MEASURING WELFARE LOSSES WITH A PRICE - ADJUSTING DIFFERENTIATED PRODUCT OLIGOPOLY

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1. INTRODUCTION

In a recent article C.A. Holt[1] indicated the difficulties involved using the Lerner equation to estimate theoretically welfare losses when the predominant industrial structure is not monopolistic but oligopolistic. Using a Cournot-type oligopoly situation, where firms produce a homogeneous good and marginal costs are constant, he showed that «the Lerner equation reuslts to a significantly greater welfare cost than the actual cost of imperfect competition in an oligopoly».

However, and regardless of the degree of accuracy the general conclusion i.e. a negative relationship between number of firms and associated welfare loss estimates, has not been discussed by Holt or others. Thus, while the literature on oligopolistic markets is by now very extensive, little attention has been paid so far to this otherwise intuitive result.

It is the main purpose of this paper to consider a differentiated Bertrand-type model and show how the degree of product differentiation alters this «otherwise intuitive result». Thus, we first investigate, in contrast to C.A. Holt's model, how the adjustment process, from quantity to price, affect the magnitude of welfare loss. Second we extent the model incorporating elements of product differentiation and conditions describing more accureteiy the degree of heterogeneity in order to show how the type of demand function used change the welfare loss measurement.

II. THE MODEL

Consider the market for a homogeneous good in which the inverse demand function is :

1. $P = a - bQ$

The supply side of this market consists by (n) firms operating as Cournot oligopolists with summetric cost functions i.e. $C_i = cQ_i$ $i = 1 \ldots n$. Defining total Surplus as :

2. $$S = \int_0^{v} (p - c)dQ = (a - c)Q - \frac{b}{2} Q^2$$

easily we can derive the maximum level of it if we use for Q the value of the quantity at the competitive solution. From the profit function of the firm (i) and under the competitive solution

$P = MC$ we derive :

3. $$Qc = \frac{a - c}{b}$$

Substituting (3) to (2) we get ;

$$4. \quad Sc = \frac{(a - c)^2}{2b}$$

which is the reference point from our analysis.

This Cournot-Nash equilibrium solution for the (n) firms case with symmetric constant average cost function is determined as follows :

$$5. \quad \Pi_i = p.q_i - cq_i \qquad Q = \sum_{i=1}^{n} q_i$$

max Π_i 6. $p + p.q_i - c = 0$

because the (n) firms are all identical we can add all over equations (i) :

$$7. \quad np + p \, \Sigma q_i - nc = 0$$

8. Given that $p = a - b\Sigma q_i$

$$9. \quad n(a - bQ) - bQ - nc = 0$$

Thus 10. $Q_{c-N} = \frac{n(a - c)}{(1 + n)\,b} \quad q_i = \frac{Q_{c-N}}{n}$ and $p_{c-N} = \frac{a + nc}{(1 + n)\,b}$

Substituting (P, Q) solution for the Cournot - Nash (C — N) case at an original profit function we get :

$$11. \quad \Pi^*_i = \left[\frac{(a - c)^2}{n + 1} \right] \frac{1}{b} \quad \forall \; i = 1 \ldots . n$$

and finally

$$S_{c-N} = \frac{(a - c)^2}{2b\,(n + 1)^2}\;(n^2 + 2n)$$

which is a measure of consumer surplus resulting under condition of Cournot-Nash equilibrium in the market. In order to calculate the magnitude of welfare loss we determine the difference :

$$12. \qquad S_c - S_{c-N} = \frac{1}{2b} \cdot \left[\frac{(a - c)}{n + 1}\right]^2 = \frac{1}{2}\;\Pi^*_i$$

a common result, and a reference point for any empirical analysis related with the measurement of welfare loss under oligopolistic conditions. However, as it is convincingly showed by C.A. Holt, this is not meaningful measure of welfare loss, because it treats all sellers as isolated monopolists.

Let's make the assumption that one firm is divided in two different firms In that case the welfare loss resulting from the difference

$$\int_{c-N}^{n+1} - \int_{c-n}^{n} \text{ is :}$$

$$13. \qquad \int_{c-n}^{n+1} - \int_{c-N}^{n} = \frac{1}{2b}\,\frac{(a-c)^2}{n + 1}\left[\frac{(n + 1)^3\,(n + 3) - (n + 2)^2 n}{(n +)^2}\right]$$

measure which is not independent from the number (n) of firms operating in the market, as the Lerner equation.

Attempting to compare the Cournot - Nash homogeneous case with the Bertrand - Nash heterogeneous result we reformulate the problem as follows.

Let the market demand function is :

$$14.\quad Q = a_i - b_i p \qquad (a_i = \frac{a}{b} \text{ and } b_i = \frac{1}{b}$$

and the individual function faced by the firm (i) is :

$$15.\quad q_i = a_n - b_n d_i + \beta_n \Sigma_{j \neq i} p_i \ (a_n,\ b_n,\ \beta_n > 0)$$

or

$$16.\quad q_i = a_n - \left\{ b_n - (n-1)\ \beta_n \right\} p_i + \beta_n \Sigma_{j \neq i} \cdot (p_j - p_i)$$

if :

$$17.\quad a_1 = na_n \quad \text{and} \quad b_1 = n \left\{ b_n - (n-1)\beta_n \right\}$$

The equilibrium solution for the Bertrand-Nash case is in general :

$$18.\quad P^*_{B-N} = \frac{a_n + cb_n}{2b_n - (n-1)\ \beta_n} \quad \text{and using equations (17)}$$

$$19.\quad P^*_{B-N} = \frac{a_1 - cb_1}{2b_1 + n(n-1)\ \beta_n} \quad + c$$

and

$$20. \quad Q^*_{B\text{-}N} = a_1 - b_1 \left[\frac{a_1 + b_1 c + cn(n-1)\beta_n}{2b_1 + n(n-1)\beta_n} \right]$$

With these results easily we can determine the welfare loss for the Bertrand-Nash case as follows :

$$21. \quad \int_{B\text{-}N}^{n} = \left(\frac{a_1}{b_1} - c\right) \left[a_1 - b_1 \frac{a_1 + b_1 c + cn(n-1)\beta_n}{2b_1 + n(n-1)\beta_n} \right]$$

$$- \frac{I}{2b_1} \cdot \left[a_1 - b_1 \frac{a_1 + b_1 c + cn(n-1)\beta_n}{2b_1 + n(n-1)\beta_n} \right]$$

or

$$22. \quad \int_{B\text{-}N}^{n} = \frac{1}{2} \left[a_1 - b_1 \frac{a_1 + b_1 c + cn(n-1)\beta_n}{2b_1 + n(n-1)\beta_n} \right]$$

$$\left[\frac{a_1}{b_1} + \frac{a_1 + b_1 c + cn(n-1)\beta_n}{2b_1 + n(n-1)\beta_n} - 2c \right]$$

Following the same process we used for equation (13) we can measure th5 welfare loss which from a first observation is not equal to $\frac{1}{2}$ of Π^*_1 given that:

$$\Pi^*_i = (p^* - c) \left(\frac{a_1}{n} - \frac{b_1}{n} \cdot p^* \right)$$

or

$$23. \quad \Pi^*_i = \frac{(a_1 - cb_1)2}{2b_1 + n(n-1)\beta_n} \left[\frac{1}{n} \quad \frac{b_1}{2b_1 + n(n-1)\beta_n} \right]$$

and

$$\int_{B-N}^{n} = a_1 \left(\frac{ad_1}{2b_1} - c \right) + \frac{1}{n} \left(c - \frac{1}{\lambda b_1 n} \right) . A + \frac{2b_1 + n(n-1)\beta_n}{b1\ (a_1 - cb_1)^2} A . \left[\frac{a_1}{2b_1} \right.$$

$$\left. \frac{a_1}{2} + \frac{1}{nb_1} A - c - \frac{1}{2} \quad \frac{2b_1 + n(n-1)\beta_n}{b_1\ (a_1 - cb_1)^2} A\Pi_i \right] . \Pi_i$$

Where : $A = a_1 + b_1 c + n\ (n-1)\ \beta_n c$

Again in order to measure the welfare loss in this Bertrand - Nash model we will work out the case for the $(n + 1)$ we get :

$$24. \quad \int_{B-N}^{n+1} = \frac{1}{2} \left[a_1 - b_1 \frac{a_1 + b_1 c + cn\ (n+1)\ \beta_n}{2b_1 + n\ (n-1)\ \beta_n} \right]$$

$$\cdot \left[\frac{a_1}{b_1} - 2c + \frac{a_1 + b_1 c + cn\ (n+1)\ \beta_n}{2b_1 + n\ (n-1)\ \beta_n} \right]$$

Comparing (24) and (22) we observe that the effects on welfare loss which a change in the number of sellers is going to have are traced easily if we compare the relative magnitude of the following two ratios.

$$\frac{a_1 + b_1c + cn\,(n+1)\,\beta_n}{2b_1 + n\,(n-1)\,\beta_n} \quad \text{and} \quad \frac{a_1 + b_1c + cn\,(n-1)\,\beta_n}{2b_1 + n\,(n-1)\,\beta_n}$$

Assuming β_n constant we conclude that increasing the number of sellers is not always synomenous with the intuition that welfare loss is going to diminish in the market. Actually for this specific model the intuition holds only when the following condition is true :

$$25. \quad \frac{a_1}{b_1} > c$$

Our first result suggests that surprisingly increasing the number of sellers in a Bertrand type model, under certain conditions will result to a decrease in social welfare.

Furthermore, and because : $a_1 = na_n$ and $b_1 = n \left\{ b_n - (n-1)\beta n \right\}$

$$(25) \quad \text{becomes} \quad \frac{a_n}{b_n - n_{\beta n}} > c$$

suggesting that the closer βn is to one (perfect substitutatility) i.e. the smaller the degree of differentiation is, the less important the cost is in order to ger peculiar results with respect to social welfare.

RUEGERECES

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