THE ROLE OF LAGGED DEPENDENT VARIABLES IN THE ESTIMATION OF A DYNAMIC PORTFOLIO MODEL

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I. INTRODUCTION

The importance of interrelations among financial markets within the context of a portfolio balance model were examined in a well known paper by Brainard and Tobin (1968). Portfolio models belong to the broad category of singular equation systems, which have been mainly studied in relation to consumer demand equations. The relevant literature abounds with theoretical and empirical specifications such as the Rotterdam model (Theil (1975)), the almost ideal demand system (Deaton and Muellbauer (1980)) or the translog models. However, theoretical restrictions are often rejected by the data as a result of dynamic misspecification. The dynamics of singular equation systems were investigated among sothers by Berndt and Savin (1975), Anderson and Blundell (1982), Ray (1985). Anderson and Blundell (1982) examined the complications of a general dynamic specification in line with Davidson et al. (1978) and Hendry and Mizon (1980). They pointed out that hypotheses suggested by economic theory apply to the long run structure of the system. The inclusion of lagged dependent variables must be decided carefully, because depending on whether there are cross equation effects of lagged dependent variables or not, estimation procedure as well as the interpretation of the results vary.

In the light of Anderson and Biundell's (1982) analysis, this paper considers the specification and empirical implementation of a dynamic portfolio balance model which includes lagged dependent variables in the right hand side of its demand equations. Empirical work conducted with quarterly data for the United Kingdom economy covers the years 1972 - 82.[1] The aggregation of assets into three broad categories has an important role to play in improving our understanding of the functioning of the system.

In the first part of section II, the portfolio model is briefly recasted. Its dynamic specification and problems related to its estimation are discussed in the second part of this section. In section III, we present the estimation results from two alternative specifications corresponding to different assumptions about the effect of the lagged dependent variables. Finally, section IV contains conclusions and a brief summary of this study.

II. THEORETICAL FRAMEWORK

IIa. The Portfolio model

Within the well known portfolio framework applying to a small open economy, the UK takes the place of the home country and the rest of the world that of the foreign country. UK portfolio investors can hold UK money M, UK government bonds B as well as foreign bonds B*, with rates of return r_m, r_b and (rb - é), respectively. é is the expected rate of appreciation of the UK effective rate. UK private financial wealth W is equal to the sum of the stock of assets held by UK residents at every point in time :

$$W = M + B + F \quad (1)$$

1. The sample period starts in 1972 : II, in order to avoid possible instabillity in the system"s underlyinq parameters caused by the disruption in financial markets brouqht about by **the** economic reforms of 1971 and the change, in the exchange rate regime in mid 1972.

F is the net stock of foregn assets held by UK residents and is equal to the difference between the value of B^* held by UK residents and the value of B held by foreigners.

Demand for each asset as a proportion of wealth is a function of the rates of return, real income y and real wealth $\frac{W}{P}$. Hence, equilibrium conditions in the UK asset markets are,

$$\frac{M}{W} = m(r_m, r_b, (r_b^* - \acute{e}), y, \frac{W}{P}) \qquad (2)$$

$$\frac{B}{P} = b(r_m, r_b' (r_b^* - \acute{e}), y, \frac{W}{P}) \qquad (3)$$

$$\frac{F}{W} = f(r_m, r_b' (r_b^* - \acute{e}), y, \frac{W}{P}) \qquad (4)$$

Equations (2), (3) and (4) describe a short run model in which the rates of return are determined by the interaction of the asset markets, while income and prices are exogenous.

lib. Dynamic specification and restrictions on the coefficients

The preferred dynamic structure follows the procedure described in Hendry et al. (1978), allowing portfolio investor decisions to be influenced not only by chances in the actual holdings of assets, but also, by previous periods discrepandes between actual and desired holdings. Although the model is undoubtedly short run, the estimated asset demands are consistent with short and long run portfolio balance. Thus, the system to be estimated has the following general form :

$$\Delta\omega_t = B_{i-1}\,\Delta\omega_{t-i-1} + \Gamma_i\,\Delta X_{t-i} - A\omega_{t-n} + ZX_{t-a} + u_t \text{ [2]} \qquad (5)$$

where Δ denotes a first other difference,

ω is a (3x1) vector of portfolio shares $\frac{M}{W}$, $\frac{B}{W}$ and $\frac{F}{W}$,

X is a (6x1) vector of the dependent variables, including the constant term c,

that is, $X = (c, r_m, r_b; r_b^* - \acute{e}, y\ \frac{W}{P})$,

i = 0, 1, 2,. . ., m, is the number of lags,

n> 0, is an indicative lag,

B_{i-1}, Γi, A and Z are coefficient matrices having dimensions $(3 \chi 3)$, (3x6), $(3 \chi 3)$ and $(3 \chi 6)$ respectively.

Ut is the vector of normally distributed white noise.

Weath constraint (J), implies the following adding up restrictions for the coefficients of (5) :

2. Apart from enablng us to distinguish long run from short rur effects, (5), is capable of encompassing various dynamic specifications such as a first order difference model, a partial ajustaient or a static equilibrium model with autoregressive errors. See Hendry et al. (1982).

R1 : Columns in each B_{i-1} matrix sum to the same number, that is,

(I)' B_{i-1} — (b_{i-1}' b_{i-1}, b_{i-1}), where (I) is the unit vector.

Similarly,

R2 : (I)' A= (a,a,a), and

R3:.(I)Ti=(0),

R4 : (I)' Z = (a,0,0), that is, columns of the matrix Z add up to zero except for the column corresponding to the constant term.

The assumption of gross substitutability among assets imply for the coefficients of Z the restrictions,

R6 : ζχλ> 0 for $\varkappa = \lambda$ where $\varkappa,\lambda$ = (1,2,3) and

$\zeta_{\varkappa\lambda} < 0$ for $\varkappa$ # λ.

A further reetriction on the coefficiente of (5) that must hold for long run equilibrium is that the diagonal elements of matrix A must be negative :

R7 : $a_{\varkappa\lambda} < 0$ for $\varkappa$ — λ.

Evidently, (5) cannot be estimated because the lagged dependent variables of the ω vector are perfectly collinear. Depending on the effect of these variables on the demand for $\frac{M}{W}$, $\frac{B}{W}$ and $\frac{F}{W}$, there are two possible ways of estimating (5) :

Possibility 1 : The estimation procedure describrd here rests on the hypothesis that there are cross equation effects from the lagged dependent variables on the asset demands that is, H1 : $b^{i-1}_{\varkappa\lambda}$, $a_{\varkappa\lambda}$ #0. To estimate (5), one of the collinear variables, supposedly $\Delta(\frac{F}{W})$t-i-l, and $(\frac{F}{W})$ t-n, must be eliminated from the $\Delta\omega_{t-i-1}$ and ωt-n vectors. As a result, the third column of the Bi_i and

A matrices must be deleted. This implies that the coefficients of the B_{i-1} and A matrices associated with the lagged dependent variables $A\left(\frac{F}{W}\right)_{t-i-1}$ and $\left(\frac{F}{W}\right)_{t-n}$ cannot be idendified. Yet, this does not have any repercussions for the long run version of the model. It should be noted, that under the H1 hypothesis, the adding up restrictions R1 and R2 become :

$$R'1 : (I)' \bar{B}_{i-1} = (0,0) \text{ and}$$

$$R'2 : (I)'\bar{A} = (0,0).$$

The bar over A and B indicates the deletion of the third column of these matrices.

Possibility 2 : The system (5) can be estimated, if there are no cross equation effects from the dependent variables on the asset demands. This requires that the B_{i-1} and the A matrices are diagonal. Moreover, from the adding up restrictions R1 and R2, it follows that the diagonal elements of each matrix must be equal. Consequently, the assumption of no cross equation effects can be expresssed as H2a : $b^{i-1}_{\varkappa\lambda} = b_{i-1}$ and

$a_{\varkappa\lambda} = a$ for $\varkappa = \lambda$ and

H2b: $b^{i-1}_{\varkappa\lambda} = a_{\varkappa\lambda} = 0$ for every $\varkappa \neq \lambda$.[3] The procedure of estimating (5) involves first the joint estimation of any two out of the three equations and then the derivation of the coefficients of the third one residually. If this third equation were estimated separately, the fitted values of its coeffciients should be equal to those derived derived.

3. Equality of diagond elements implies that all the three asset markets approach long run equilibrium with the same speed. This makes H2 a very restrictive hypothesis.

III. ESTIMATION RESULTS

Since there is no theory indicating which of the hypotheses H1 or H2 is preferable, both hypotheses must be tried. However, there is no rigorous test enabling us to chose with certainty between the two. Nevertheless, there are indications such as the goodness of fit tests of the estimated equations and the satisfaction or not of the restrictions implied by theory, that are helpful in taking decisions about accepting or rejecting the hypotheses.

According to possibilities (1) and (2), estimation of (5) yeilded the results presented in Tables 1 and 2. Although the ordinary least spare estimates are reported, the full information maximum likelihood estimates are almost identical[4]. Regressions in both Tables are free of first order autocorrelation as suggested by the Durbin's h tests, which have values well below the critical value of 1.96, a.t the 5 % level of significance. They are also free of fourth order autocorrelation as indicated by the Lagrange multiplier statistics LM(4), which are lower that the critical value of 9.49, at the 5 % level. Unfortunately, few of the fitted values are significantly different from zero. Still it must be reminded that the criterion or retaining a variable in the regression is not the statistical significance of its coefficient, but its importance to the interrelations among markets[5] Real income, being highly correlated with real wealth, is excluded from the estimation. Assuming rational espectations, é is replaced by its realized value.

In Table 1, where $\frac{F}{W}$ is deleted from the lagged dependent variable vectors, estimation is carried out under the hypothesis H1. As can be observed, the estimated coefficients of the long run variables have the expected signs and thus the restrictions R6 and R7 are satisfied. Besides, from column 4 presenting the sum of the estimated parameters,|it is inferred that the adding up restrictions R'l, R'2, R3 and R4 for portfolio balance, are satisfied too.

4. Barren (1969) indicates that if ail the variaàles on the right hand side are equal for all commodities (assets, in our case) the OLS method will give maximum likelihood estimates satifsfying the adding up restrictions.

5. «It possible that cross effects are so diffused that none of them appears significant in empirical regressions. Yet, it is a mistake to drop them out...» (Brainard and Tobin (1968)

TABLE 1

OLS estimates of the UK portfolio demand equations with cross equation effects of lagged dependent variables (sample period 1972: II - 82 : IV)

Dependent Variables / Independent Variables	(1) $(\frac{M}{W})_t$	(2) $(\frac{B}{W})_t$	(3) $(\frac{F}{W})_t$	(4) (1)+(2)+(3)
$\Delta(\frac{M}{W})_{t-1}$	8.105 (0.629)	−0.045 (0.003)	−8.060 (0.413)	−0.000
$\Delta(\frac{B}{W})_{t-1}$	0.026 (0.002)	7.823 (0.507)	−7.848 (0.403)	−0.000
Δr_{mi}	0.102 (0.464)	−0.518 (1.959)	0.416 (1.247)	−0.000
Δr_{bi}	−0.443 (1.964)	0.364 (1.242)	0.079 (0.230)	−0.000
$\Delta(r^{*}-\acute{e})$	0.027 (0.368)	−0.066 (0.733)	0.038 (0.339)	−0.000
$\Delta(\frac{W}{P})$	−0.071 (4.537)	0.007 (0.384)	0.064 (2.685)	−0.000

TABLE 1 (continued)

	(1)	(2)	(3)	(4)
$\frac{M}{W}_{t-2}$	−11.161 (1.514)	5.051 (0.569)	6.110 (0.546)	−0.000
$\frac{B}{W}_{t-2}$	−2.513 (1.227)	−3.631 (1.474)	6.130 (1.977)	−0.000
r_{mt-3}	0.167 (1.344)	−0.001 (0.009)	−0.166 (0.878)	−0.000
r_{bt-2}	−0.302 (1.776)	0.379 (1.855)	−0.077 (0.300)	−0.000
$(r^{*}-\acute{e})_{t-3}$	−0.088 (1.010)	−0.152 (1.440)	0.240 (1.807)	−0.000
$(\frac{W}{P})_{t-4}$	0.011 (0.882)	−0.003 (0.195)	−0.008 (0.426)	−0.000
C	5.832 (0.882)	−1.253 (0.152)	−4.578 (0.441)	0.000
T	41	41	41	41
S	0.013	0.015	0.019	
$\bar{R}^2$	0.587	0.545	0.049	
h	0.719	0.454		
d			2.177	
LM(4)	6.854	5.700	5.547	

* All estimates are multiplied by 100.t ratios are recorted in parentheses. S is the standard error of the regressions. h is the Durpin's test for first order autocorrelation. The standard Durbin-Watson test d is reported where the calculation of his not possible. LM(4) is the Lagrange mitiplier test for 4rst order autocorrelation.

The horizontal sum of the estimates may not be zero due to rounding.

TABFE 2

OLS estimation of the UK portfolio demand equation without cross equation effects of lagged dependent variables (samble period 1972= 11 - 82 : IV)*

	(1)	(2)	(3)	(4)
Dependent Varbiablo	$\left(\frac{M}{W}\right)_t$	$\left(\frac{B}{W}\right)_t$	$\left(\frac{F}{W}\right)_t$	$\left(\frac{F}{W}\right)_t$
Independent variables				
$\Delta\left(\frac{M}{W}\right)_{t-1}$	10.998 (0.879)			
$\Delta\left(\frac{B}{W}\right)_{t-1}$		10.127 (0.703)		
$\Delta\left(\frac{F}{W}\right)_{t-1}$			10.127	7.964 (0.590)
Δr_{mt}	0.107 (0.495)	–0.532 (2.089)	0.425	0.416 (1.318)
Δr_{bt}	–0.428 (1.918)	0.363 (1.378)	0.065	0.079 (0.240)
$(\Delta r^*-\acute{e})_t$	0.023 (0.314)	–0.061 (0.717)	0.038	0.038 (0.353)
$\Delta\left(\frac{W}{P}\right)_t$	–0.078 (5.354)	0.009 (0.503)	0.069	0.064 (2.795)

TABLE 2 (continued)

	(1)	(2)	(3)	)4(
$\left(\frac{M}{W}\right)_{t-2}$	−4.635 (0.989)			
$\left(\frac{B}{W}\right)_{t-2}$		−4.635 (2.878)		
$\left(\frac{F}{W}\right)_{t-2}$			−4.635	−6.103 (2.395)
r_{mt-3}	0.156 (1.299)	0.005 (0.035)	−0.161	−0.166 (0.927)
r_{bi-2}	−0.299 (1.907)	0.344 (1.819)	−0.045	−0.077 (0.337)
$(r^* - e)_{i-3}$	−0.070 (0.874)	−0.150 (1.586)	0.221	0.240 (1.884)
$\left(\frac{W}{P}\right)_{t-4}$	0.011 (0.893)	−0.006 (0.503)	−0.005	−0.008 (0.580)
C	1.284 (0.230)	2.975 (1.057)	0.375	1.553 (0.418)
T	41	41	41	41
S	0.013	0.015		0.019
$\bar{R}^2$	0.594	0.386		0.525
h	0.382	1.042		1.089
LM(4)	6.791	4. 040		5.399

*A11 estimated parameters are multiplied by 100.

The estimation results of Table 2 hold under the hypothesis H2. The coefficients of the foreign bonds equation are derived residually in column 3 of this table. In column 4 the estimated coefficients of the foreign bonds equation are shown: We observe that b_{11} and b_{22}—the hat denoting fitted values are almost identical, while a_{11} and a_{22} are equal. However, b33=7.964 in column 4 of Table 2 does not coincide with $\hat{b}_{11}$ $\hat{b}_{22}$ = —10.127. Moreover, a_{33}= —6.103 in column 4 differs from a n = $\hat{a}_{22}$ = —4.635. In addition, although the values of some coefficients in column 3 and 4 are similar, some others differ notably, suggesting mispecification. From these, it is implied that we cannot accept H2a hypothesis about equality of the diagonal elements of the lagged dependent variables coefficient matrices. Besides, H2b hypothesis cannot be tested directly[6]. The long run coefficients have the expected signs except for the coefficient of the third lag of r_m in the domestic bonds equation, which is positive instead of negative. Hence, the restriction R6 is not thoroughly satisfied while restriction R7 actually holds.

It is therefore concluded that the estimated asset demand equations of Table 1 are preferable to those of Table 2, as they satisfy the theoretical preconditions for portfolio balance both in the short and long run as well as the assumption of gross substitutability among assets. Consequently, between H1 and H2 we must choose H1 accorting to which there are cross equation effects of the lagged dependent variables on the asset demands. In the short run, the values of the estimated parameters do not seem to differ markedly under the two alternative hypotheses, as can be inferred from a comparison of the estimates in Tables 1 and 2. On the, contrary, it can easily be checked that the coefficients describing the effect of the independent variables, in the long run equilibrium, are influenced by the choice of the lagged dependent variables.

IV. SUMMARY AND CONCLUSIONS

There are two alternative but not equivalent ways of proceeding with the estimation of a singular equation system, depending on whether there are cross equa-

6. indeed hypothesis H2b cannot be tested directly. Instead we can test the hypothesis, a_{12} = a_{21}== b_{21}= 0, wich cannot be rejected with the standard F test. However, the high standard errors of the estimated coefficients renders our tests unreliable.

tion effects of the lagged dependent variables. Therefore, before deriving conclusions from empirical findings it is necessary to clarify which of the dependent variables are essential for the estimation.

In this paper, we estimated a portfolio balance model with three assets for the UK, following two distinct estimation procedures. The first was realized under the hypothesis that demand for each asset as a proportion of wealth is influenced by the lagged values of all the dependent variables. In this case, the coefficients describing the effect of one of the three lagged dependent variables could not be identified. The second estimation was carried out under the hypothesis that each asset demand as a proportion ofwealth if influenced by its own lagged value. Indications incuding the goodness of fit tests and the satisfaction of the restrictions implied by portfolio balance and the assumption of gross substitutability among assets favoured the hypothesis of cross equation effects of the lagged dependent variables. Conclusions about the role of the independent variables in the long run equilibrium vary according to alternative hypothesis. Hence, in order to enhance our understanding of the functioning of a singular demandequation system, the choice of lagged dependent variables must be made with caution.

DATA APPENDIX

M is the UK MI stock.

B is the difference between the UK monetary base and the cumulated sum of the UK PSBR.

F is the difference between the cumulated sum of the UK current account -purged from its component corresponding to the public sector- and the UK official reser-ves.

P is the UK consumer price index,

r_m, i'_b and r^*_b are the interest on 7 day deposits in the UK, the UK Treasury Bill rate and the Eurodollar 3 month rate, respectively.

é is the UK effective rate.

Data come from the OECD Main Economic Indicators. Those not available there, are taken from the UK Monthly Digest of Statistics (CSO), excelt for the UK monetary base taken from the Bank of England Quarterly Bulletin.

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