THE EXPECTED RETURN-RISK LINEARITY OF AN INDIVIDUAL INVESTOR UNDER DIFFERENTIAL AND CAPITAL GAINS

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The Capital Asset Pricing Model (CAPM) constitutes one of the major extensions of the two - parameter, one - period portfolio analysis model of Markowitz (1952). In a perfect capital market given the existence of a riskless security and homogeneous expectations among risk - averse investors, Sharpe (1964), Lintner (1965) and Mossin (1966) have shown the theoretical validity of a single-period ex-ante equilibrium relationship between the return on a security of portfolio end its risk in the market portfolio (the beta coefficients). More recently Black (1972) considered the case with short selling but without riskless borrowing or leading and he proved the zero - beta version of the CA.PM.

Athough both versions of the CAPM have been the basis for many theoretical and empirical studies in the capital markets, Roll (1977) raised some legitimate questions regarding their testability. He argued that any version of the CAPM may be valid, but it cannot be tested unambiguously unless the exact composition of the market portfolio is known and used in the empirical tests.

Because the CAPM assumes several real world influences away, a considerable amount of effort has been expended in modifying the model in order to bring theory in closer conformity with reality. Mayers (1972, 1973, 1976) has proposed an extension of the CAPM for the case where the entire number of securities can be divided into two groups : one containing costlesly marketable securities and the other containing non - marketable securities. Brennan (1970), Long (1979), and Elton and Gruber (1978) extended the basic model to a situation where differential taxation of dividends and capital gains is taken into consideration. Theobald (1979) and Milne and Smith (1980) incorporated variable transaction costs into the setting of the CAPM. Gonedes (1976) developed an equilibrium model by relaxing the assumption of homogeneous expectations, while Lindenberg (1979) derives equilibrium conditions by assuming a non - price taking behaviour. Finally Merton (1973), and Constadinedes (1982) offered continuous times intertemporal asset pricing models. However, these extensions of the CAPM rely heavily on the identification of the market portfolio and so in view of Roll criticisms they reveal the lack of empirical verification.

The objective of this study is to develop an expected return-risk linearity incorporating differential taxation of dividends and capital gains. It is worthwhile emphasizing that such a linearity is analoguous to previous asset pricing models which are based upon differential taxation, but in the present procedure no use is made of the market portfolio nor are estimates of market portfolio returns necessary to its tests. The present approach is a model that applies to an individual investor and it utilizes only his optimal portfolio which is observable. As a consequence it does not share the criticisms of Roll regarding the testability of the CAPM.

This paper is organised as follows. The first section is devoted to the mathematical proof of the individual investor's after-tax expected return-risk linearity and it summarizes the advantage of using such a relationsihip. The next section discusses the conditions for the equivalence of the individual investor's before - and after-tax risk-return linearities. The third and final section, contains a summary of the paper.

I. THE DERIVATION OF THE AFTER-TAX EXPECTED RETURN-RISK LINEARITY OF AN INDIVIDUAL INVESTOR

In the following pages a single - period expected return-risk linerarity that takes into account an individual investor's marginal tax rates is derived. Consider an individual investor who selects a finite number of N risky securities, Investors are assumed to be single - period expected utility maximazers, whose expected utility functions are of the form $U(r\ ,\ \sigma\ )$ with $dU/dr > 0$ and $dU/da_p{}^{*2}p<0$.

The investor portfolio selection problem can be described as follows :

$$\max_{X_p} U(rp_p^{*},\ \sigma_p^{*2})$$

subject to $x'\ i = 1$

The first - order condition for maximum values give

$$\frac{dU}{dr_p{}^*}\ \frac{dr_p{}^*}{dX_p} + \frac{dU}{da_p{}^*2}\ \frac{d_{\sigma p}{}^*2}{dXp} - Ul\ l\ \_0 \qquad (3)$$

and $X\ \ i = 1$

where

λi is a Lagrangian multiplier.

Differentiating equations (1) and (2) partially with respect to X_p it follows respectively that

$$dr_p^*/dX_p = (1 - t_g)\,R + D\,(1 - t_d) \tag{4}$$

and

$$d\sigma_p^{*2}/dX_p = 2\,(1 - t_g)\,VX_p \tag{5}$$

Substituting equations (4) and (5) into equation (3) and manipulating this becomes

$$R^* = c_1\,i + c_2 VX_p{}^2 \tag{6}$$

where

$$R^* = (1 - t_g)\,R + D\,(1 - t_d) \tag{7}$$

$$c_1 = \lambda_1/(dU/dr_p^*) \tag{8}$$

$$c_2 - (1 - t_g)\,dU/d\sigma_p^{*2}/dU\,dr_p^* \tag{9}$$

2 (a). The investment proportions vector of an after-tax efficient portfolio is derived in Appendix I.

(b). A portfolio, other than the global efficient portfolio (see c below), is said to be an after - tax efficient if :

(i) no other portfolio with the same after tax expected return can have lower after - variance of return and

(ii) no other portfolio with the same or lower after-tax variance of return can have higher after - tax expected return.

(c). The global efficient portfolio is the efficient portfolio with the smallest variance of returns. The investment proportions vector defining such a portfolio is given by $X_{fl} = V^{-1}\,i/i'\,V^{-1}i$ and it is independent of tax rates. This is the reason of excluding the global efficient portfolio from the after - tax efficient portfolio set.

Suppose that $X^{3}z$ is the investment proportion vector defining the orthogonal portfolio of p, then by postmultiplying equation (5) by X_{ζ} it can be seen that

$$r_z = C_1$$

where

rz is the expected after - tax retourn on the orthogonal portfolio of p[4]

Substituting c_1 from the previous equation into equation (6) it can be established that

C

$$R^* = r_z i + c_2 VX \qquad (10)$$

After postmultiplying both sides of equation (6) by X ρ and rearranging terms is easy to derive the following expression

$$C2 = r_p - r_z \qquad (11)$$

Substituting the expression for C_2 into equation (10) yields

$$R^* = r_z i + (r_P - rz)\frac{VX_P}{\sigma_p^{*2}} \qquad (12)$$

3. The investment proportions vector defining the after - tax efficient portfolio orthogonal to the investor's after - tax efficient portfolio is given in Appendix I.

4. The derivation of equation (13) of Appendix I from the investor's selection problem also proves the after - tax efficient set theorem, that is : In the presence of taxation the optimal portfolio of an investor is after - tax efficient and different from the global efficient portfolio.

The i^{th} th row of the vector equation (12) is written as

$$r_i^* = (1 - t_g)\ r_i + (1 - t_d)d_i = (1 - t_g)r_Z + (1 - t_d)d_Z +$$

$$[\ (1 - t_g)\ (r_p - r_Z) + (1 - t_d)\ (d_p - d_Z)\ ]\ b_i \qquad (13)$$

$$\text{where} \quad b_i = \sigma^*_{ip} / \sigma^{*2}_p \qquad (14)$$

with σ_{ip} = the covariance between the after - tax returns of a security i and the investor's optimal portfolio p.

σ^{*2}_p = the variance of the inverstor's after-tax optimal portfolio p.

Since it has been assumed that the next period' s dividend is certain, then it can written

$$\sigma^*_{ip} = (1 - t)^2\ \sigma_{ip}$$

$$\sigma^{*2}_p = (1 - t)^2\ \sigma^2_p$$

Substituting in equation (14),

$$b_i = \frac{\sigma_{ip}}{\sigma^{2}_{p}} \qquad (15)$$

Equation (13) represents the after-tax expected return- risk relationship of an individual investor. It states that the expected after - tax return of a security i is related linearly and exactly to its risk relative to the investor's optimal portfolio. In the return - risk plane the after - tax expected return - risk relationship of an investor defines a straight line intersecting the expected after - tax axis at i_z. The advantages of the relationship derived in this paper can be described as follows :

(a) The individual investor's after-tax expected return - risk linearity does not involve any use of the market portfolio. It is based only upon the personal portfolio of a specific investor which can be identified and utilized in the tests. As a consequence the criticisms of Roll (1977) concerning the empirical validity of the CAPM are not applicable to the model of this study. The coefficient of the present model can be estimated via the slope coefficient of a linear regression between the return of a security and the return of the investor's portfolio and is defined by equation (15) above.

(b) The individual investor's after-tax risk-return linearity takes into consideration his differential taxation of dividends and capital gains and so provides a more realistic expected return - risk relationship that those which assume a taxless world.

(c) The individual investor's after - tax expected return - risk linearity does not require the stringent assumption of the homogeneous expectations of the CAPM. Given that an investor has selected a number of N individual securities, where $N = 1, 2, \ldots, \eta$, his efficient frontier can be determined. Such a frontier shows the relationship between expected portfolio return and portfolio risk and is unique to him. This reveals that the relationship derived in this paper describes expected returns on the micro level in contrast to the CAPM which provides a description of expected returns on a marco level.

(d) The individual investor's after - tax risk - return linearity can be tested by considering only a subset of marketabie securities, namely the total number of individual securities held by the investor. Consequently the criticisms concerning the unrealism of the CAPM assumption that assets are readily marketable are not shared by the present approach.

1 EQUIVALENCE OF THE INDIVIDUAL INVESTOR'S BEFORE-AND AFTER-TAX EXPECTED RETURN - RISK LINEARITIES

The individual investor's before-tax expected return-risk linearity can be easily derived from equation (13) by setting $t_g = ta = 0$.

$$\eta \ + di = r_z \ + dz \ + f \ (r_p - r_z) \ + (dp - dz) \ ] \ bi \qquad (16)$$

Equation (13) and (16) are not generally equivalent. There are, however, three circumstances which imply the equivalence between the two ralationships.

Proposition 1.

If the portfolio of an investor is after - tax efficient then the following statements are equivalent :

(i) The marginal capital gain tax rate and the marginal income tax rate are equal.

(ii) The individual investor's after - and before - tax expected return -risk leariities are equivalent.

Proposition 2

If the portfolio of an investor is after - tax efficient and his marginal before - and after tax rates are not equal and both are different from zero then the following statements are equivalent :

(i) The individual investor's before - and after - tax expected return - risk linearities are equivalent.

(ii) There exist constants a_1 and a_2 such that

$$di = a_1 Ri + a_2, \text{ for all } i = 1,2 \ldots\ldots N,$$

where R is the expected after - tax total return on security i.

Corollary 1

If the portfolio of an investor is after - tax efficient and his marginal before - and after - tax not equal and both different from zero then the following statements are equivalent :

(i) The individual investor's before - and after - tax expected return - risk relationships are equivalent.

(ii) Security dividends are related to betas by the linearity

$$di = d_z + (di - d_z)\ bi, \text{ for all } i = 1,2,\ldots N\ s.$$

The sufficient condition of proposition 2 is in line with Long's (1972) proposition 2. He has proved that the before- and after- tax CAPM's are equivalent if there exists an exact cross - sectional linearity between the dividends yields and total returns. However, he restricted the value of the coefficient of the total returns, proposition 2 of this study does not impose any restriction on the value of a_1. This proposition also shows that the linearity between yield and return is implied by the equivalence between the before - and after tax expected return - risk relationships and so it provides a more general result than that of Long.

A portfolio is before-tax efficient if and only if equation (14) is valid[6]. In this study it was proved that a necessary and sufficient condition for the after - tax efficiency of the portfolio is the validity of equation (12). The before - and after - tax expected return - risk linearities are not general equivalent which in turn reveals that the before - tax optimal portfolio of an investor is generally different from his after - tax optimal portfolio. However, investors seek to hold

5. (a) See Appendix II for the proofs of Propositions 1 and 2, and Corollary 1. (b) The validity of the before - (after-) tax linearity implies before - (after-) tax optimality. So can be inferred that the three circumstances given in Proposition 1 and 2, and Corollary 1, respectively, also imply equivalence of before - and after - tax optimality.

6. See Roll (1977), Corollary 6, p. 165).

after-tax optimal portfolios, and hence by considering no taxes it is most likely they will chose portfolios that are not optimal [7].

III. CONCLUSIONS

In this paper the individual investor's after - tax expected return - **risk** linearity has been developed. Unlike the CAPM which describes expected returns on the macro level, the present approach provides a description of expected returns on the micro individual investor behaviour. The empirical examination of this procedure can be achieved by only using in the tests the specific investor's portfolio which can be identified. As a result, the present procedure does not share the criticisms associated with testing models which rely upon the identification of the market portfolio. In the presence of personal taxes the individual investor's after - and before - tax linearities are not generally equivalent. Only under three restrictive conditions are the two linearities equivalent. These are : (1) when the marginal capital gains tax rates is equal to the marginal income tax rate, (2) when the dividends yields are related linearity to expected total security returns, or (3) when the dividends yields are related linearly to security betas. However, these three situations are rarely met in reality. As a consequence the after - tax expected return - risk linearity must be taken into consideration since it recognizes an influence of the real world, namely the existence of differential taxation of dividends and capital gains.

7. All the previous studies concerning the empirical validity of the before - tax expected return - risk linearity were based upon selected proxies for the market portfolio. In the presence of taxes, which is the case of the real world, there was a large possibility that the chosen proxies were before - tax inefficient. This is a possible explanation of the rejection of the before - tax expected return - risk relationship.

APPENDIX I

DERIVATION OF THE INVESTMENT PROPORTIONS VECTOR OF AN AFTER-TAX EFFICIENT PORTFOLIO

By rearranging terms equation (6) is written as

$$X_P = V^{-1} (R^* \ i) \begin{pmatrix} 1/c_1 \\ -c_1/c_2 \end{pmatrix} \qquad (I\ 1)$$

Where it was assumed that the covariance matrix is not singular.

Premultiplying both sides of equation (I 1) by (R*i)', the result is reduced to

$$\begin{pmatrix} r_p^* \\ 1 \end{pmatrix} = M \begin{pmatrix} 1/c_1 \\ -c_1/c_2 \end{pmatrix} \qquad (I\ 2)$$

where

$$M = \begin{bmatrix} R'^* V^{-1} R^* & R'^* V^{*-1} i \\ R'^* V^{-1} i & i' V^{-1} i \end{bmatrix}$$

Assuming that the vector R contains at least two different entries a combination of equation (I 1) and (I 2) produces the investment proportion vector of an after - tax efficient portfolio [8]

$$X_p = V^{-1} (R^* \; i) \; M^{-1} \begin{pmatrix} r^*_z \\ 1 \end{pmatrix} \qquad (1\ 3)$$

Similarly it can be proved that the after-tax efficient portfolio which is orthogonal to the optimal portfolio ρ is given by

$$X_Z = V^{-1} (R^* \; i) \; M^{-1} \begin{pmatrix} r^*_z \\ 1 \end{pmatrix} \qquad (1\ 4)$$

EQUIVALENCE BETWEEN THE AFTER-TAX EFFICIENCY OF PORTFOLIO AND THE AFTER - TAX EXPECTED RETURN - RISK RELATIONSHIP

Suppose that ρ is an after - tax efficient portfolio. Then there exists an infinite number of portfolios haying the same expected return and being uncorrected with p. Let X_{z1} and X_{z2} be two distinct investment proportions vectors defining two portfolios Z_1 and Z_2, respectively, where

$$X'_{Z1} (VX_p) = 0 \qquad (1\ 5)$$

$$X'_{Z_2} (VX) = 0 \qquad (1\ 6)$$

with

$$X'_{Z1} \; i = 0 \qquad (1\ 7)$$

8. Equation (1 3) derived in a similar fashion to the results proved by Roll (1977, Theorem 1, p. 160).

$$X_{Z2}\ i = O \qquad (18)$$

It is also hold

$$X_{Z1}\ R^* = X_{Z2}\ R^* \qquad (1\ 9)$$

Using equations (15), (16), (17), (18), and (19) it can be obtained

$$(Xzi\text{-}Xz2)'\frac{VX_P}{{}^*2}=0 \qquad (110)$$

$$(X_{Z1}\ -\ Xz_2)'\ i = 0 \qquad (111)$$

$$(Xz_1\ -\ Xz_2)'\ R^* = 0 \qquad (I\ 12)$$

From equations (I 10), (I 11), and (I 12) it can be seen that the vectors $VX_{P;}$ i, and R* are othogonal to the vector (Xz1— Xz2). Therefore applying ap well-known theorem of the linear algebra [8] it can be written

$$R^* = \lambda_1\ i \qquad + \lambda_2 VXp/a_p^{*2} \qquad (I \qquad 13)$$

where λ_1 and λ_2 are real numbers.

After some manipulation equation (12) can be derived from the last equation. Conversely, assume that equation (12) is valid. If the portfolio ρ is not after - tax efficient then there always exists an after - tax efficient portfolio, call it p, such $r^*_p = r_p i$ and $\sigma^{*2}_\rho > \sigma^{*2}_{\rho 1}$.

Since ρ is an after - tax efficient portfolio the following equation holds

$$R^* = r_z i + (r_p - r_z) \frac{VX_{P1} \quad s}{\sigma^{*2}}$$

Since the portfolios ρ and p_1 have the same expected returns it can be concluded that $\sigma_t = \sigma_{ip1}$, for each i = 1, 2, ... N. Hence a comparison of equations (12) and (I 13) gives $\sigma^{*2} = \sigma^{*2}$. This is a contradiction and so ρ is an after-tax efficient portfolio.

9. This theorem states : If there vectors are all orthogonal to a single vector, then any two of the three can be expressed as a linear combination of the third.

APPENDIX II

Proof of Proposition 1

(i) => (ii) Suppose that t_g t_d 0. Then the after-tax relationship shown in equation (13) can be rewritten as

$$r_i + di = r_z + d_z + (r_p - r_z) + (d_p - d_z) \ bi \ +$$

$$tg \ [\ r_i - r_z - (r_p - r_z) \ bi \ + di - d_z - (d_P - d_z) \ bi \ ] \qquad (II \ 1)$$

Rearranging terms the last equation produces

$$[\ ri + di - r_z - d_z - [\ (r_p - r_z) - (dp - d_z) \ ] \ bi] \ [\ 1 - tg] = 0 \qquad (II \ 2)$$

Since t < 100 % from equation (I I2) the before-tax expected return - riks linearity can be established.

(ii) =>(i) If the after - and before-tax risk-return relationships are equivalent then a combination of equations (I I1) and (I 12) implies

$$tg \ [\ r_i - r_z - (r_p - r_z)bi \ ] + t_d[\ di - d_z - (d_P - d_z)bt] = 0 \qquad (1I \ 3)$$

It is convenient to rewrite equation (14) as

$$r_i - r_z - (r_p - r_z)\, b_i = - [d_i - d_z - (d_p - d_z)\, b_i] \qquad \text{(II 4)}$$

Making use of equations (11 3) and (11 4) it can be easily confirmed that $t_g = t_d$.

Proof of Proposition 2

(i) => (ii) The epuivalence between the before- and after - tax risk - return relationship ensures the validity of epuation (14) and (II 3). Since tg 0, let td = k + t_g, where k 0. Then equation (II 3) takes the form

$$t_g [r_i - r_z - (r_p - r_z)\, b_i + d_i - d_z - (d_p - d_z)\, b_i] + k [d_i - d_z - (d_p - d_z)\, b_i = 0 \qquad \text{(II 5)}$$

With the aid of equation (14) equation (II 5) is reduced to

$$k [d_i - d_z - (d_p - d_z)\, b_i] = 0$$

from which, since k 0 by assumption, it immediately follows that

$$d_i - d_z - (d_p - d_z) b_i = 0 \qquad \text{(II 6)}$$

A combination of equations (15) and (II 6) leads to

$$\frac{d_i - d_z}{d_p - d_z} = \frac{r_i - r_z}{r_p - r_z} = b_i \qquad \text{(II 7)}$$

After some manipulation the last equation may be put in the form

$$d_i = a_1 \; r_1 + a_2 \qquad \text{(II 8)}$$

where

$$a_1 = \frac{d_p - d_Z}{r_p - r_Z} \qquad \text{(II 9)}$$

$$a_2 = \frac{d_Z r_p - d_p r_Z}{r_p - r_Z} \qquad \text{(II 10)}$$

(ii) ⇒ (i) Suppose that

$$D = a_1 \; R^* + a_2 \qquad \text{(II 11)}$$

where

R* = the (Nx 1) column vector with entries the expected before - tax total security returns.

The vector R* can be expressed as

$$R^* = \frac{1}{1 - t_g} R - \frac{1 - t_d}{1 - t_g} D \qquad \text{(II 12)}$$

Substituting equation (11 12) into equation (11 10) and collecting terms it can be deduced

$$D = c_1 R + c_2 \qquad \text{(II 13)}$$

where

$$c_1 = \frac{a_1}{(1 - t_g) + a_1 (1 - t_d)}$$

$$c_2 = \frac{(1 - t_g)a_2}{(1 - t_g) + a_2(1 - t)_d}$$

If X_p and X_z are the investment proportion vectors defining the optimal after-tax portfolio of an investor and its orthogonal portfolio, respectively, then with the help of equation (II 11) the validity of the following equations can be prove :

$$c_1 = \frac{d_p - d_Z}{r^*_p - r^*_Z} \qquad \text{(II 14)}$$

$$c_2 = \frac{d_Z \, r^*_p - d_p \, r^*_Z}{r^*_p - r^*_Z} \qquad \text{(II 15)}$$

Upon applying equations (II 14) and (11 15) to equation (II 13), it is found that

$$\frac{d_i - d_Z}{d_p - d_Z} = \frac{r_i^* - r_z^*}{r_p^* - r_z^*} \qquad \text{(II 16)}$$

Because the portfolio ρ is aftex - tax optimal, the after - tax expected return - risk linearity can be derived. That is

$$r_i^* = r_Z^* + (r_p^* - r_Z^*)\, b_i \qquad \text{(II 17)}$$

A comparison of equations (II 16) and (It 17) results in

$$di = d_z + (d_P - d_z)\, b_i \qquad \text{(II 18)}$$

Using equation (II 17) and the fact that $t_g = k_d + t$, where k 0, the result is easily seen to be

$$[\, r_i + d_i - r_z - d_z - [\, (r_P + d_p) - (r_z + d_z)\,]\, b_i \,]\, [1 - t_g] - k\, [\, d_i - d_z - (dp - d_z)\, b_i] = 0 \qquad \text{(II 19)}$$

Taking into account equation (II 18) and that t< 100 % the last equation implies the before-tax expected return-risk linearity.

Proof of Corolry 1

(i) => (ii) Suppose that the portfolio of an investor is after-tax efficient and the before - and after-tax expected return-risk relationships are equivalent. Then by following a similar argument to this of proposition 2 equation (II6) can be derived.

(ii) => (i) assuming that equation (II 6) is true and making use of equation (II19) it can be written

$$fa + di - r_z - d_z - [(r_p + dp) - (r_z + d_z)]\ b_i\ [1 - t_g] = 0$$

Since $t_g < 100$ % the last equation easily produces equation (15).

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