ESTIMATING WATER NEEDS FOR THE IRRIGATION OF POTATOES: A RENEWAL THEORY APPLICATION

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SUMMARY

In this work a method is proposed for the estimation of water needs during the cultivation of a crop. The method is based on renewal theory considerations, and its application is examined on the irrigation of spring potatoes using soil water negative pressure measurments.

1. INTRODUCTION

Irrigation with the use of tensiometers is based on the changes of the mean negative pressure of the soil moisture or in the changes of the negative pressure in a certain depth. An irrigation is applied when the soil negative pressure reaches a certain value. It is known that maximum yield is obtained when the soil negative pressure remains at a certain level during the cultivation period. (Taylor, A.S. (1972), Marsh, A.W. (1961), Wittam, M.T. et al (1963).

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In this work a method is presented for estimating the total amount of water needed for the irrigation of the crop from the start of the cultivation period till the end. The model is based upon renewal theory considerations and its application is examined on a set of date of successive irrigations of potatoes based on soil water negative pressure measurements. In section 2 the renewal theory model is developed while in section 3 the application of the model is examined.

Previous research in this area is described in references 4,5,6,7 and others. In these are used simulation techniques, data based regression models and expen sive, complex energy - mass exchange models.

The method proposed here is characterized for its simplicity in comparison to previous work. The only data it requires is the time intervals between successive irrigations which can be obtained from previous cultivations.

2. THE MODEL

According to the method of irrigation which is based on soil water negative pressure measurements, an irrigation is applied each time the soil negative pressure reaches a certain value. In the sequel, irrigations are considered as instant events, that is, all the water in the irrigation joins the soil at the beginning of the irrigation. This assumption seems realistic due to the short time duration of irrigation in relation to the long time periods this model examines. The time interval between two successive irrigations is a random variable which depends on several parameters as the vegetative cover, the amount of the soil water in the root zone, the vegetative state of the crop, the physical properties of the soil and the evaporative power of the atmosphere.

Let X_1 be a random variable denoting the time interval between the start of the cultivation (at the start of the cultivation an initial irrigation is applied) and the first irrigation, x_2 the time elapsing between the first and the second irrigation and so on x_i the time interval between the i - 1 and i irrigations.

The system $(x_1, x_2, x_3, \ldots)$ is a system of independent and identically distributed random variables all with probability density function (pdf) f (x). This is because the time length of the i irrigation does not affect in any way the duration of the rest irrigations. Furthermore all the x_1s represent time intervals between successive irrigations and are assumed to follow the same pdf f(x). This assumption is true provided that the duration of cultivation is not to long so that the pdf of the interval between successive irrigations will change significantly. This assumption is fullfilled in this study. Thus the system $\{x_1, x_2, x_3, \ldots\}$ forms an ordinary renewal process.

Let I_t be the number or irrigations in $(0,t)$, then the mean value of I_t, denoted by $E(I_t) = H(t)$, is given by

$$H(t) = E(I_t) = \sum_{r=1}^{\infty} k_r(t)$$

Where $k_r(t)$ is the cumulative distribution function (cdf) of the random variable of the time up to the rth irrigation. Alternatively H (t) can be evaluated by inverting the Laplace transform of H (t) given by

$$H^*(s) = \frac{f^*(s)}{s(1 - f^*(s))}$$

where $f^*(s)$ the Laplace transform of $f(x)$. (Cox, D.R., Chapter 4).

The variance of the number of irrigations, $V(I_t)$, is given by

$$V(I_t) = 1(t) - H(t) = H^2(t)$$

where

$$1(t) = E[I_t(I_t + 1)]$$

The function I (t) can be found by inverting its Laplace transform

$$1^*(s) = \frac{2\ f^*(s)}{s\ (1 - f^*(e))^*}$$

(Cox, D.R., Chapter 4)

Assymptotically for large t, the mean number of irrigations in (0, t) is given by

$$H\ (t) = E(I_t) = \frac{t}{\mu} + \frac{\sigma^2 - \mu^2}{2\mu^2} + 0\ (1)$$

and the variance of the number of irrigations is given by

$$V\ (I_t) \quad \frac{\sigma^2\ t}{\mu^3} + (\frac{1}{12} + \frac{5\sigma^4}{4\mu^4} - \frac{2\mu^3}{3\mu^3}) + 0\ (1)$$

where 0 (1) denotes a function of t tending to zero as $t \rightarrow \infty$ and μ, σ and μ_3 are respectively the mean, standard delviation and the third moment about the mean of the interval between successive irrigations (Cox, D.R., 1961, Chapter 4).

Knowing the mean μ and variance σ^2 of the number of irrigations I_t in (0, t), it is possible to evaluate the neccessary quatity of water that guarrantees that there will be enough stock (of water) for the cultivation with a certain probability. This is possible, since the number of irrigations I_t is assymptotically normally distributed (Cox, D.R., Chapter 3).

3. APPLICATION OF THE MODEL

Date Collection and Analysis

Tensiometers were used to follow the negative pressure of the soil moisture so that to apply *irrigation water on an experimental site of potatoes in the Attiki* area. The soil of the experimental site was homogeneous on the surface and in depth. According to the soil Texonomy it could be classified as Typic Xeroflurents. An irrigation of 40 mm or 40 m^3/acre was applied when the soil negative pressure at 45 cm dropped to —0.50 atm.This is because this pressure is within the limits that give maximum yield. The experiment was done concurently on three different sites. More details about this roject can be found in Aggelidis, S.M. et al.

For each of the three sites, notes were kept of the time and date of successive irrigations during the cultivation of potatoes. This permitted the evaluation of the time intervals between successive irrigations, that is of the values of

$$x_i, \quad i = 1, 2 \ldots\ldots$$

The mean number of irrigations for the three sites is 10.7 and the standard deviation is 1.53. The mean value $(\bar{x})$, standard deviation (σ) and third moment about the mean (μ_3), of the 32 observations on the time intervals between successive irrigations, from the three sites are respectively :

$$\bar{x} = 5.06 \qquad \sigma = 2.25 \qquad \mu_3 = 260.85$$

The negative exponential distribution with pdf given by

$$p \exp(-px) ; x > 0 \text{ parameter)}$$

mean and standard deviation 1/p and cdf given by

$$1 - \exp(-pt), \qquad t > 0$$

was fitted to the above data. The value of the parameter p was estimated with the method of moments as p = 0.20. In the continuation Table 1 was constructed which gives the theoretically expected number of observations, the observed and the difference between them.

TABLE 1.— Fitting of Exponential distribution

X	1	2	3	4	5	6	7	8	9	10
Expected	6	10	14	17	20	22	24	25	27	28
Observed	0	7	10	18	22	23	26	29	30	32
Difference	6	3	4	1	2	1	2	4	3	4

The maximum observed difference is 6 and $6/32 \simeq 0.19$ which is less than the critical value of the Kolmogorov - Smirnov criterion at 0.05 significant level and 32 observations which is 0.24 (Hoel, P.G.). So it can be hypothesized that the time interval between successive observations is adequately described by the negative exponential distribution with p = 0.20.

Application of the model

It was shown that the interval between sucessive irrigations follows the negative exponential distribution, so the number N_t of irrigations in (0. t) has the Poisson distribution with mean pt

i. e.

$$\Pr\,(It - m) = \frac{(pt)^m\, e^{-pt}}{ml}\;;\quad m = 0, 1, \ldots\ldots$$

Furthermore the mean number of irrigations in (0. t), by inverting the Laplace transform, is given by

$$H(t) = E(I_t) = tp$$

while

$$1(t) = p^2 t^2 + 2pt$$

so

$$V(It) = pt$$

(Cox, D.R.)

The estimation of the mean and variance of the number of irrigations, during the mean period of cultivation of the three different sites, is 10.1 and 10.1 respectively.

Using the asymptotic formulas, the estimation for the mean and variance of the number of irrigations is 9.7 and 2.0 respectively.

This must be compared with the mean number of irrigations as evaluated from the data which is 10.7 while the variance is 2.3. It must be noted, that there are only three observations to evaluate these figures.

Since the number of irrigations I_t is asymptotically normally disttributed, the probability p that no more than Ip x 40m³/acre of water will be needed, during the cultivation period (0, t), is such that

$$Pr \quad (It \leqslant I_P) = p$$

Using table for the normal probability distribution the quantity I_p can be evaluated. For $p = 0.70$, for examble

$$I_p = \mu_t + 0.52\, \alpha_t$$

which means that having I_p 40 m³/acre of water available during the cultivation period, reassures, that there will be enough stock of water with probability 0.70.

4. CONCLUSIONS

In this work a renewal theory based model is examined for estimating the amount of water needed during the whole peroid of cultivation of spring potatoes. The fit of the model was found to be satisfactory as far as the mean is concerned. The fit of the variance was not very satisfactory.

This may be due to the lack of a sufficient amount of data to estimate the variance of the number of irrigations or the assumptions implicit in the model, such as the incapability of the model in taking explicitly into consideration the rainfalls during the period of irrigation. If some of these assumptions will be relaxed, the fit of the model may be improved. But for this, more research is needed to built more detail into the model.

The model can find useful application, since it takes into account modern irrigation practice in estimating the need for irrigation water. It helps as well, in estimating the amount of water needed that guarantees that there will be enouqh stock of water with a certain probability.

REFERENCES

1. Taylor A. S., 1972. : Physcal Edaphology. The physics of Irrigated and nonirrigated soils, W. H. Freeman and Co., San Francisco. pp. 434 - 435.

2 . Marsh, A. W., 1961 : Tensiometers : Key to increase profits. Western Grower and Shiper. February 1961. pp. 15-17, 34.

3. Wittam B. E., Reynolds C.W., Struchtemeyer, R. A. 1963 : Soil-plant -water relationships, as a basis for irrigation. Crop responds to irrigation in the Northeast New York Agr. Exp. Sta. Geneva, Bull, 800.

4. Stewart, J. I., Hagan, R.M. and Pruitt,W. D., 1974. Functions to predict optimal Irrigation Programs. Journal of the irrigation and Drainage Division, Vol. No 2, June 1974, pp. 179 - 199.

5. Smith, M. 1985., : FAD, Land and Water Development Division, Rome, Italy. Irrigation scheduling and water distribution. Les besoins au eau des cultures. Conference Internationale, Paris, 11-14 Sept. 1984, INRA, Paris.

6. Cuenca, R. H., ASCE, M. and Nicholson, M. T. 1982 Application of Penman Equation Wind Function, ASCE Ir, No 1. 8, March 1982.

7. Burt, J. E., Hayes, J. T., O' Rourke, P. A., Terjung, W. H. and Todhunter, P. E. 1981., Parametric Crop Water Use Model.Water Resources Research, Vol. 17, No 4, pp. 1095-1108. August 1981.

8. Cox, D. : Renewal Theory. Methuen & Co. Ltd.

9. Aggelidis, S.M., Chardas, G. K. Tsakalaris, P. D. and Stamos G. I., 1981 : Irrigation, potatoes based on soil, water negative pressure measurements, Agricultural Research 8, 45 - 55.

10. Hoel, P.G. : Introduction to Manhematical Statistics. Third Edition John Wiley & Sons Inc.