities or portfolios and factor beta coefficients, but neither of these expected returns nor the factor beta coefficients are directly observable. As a result the APM is tested with the aid of ex - post data, the use of which implies the substitution of ex - post distributions for ex - ante distributions. This necessitates the assumption that portfolio returns obey a stationary multivariate distribution during the samble period.

Also the K - factor returns generating model shown in equation (2) is a static (single period), although in tests is treated as if it holds intertempo rally. The intertemporal validity of the model, however, is guaranteed if the joint distribution of portfolio returns is stationary during the samble period.

The third row of Table 1 provides the F - test values for the interternpo-

rai stationarity of the covariance matrix[1]. These results reveal that the covariance matrices of portfolio returns do not remain intemporally stationary and thus uniformaliy suggest that the joint distribution of portfolio returns is not intertemporally stationary. By factor analyzing the covariance matrix, heteroscedastic specific variances are estimated which in turn imply the asymptotic inefficiency of the factor loadings and so question the results which are based upon such estimated factor coefficients. Reinganurn (1981) provided a test of the APM by using the covariance matrix of security returns, but he left unverified the assumption regarding its intertemporal stationarity, his results, hence, may be questionable on this ground alone.

Table 1 also reports the values of chi - square test concerning the intertemporal stationarity of the correlation matrix 2. These results support the the hypothesis that the correlation matrix of porftolio returns remains stationary through time. Therefore the correlation matrix should be used for testing the APM to correct for heteroscedastic portfolio specific variances. These findings are in line with the empirical evidence produced by Gibbons (1981) using US portfolio groups, and with those of Diacogiannis (1986a).

There is another assumption which is made implicitly in factor analysis and it again concerns the correlation matrix. Such an assumption states that a correlation matrix is suitable for factor analysis if it includes several sizable correlations. Dziuban and Shirkey (1974) discuss three different techniques for the assessment of the adequancy of the correlation matrix for factor analysis. These techniques are (a) computation of the Bartlett's test of sphericity, (b) inspection of the off- diagonal elements of the anti - image correlation matrix, and (c) computation of the Kaiser - Meyer - Olkin measure of sampling adequancy. The first technique, that, proposed by Bartlett (1954) is very sensitive to the number of observations in the gamble. The test statistic of the approach is significant "with sarnbles having substantial size even if the correlations among variables are very small, and thus not recommended. The suitability of the correlation matrices was tested by embloying the second method (the third technique could also be used since it produces similar results. The findings presented in Table III of Appendix II reveal the adequancy of the correlation matrices for factor analysis.

1. This test was proposed by Box (1949).
2. This test was established by Jennrich (1970).

TABLE 1

AN F - TEST/CHI - SQUARE TEST FOR THE INTERTEMPORAL STATIONARITY OF THE COVARIANCE/CORRELATION MATRIX OF PORTFOLIO RETURNS [a]

Subperiods	Group of size 30 Containing Portfolios of size	F - Test[b]	Chi - Square Test[c]
11/56 - 1/63	5	1.48	312.87
and	10	1.51	315.14
2/63 - 4/69	15	1.54	321.17
	20	1.62	324.11
	25	1.58	348.00
5/69 - 7/75	5	1.52	320.14
and	10	1.54	332.18
8/75 - 10/81	15	1.68	341.44
	20	1.72	354.23
	25	1.89	364.21
11/56 - 1/63	5	1.40	289.27
and	10	1.54	290.28
5/69 - 7/75	15	1.58	298.35
	20	1.61	302.16
	25	1.68	303.80
2/63 - 4/69	5	1.38	340.28
and	10	1.42	345.44
8/75 - 10/81	15	1.49	350.65
	20	1.61	352.18
	25	1.73	356.77
11/56 - 5/69	5	1.74	320.11
and	10	1.86	323.19
6/69 - 12/81	15	1.84	329.48
	20	1.93	334.51
	25	2.01	338.19

a The assumed level of significance is 1 %.

b The null hypothesis that the covariance matrix of security returns is intertemporally stationary is rejected in all the cases at the 1 % level of significance. The critical value of the F - statistic is approximately 1.

c The null hupothesis that the correlation matrix of security returns is intertemporally stationary is accepted in all the cases at the 1 % level ofsignificance. The critical value of the x^2 statistic is approximately 506.55.

II_2. The instability of the number of factors acrorss fvariou time periods

In principle the portfolio returns generating model is static. A usual assumption made when time series data is use to test the APM is that the portfolio returns generating model holds in each required time interval (e.g. day, month) of the sample period. In this case except of the assumption regarding the intertemporal stationary distributions of portfolio returns an additional assumption is needed. This assumption is that the common factors affecting the portfolio returns remains unchanged across the various time intervals of the sample period. Due to the identification problem of the common factors, there is no way to ascertain whether the factors having influence on portfolio returns are repiicabie across varius time periods. This assumption can, however, be rejected if the number of factors affecting the portfolio returns changes across various time periods for the same portfolio group.

Table 2 shows the number of factors emerged by factor analyzing various portfolio groups across two different subperiods[3]. From these findings it can be observed that the number of factors does not remain the same across the subperiods 1/72 - 12/77, and 1/78 12/83. The consequences of the results are as follows :

a) In view of the identification problem of the portfolio returns generating model of the APM, it is very fihgicult to ascertain which is the appropriate time lenght that has to be utilized for the empirical examination of the APM. By using a given sample period it cannot be concluded that the producing portfolio returns generating model is the unique model of the APM, since if such a model exists it cannot be identified.

b) Portfolio returns generating models estimated with the aid of factor analysis cannot be considered as forecasting tools.

c) One of the basic assumptions required to give to the APM an empirical context is violated and thus the model cannot be tested unambiguously utilizing a portfolio methodology.

It is evidence that these conclusions do not necessarily imply the invalidity of the APM, they simbly show our inability to provide a rigorous statistical methodology to verify empirically the model.

3. (a) Factor analysis was performed using the statistical package of SPSS.
   (b) Details of the goodness of fit of the factor model are given in Rao (1955).

TABLE 2

NUMBER OF FACTORS ACROSS TWO NONOVERLAPPING SUBPERIODS FOR THE SAME GROUP OF PORTFOLIOS

Portfolio Size : 30 securities

	Group size 10		Group size 20		Group size 30	
Subperiod	Number of factors	% Variance	Number of factors	% Variance	Number of factors	% Variance
1/72 - 12/77	1[a]	45.8	4	55.4	5	59.4
	2	41.7	5	65.8	7	61.3
	2	42.5	3	49.1	7	62.7
	1	39.2	3	51.4	7	60.1
	1	37.1	4	57.8	8	68.1
1/78 - 12/83	2	58.1	3	64.9	7	70.2
	1	54.4	4	67.4	8	72.1
	2	60.1	5	69.2	6	69.4
	2	61.1	5	68.7	7	71.1
	1	42.8	2	58.3	6	68.9

a The null hypothesis that there exist exactly K factors is accepted at the 1 % level of significance.

II_3. The existence of a significant positive relationship between the number of factor and the size of the group being factored

The aim of factor analysis is to explain the interrelationships among a relatively large number of variables (portfolio returns) with a minimal number of factors. As a consequence a portfolio return generating model would be considered as satisfactory if the appropriate number of factors is minimal. Table 3 shows that the number of factors emerging via Rao's factor analysis is positively related to the group size. By averaging the number of factors across port-

TABLE 3

THE NUMBER OF FACTORS VERSUS THE GROUP SIZE [a]

	Portfolios comprised of 5 securities		Portfolios comprised of 10 securities		Portfolios comprised of 15 securities	
Group size	Number of factors	% variance	Number of factors	% variance	Number of factors	% variance
5	1	51.4	1	53.6	1	54.9
10	2	63.4	2	64.7	2	65.9
15	3	64.0	2	59.3	3	66.0
20	5	68.3	4	66.1	4	67.8
25	7	67.4	6	70.9	7	69.1
30	8	68.3	9	69.1	8	67.2

	Portfolios comprised of 20 securities		Portfolios comprised of 25 securities		Portfolios comprised of 30 securities	
Group size	Number of factors	% variance	Number of factors	% variance	Number of factors	% variance
5	1	56.6	1	59.4	1	61.6
10	1	47.7	1	49.2	2	68.7
15	2	58.9	3	64.2	3	65.4
20	3	66.1	3	67.7	4	65.8
25	6	65.6	5	64.3	6	68.9
30	7	67.9	7	68.8	7	77.3

Group size	5	10	15	20	25	30
Average number of factors	1	2	3	4	6	8
Average % variance	56.2	59.9	62.9	66.9	67.7	69.7

[a] Th null hypothesis that there exist exactly K factors is accepted at the 1% level of significance.

folio groups of the same size it can also be seen that the number of factors increases with group size. These findings show that the number of factors which influence portfolio returns, does not remain the same across various portfolio groups. The results presented here are in line with those of Gibbons (1981), Kryzanowski and To (1983), Dhrymes, Friend, and Gultekin (1984), and Diacogiannis (1986b).

The evidence of Table 3 have the following implications :

I) It is very difficult to assess which is the appropriate group size that has to be used in order to investigate the empirical validity of the APM. By utilizing portfolio groups having a given size it cannot be asserted that the producing portfolio return generating model is the unique model of the APM, since if such a model exists is unobservable.

II) A basic assumption concerning the uniqueness of the portfolio return generating model is violated. Hence the APM cannot be tested unambiguously using portfolios from the London Stock Exchange. As a consequence its introduction into the literature as a testable alternative of the CAPM may be challenged.

It is clear that these conclusions do not necessarity imply the invalidity of the APM, they simply show our inability to provide a rigorous statistical methodology to the model.

II4. Â problem in extracting the number **of** common factors **from** large **portfolios**

When portfolios are used in factor analysis an additional problem arises with respect to the eigenvalue-one criterion for extracting the number of common factors which influence returns. In this case the eigenvalue-one criterion reflects the existence of a single factor model when in fact a multifactor model exists. Table 4 shows the eigenvalues and the percentage of the variance accounted for by the common factors utilizing portfolios of size 5, 15, and 30 securities. These clearly indicate that the importance of the first (other) factor (factors) increases (decreases) with the portfolio size. A comparison of Tables 3 and 4 reveals that Rao's goodness of fit of the factor model statistical test gives more that one factor.

The findings of Table 4 may be considered as a potential explanation of some previously reported results regarding the use of portfolios in factor analysis. Gehr (1978) factor analysis large portfolios and concluded that the first factor

TABLE 4

PORTFOLIO SIZE AND THE VARIANCE EXPLAINED BY THE COMMON FACTORS
GROUP SIZE : 30 PORTFOLIOS

Portfolios comprised of 5 securities			Portfolios comprised by 15 securities		
Factor	eigenvalue	proportion of total variance accounted by the common factor	Eactor	eigenvalue	proportion of total variance accounted by the common factor
1	10.21	41.1	1	15.8	48.9
2	3.41	16.3	2	2.4	12.8
3	2.12	4.1	3	1.9	3.7
4	1.81	3.0	4	1.5	2.5
5	1.69	2.4	5	1.1	1.9

Portfolios comprised of 30 securities

Factor	eigenvalue	proportion of total variance accounted by the common factor
1	20.1	68.0
2	1.8	3.6
3	1.2	2.2
4	0.9	1.7
5	0,6	1.0

explained 64 % of the total variance, whereas the explanatory power of the second one was only 4 %. Due to the unreability of the eigenvalue-one criterion he used, his results may be misleading and should interpreted with caution. Brennan and Schwartz (1980) factor analyzed the residuals of bond portfolios and found that the first factor explained 83 % of the total variance, while the explanatory power of the second one reduces only to 3 %. In view of their results they concluded the existence of only one factor, a conclusion which receives the same criticisms of that of Gehr (1978).

CONCLUSIONS

In this paper the portfolio return generating model of the APT was examined, utilizing portfolio of London Stock Exchange stocks. À combination of the empirical findings of the present study produces several conclusions concerning the portfolio return generating model. These include :

a) The variance matrix of portfolio returns does not remain stationary through time, whereas the correlation matrix is intertemporally stationary. As a consequence the latter matrix should be used estimating the portfolio return generating model to correct for heteroscedastic portfolio - specific variances.

b) The number of factors which influence the portfolio returns changes through time. This imlies that the portfolio return generating model cannot be used for predictive purposes and indicates the impossibility of contacting an empiri-cal test regarding the intertemporal stationarity of the factor beta coefficients.

c) The number of factors increases with group size which in turn shows thai the portfolio return generating model is not unique as the APT requires. This higlights the fact that the APM may be true, but the existing statistical methodology does not provide an unambiguous test of the model using LSE stocks.

d) The use of large portfolios in factor analysis reflects the existence of a single factor model when in fact a multifactor model is true.

These conclusions clearly indicate that the utilization of factor analytic techniques in research on portfolio returns will always yield ambiguous results which in turn casts considerable doubts on the usefulness of the techniques. Therefore it is not the appropriate methodology that can be employed for testing financial models of porfolio returns.

APPENDIX I

A TEST OF THE HOMOGENEITY OF TWO COVARIANCE MATRICES PORTFOLIO RETURNS

This test examines the intertemporal stationarity of the covariance matrix of portfolio returns. The test statistic is represented by the following equation :

$$F_1 = \frac{f_1 (T-1)}{1 - \dfrac{2p^2 + 3p + 1}{4(p+1)(T-1)} \dfrac{f_1}{f_2}} \ln \frac{(\mid S_g + S_{g*})^{2}/_4}{\mid S_g \mid \mid S_{g*} \mid}$$

where

F_1 is approximately distributed as a central F— variate with f_1 and f_2 degrees of freedom.

$f_1 = p \ (p + 1)/2$

$$f_2 = \frac{24[(p+1)^2 \ (T-1)^2] \ [p \ (p+1) \ + 4]}{14 \ (p-1) \ (p+2) \ (p+1)^2 - 3(2p^2 + 3p - 1)^2}$$

p = the number of porttfolios in the group.

ln = the natural logarithm operator.

S, S = the unbiased covariance matrices estimated using the first and second T observations respectively.

The null hypothesis of the homogeneity of the two covariance matrices is accepted if $F_1 < F_{a,f1,f2}$, where $F_{a,f1,f2}$, is critical value of the test statistic.

A TEST OF THE HOMOGENEITY OF TWO CORRELATION MATRICES OF PORTFOLIO RETURNS

This test investigates the intertemporal stationarity of the correlation matrix of portfolio returns. The statistic is given by the following equation :

$$C = \tfrac{1}{2}\ \mathrm{tr}(Z^2) - dg'\ (Z)S^{-1}\ dg'(Z)$$

where

C is distributed as a central chi - square variate with p (p— 1) / 2 degrees of freedom.

P = the number of portfolios in the group.

$$Z = \sqrt{\frac{T}{2}\left[\frac{R_g + R_{g*}}{2}\right]^{-1}}\quad (R_g - R_{g*}).$$

T = the number of observations.

R_g, R_{g*}, = the unbiased correlation matrices estimated using the first and second T observations respectively.

$Z^2 = ZZ.$

tr (.) = the trace of the square matrix Z.

$S = I + r_{ij} r^{ij}$.

I = the identity matrix.

r_{ij} = the entry of the matrix $(R_g + R_{g*})/2$ appearing in the ith row and the j^{th} column.

r^{ij} = the entry of the matrix $[(R_g + R_{g*})/2]^{-1}$ appearing in the i^{th} and the j^{th} column.

$dg'(Z)$ = the vector whose entries are the diagonal elements of the matrix Z.

$dg(Z)$ = the transpose of dg(Z).

The null hypothesis regarding the homogeneity of the two correlation matrices is accepted if $C < X^2_{a,p(p-1)/2}$, where $X^2_{a,p(p-1)/2}$ is the critical value of the test statistic.

APPENDIX II

TABLE 5

THE ANTI-IMAGE TEST INDICATING THE ADEQUACY OF THE CORRELATION MARTIX OF PORTFOLIO RETURNS FOR FACTOR ANALYSIS

Portfolio size	Groups containing 10 portfolios	Groups containing 20 portfolios	Groups containing 30 portfolios
5	18.4%[a]	8.4%	4.6%
10	17.6%	7.3%	3.8%
15	16.9%	7.7%	3.2%
20	19.3%	6.9%	3.3%
25	19.1%	7.1%	3.5%

a. They indicate the percentage of the off - diagonal elements of the anti - image covariance matrix which are greater than .09.

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