THE LINEAR EXPENDITURE SYSTEM AND ITS APPLICATION TO GREEK IMPORTS AND EXPORTS

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I. INTRODUCTION

The purpose of this paper is twofold. Firstly, a brief discussion of the Linear Expenditure System (L.E.S.) and secondly its application to data on Greek exports and imports.

We first present the basic model and the implications of the imposition of certain restrictions required by the standard demand theory, as well as additional implications brought about by the imposition of these restrictions followed by a brief reference to econometric considerations.

Secondly the LES was fitted to Greek data on imports and exports for the years 1958 - 1987. The technique of the estimation and the results are given in part II.

The LES is a relatively simple model presenting the quantity of goods demanded as a function of the prices of goods and income, which is assumed to be equal to total expenditure. These are the basic determinants of demand. Although some modifications have been introduced to incorporate changes in tastes and income which are supposed to be constant in basic LES.

1.1. The Basic L.E.S.

We consider the system :

$$P_i\ x_i \quad P_i\ c_i + b_i\ i_v - \sum_{j-1}^{n} \quad P_i\ c_i),\ i \quad 1,2.\ldots\ldots,\ n \tag{1}$$

Where :

$P_i\ x_i$ is the expenditure on the ith commodity

c_i is a certain basic amount of consumption on the ith commodity

Income is measured by total expenditure y p′ x where p′ is the ixn row vector of prices and x the nx1 vector of quantities, or

$$\sum_{i-1}^{n} \quad p_i\ x_i = y.$$

y is the total expenditure which we cell income.

$y = \sum_{j-1}^{n} p_j\ c_j$ is what is left from income after the allocation of a part of it to the consumption of the basic amounts of commodities.

b is a vector of constants with $\sum_{j-1}^{n} b_i - 1$ and it reprents the marginal propensity to consume.

System (1) make sense if $y = \sum_{j-1}^{n} p_j\ c_j < 0$, which means that the total expenditure should be greater than the expenditure on basic amounts of commodities, or that consumers should be able to spend more than just basic expenditure or that this system can be applied in situations where consumption is above a certain level. This is the basic hypothesis on which LES is based. The basic LES can be obtained by constrained maximization of a certain utility function. This utility function has been derived by R.G. Geary [1].

1.2. Restrictions placed on the L.E.S. and their implications

The system presented so far is just a system of equations. To make this a theoretically acceptable system we must place upon it the following requirements of demand theory.

a. Adding Up.

The adding up restriction implies that

$$\sum_{i=1}^{n} p_i x_i = y \quad \text{and} \quad \sum_{i=1}^{n} b_i = 1$$

The condition $\sum_{i=1}^{n} b_i = 1$ is automatically satisfied if the demand system is derived by constraint maximization of an appropriately specified utility function [1],

b. Homogeneity

In mathematical terms this condition is expresed as

$$\sum_{i-1}^{n} \frac{\partial x_i}{\partial p_j} \cdot \frac{P_j}{x_i} + \frac{\partial x_i}{\partial y} \cdot \frac{Y}{x_i} = 0$$

i.e the sum of all direct and cross elasticities of any commodity plus the income elasticity equals zero [3].

c. The Slutsky Restriction

This property requires the symmetry of the substitution effects. i.e

$$\frac{\partial x_i}{\partial y} = \frac{b_i}{p_i} \text{ and } x_j \frac{\partial x_j}{\partial y} = \frac{b_i}{p_i} \left(c_j + b_j \frac{y}{p_j} - b_j \sum_{i=1}^{n} \frac{P_i}{p_j} c_i \right)$$

$$\frac{\partial x_j}{\partial y} = \frac{b_j}{p_j} \text{ and } x_i \frac{\partial x_i}{\partial y} = \frac{b_j}{p_j} \left(c_i + b_i \frac{y}{p} - b_i \sum_{i=1}^{n} \frac{p_j}{p_j} c_j \right)$$

d. The Negativity of the Direct Substitution Effect

This condition requires that

$$k_{ii} = \frac{\partial x_i}{\partial p_i} + x_i \frac{\partial x_j}{\partial y} < 0$$

The LES after the imposition of the previous restrictions exhibits the following charecteristics

i. Positivity of Cross—Substitution Effects

This means that

$$k_{ij} = \frac{\partial x_i}{\partial p_j} + x_j \frac{\partial x_i}{\partial y} > 0$$

ii. Positivity of all Income Elasticities

This is

$$ey = \frac{\partial x_i}{\partial y} \cdot \frac{y}{x_i} = \frac{b_i}{p_i} \cdot \frac{y}{x_i} = b_i \frac{y}{p_i x_i} = \frac{b_i}{W_i}$$

which is positive because $0 < b_i < 1$, $P_i x_i > 0$ and $y > 0$.

This result means that the case of inferior goods is ruled out by the systems specification

iii. Inelasticity of Own Price Elasticity

The own price elasticity is

$$e_{ii} = -\frac{\partial x_i}{\partial p_i} \cdot \frac{p_i}{x_i} = 1 - \frac{(1 + b_i)c_j}{x_i}$$

and since $0 < b_i < 1$, $c_j < x_i$ we have that $e_{ii} < 1$

This result restricts the applicability of the system to broad categories of commodities rather than to individual commodities.

1.3. Limitations of the system Econometric and estimations Considerations

The empirical specification and estimation of the LES is usually difficult.

An econometric problem that arises is the constancy of bi's and ci's which are usually estimated from time series data.

This implies that tastes remain unchanged. The solution is to let bi's and Ci's vary over time [2].

Another problem refers to the structure of the error terms.

We have in the system as many error terms as commodity groups. These terms cannot be indepedent from each other. This means that the covariance matrix of the error terms is singular. There exist several ways of specifying the distribution of the error terms so that the matrix is singular [4].

The estimation technique that is most widely used is the least squares. However OLS cannot be applied directly since there is no independence among the regressors. To avoid the problem Stone [5] introduced a two - step iterative procedure which is described in the next chapter. The problem with this method is, as already indicated the singularity of the covariance matrix of the error terms, which means that the b_i and c_i estimates are not maximum likelihood ones. The above imply that the LES has certain limitations in describing expenditure patterns. For example as we have already seen $b_i [y - \sum_j P_i c_j] < 0$, cannot have an economic interpretation.

Also the assumption of additivity and the absence of complementary as well as inferior goods is unrealistic. To overcome this the LES is usually applied in the case of commodity groups.

Moreover, it might not be appropriate to use the theory of individual consumer — demand on the aggregate lever, since in this case it is implied that the same utility function applies to all individuals.

II. AN APPLICATION OF THE L.E.S.

The L.E.S. was fitted to annual Greek data, 1958 - 1987, on per capita imports and exports at current and constant prices (1970 = 100) using the GDP deflator.

The data are reported by the National Statistical Service of Greece according to the Standard International Trade Classification and were combined into six commodity groups (see tables 1 and 2). The method of estimation was a slight variation of Stone's two — step iterative procedure [5].

The initial computations involve separate OLS estimation of each of the following six equations :

$$P_i x_i = b_i y, \quad i = 1, \ldots\ldots, 6 \tag{1}$$

This allowed us to obtain provisional estimates of b_i' s, indicated by b_i^*, which sum up to unity since $\sum_{i=1}^{6} P_i x_i = y$.

These estimates were used in the fisrt step of the iterative procedure which involved estimation of provisional c_i' s, indicated by c_i^*, by succesive OLS estimation of the following six equations :

$$P_i x_i = A + c_{1i} P_i (1 - b_i^*) + c_{2i} b_i^* \left(\sum_{i=j}^{6} P_i \right) + U_i \tag{2}$$

where c_{i1} was used as c_i^* and the second term in (2) appears as a sort of weight so that c_{2i} is not used. The constant term A also includes $b_i^* y$.

The provisional c_i^*,s were in turn used to estimate provisional b_i^*, s by succesive OLS estimation of each of the following six equations :

$$P_i x_i = B_i + b\,(y - P_i c_i^*) + U_i \tag{3}$$

The constant term B includes $P_i c_i^* - b_j \sum_{i=j}^{6} P_i c_j^*$

The estimated b_i^*, s were used in (2) again and this itteration was repeated several times.

Since there was no guarante that this scheme would converge towards stable estimates when this occured as in the case of both imports and exports at current prices we ran several iterations to establish empirically a periodic pattern after which we selected the iteration with the highest system R^2.

After selecting the four iterations exhibiting the highest R^2 for each case, we intervened manually and scaled b_i, s up or down accordingly so that $\sum b_i = 1$, while at the same time we used alternative values of c_i, s frcm one case to the other (separately for imports and exports) observing that the system R^2 changed positively with the introduction of each different c_i. Otherwise we retained the original C_i value.

This had as a result a final common set of b_i, s for imports at both constant and current prices, and another set for exports as well as four sets of c_i, s (see tables).

This intervention increased R^2 as follows : For imports at constant prices from 0.64 to 0.76, for imports at current prices from 0.79 to 0.90, for exports at constant prices from 0,66 to 0.85.

For exports at current prices however R^2 decreased from 0.92 to 0.91.
$\sum_{i=1}^{6} b_i$ before the adjustment ranged for all four cases from 0.72 to 1.36.

The final empirical estimates are shown in tables 1 and 2.

TABLE 1

Imports at Constant and Current Prices

Commodity Group	b_i	C_i (Constant)	C_i (Current)	R^2 (Constant)	R^2 (Current)
1. Food Drinks Fats	0.152	0.091	0.091	0.71	0.75
2. Materials	0.057	0.033	0.041	0.50	0.35
3. Minerals	0.294	1.682	1.682	0.88	0.90
4. Chemicals	0.097	0.082	0.082	0.78	0.84
5. General Industrial	0.051	0.178	0.220	0.89	0.97
6. Machinery	0.349	0.042	0.042	0.60	0.96
			System R^2	0.76	0.90

TABLE 2

Exports at Constant and Current Prices

Commodity Group	b_i	C_i (Constant)	C_i (Current)	R^2 (Constant)	R^2(Current)
1. Food Drinks Fats	0.409	0.200	0.305	0.93	0.97
2. Materials	0.046	0.372	0.372	0.61	0.99
3. Minerals	0.167	0.166	0.166	0.71	0.69
4. Chemicals	0.063	0.011	0.011	0.59	0.46
5. General Industrial	0.254	0.768	1.184	0.78	0.42
6. Machinery	0.061	0.002	0.002	0.63	0.42
			System R^2	0.085	0.91

OBSERVED AND ESTIMATED IMPORTS CONSTANT

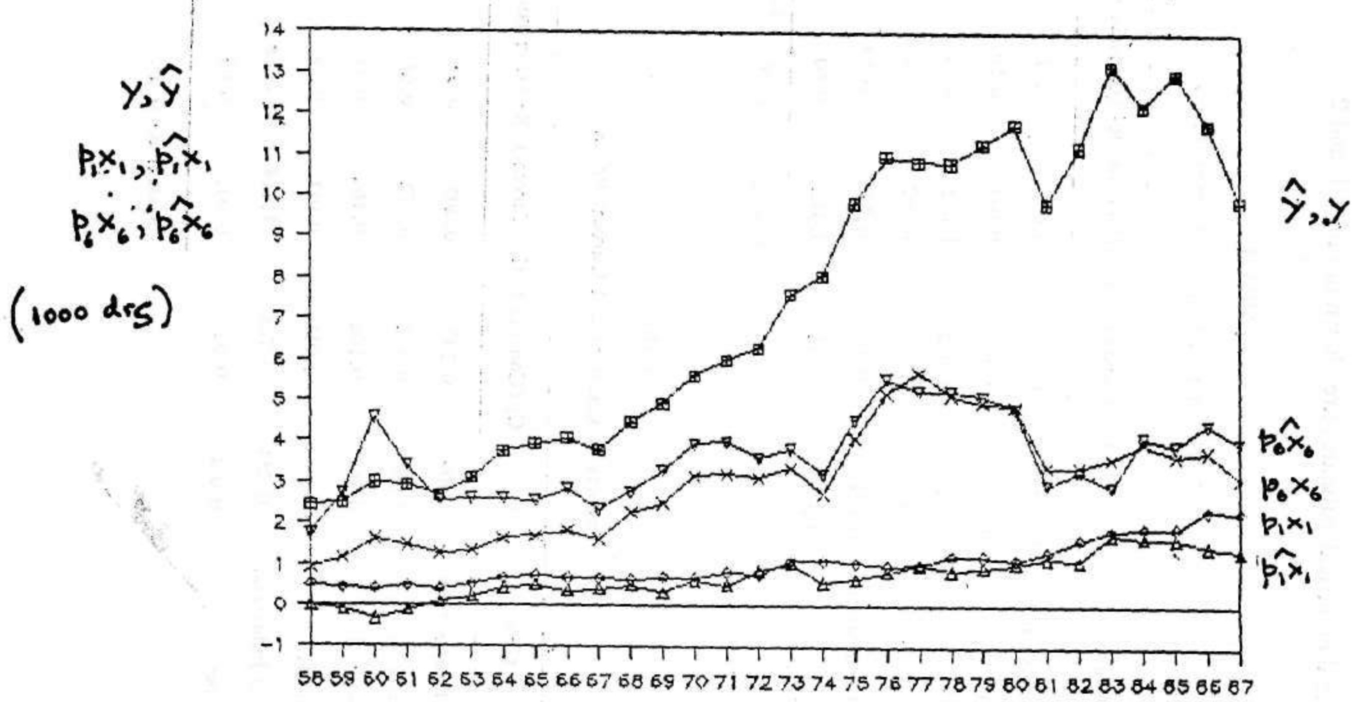

FIG. 1

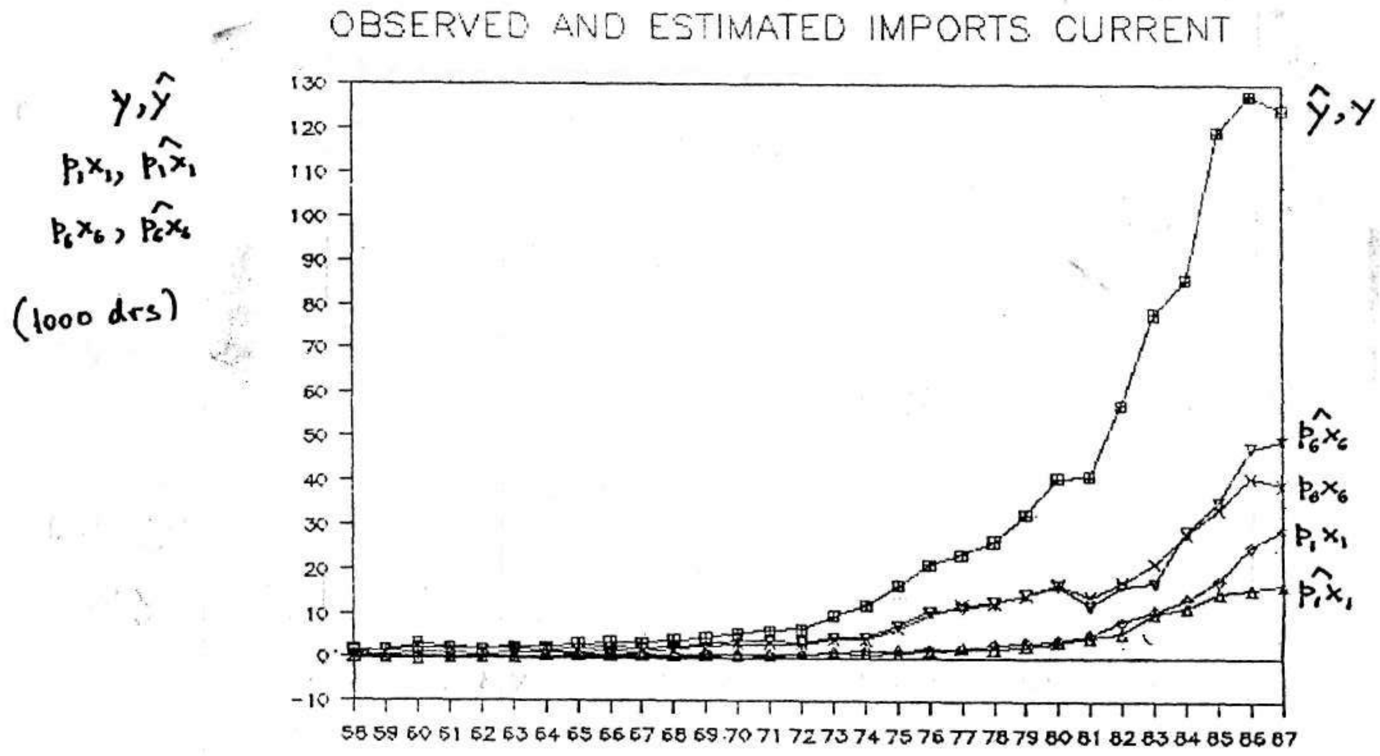
OBSERVED AND ESTIMATED IMPORTS CURRENT
$y, \hat{y}$
p_1x_1, $p_1\hat{x}_1$
p_6x_6, $p_6\hat{x}_6$
(1000 drs)
130
120
110
100
90
80
70
60
50
40
30
20
10
0
-10
58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87
$\hat{y}, y$
$p_6\hat{x}_6$
p_6x_6
p_1x_1
$p_1\hat{x}_1$

FIG. 2

$y, \hat{y}$

$p_1 x_1, \hat{p_1 x_1}$

$p_6 x_6, \hat{p_6 x_6}$

oo drs)

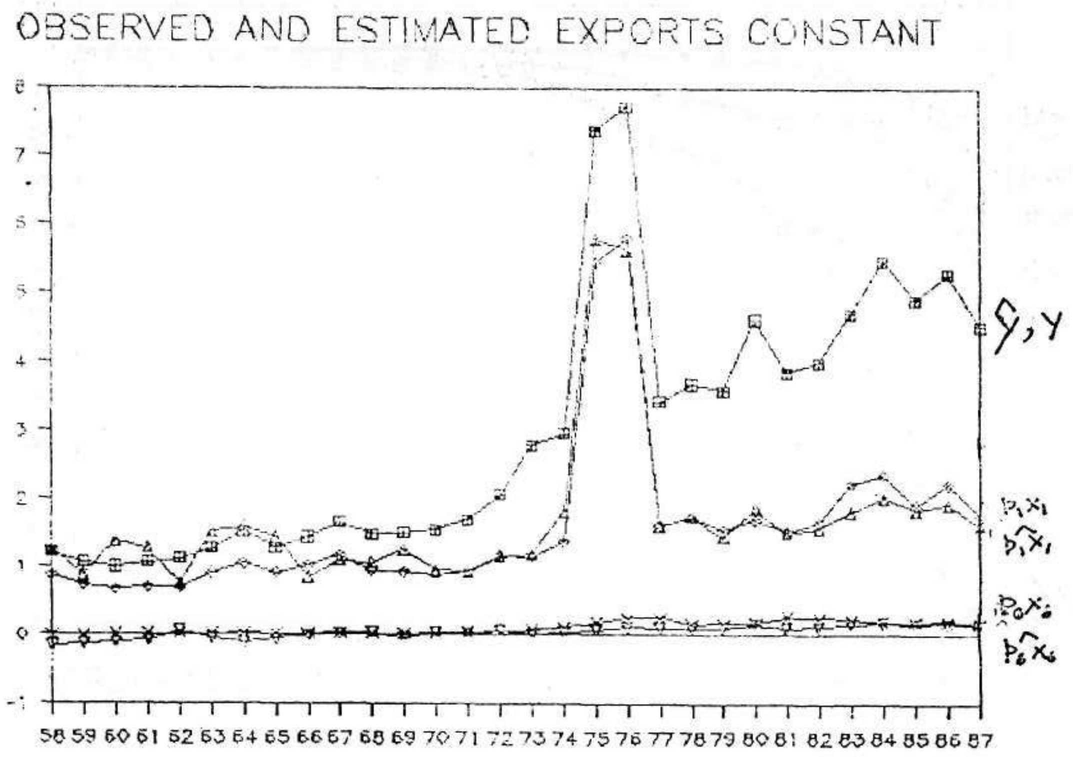

FIG. 3

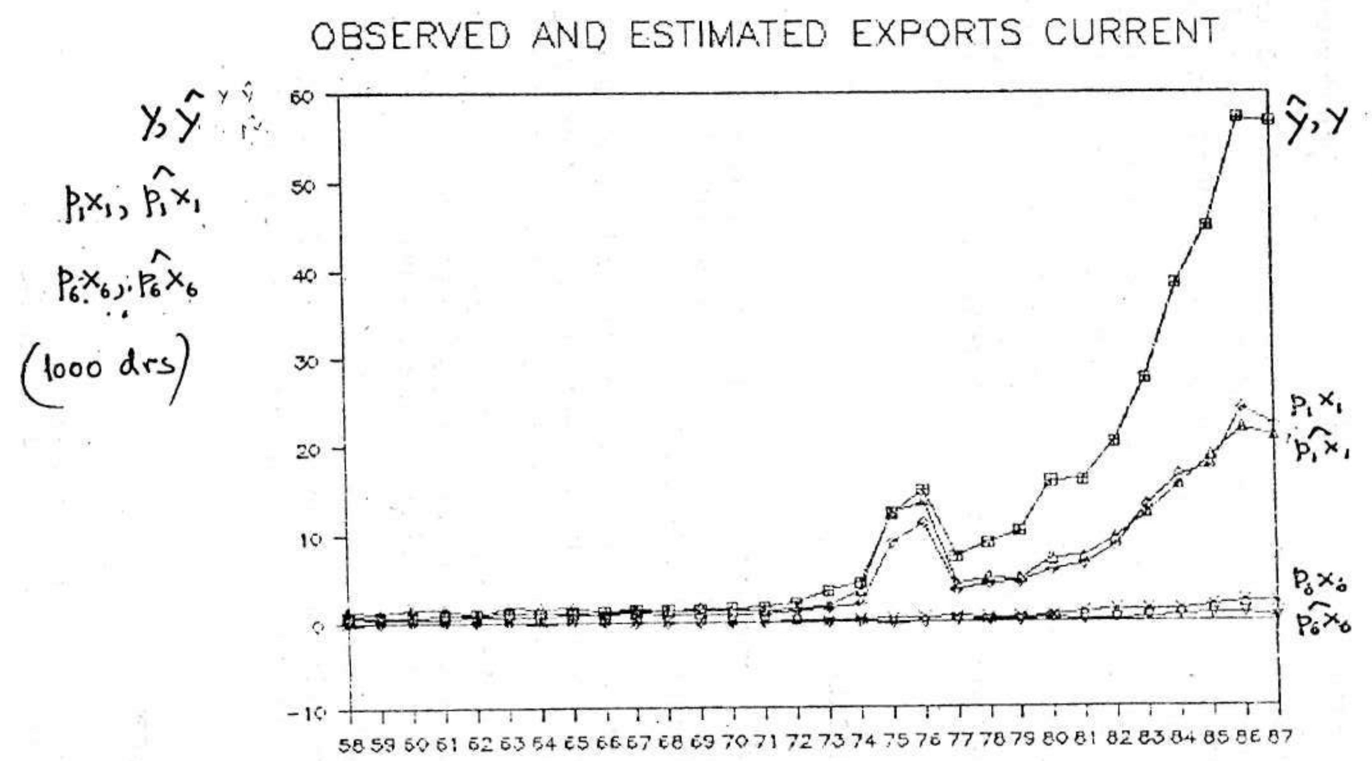
OBSERVED AND ESTIMATED EXPORTS CURRENT
$Y, \hat{Y}$
$P_1X_1, \hat{P_1}X_1$
$P_6X_6, \hat{P_6}X_6$
(1000 drs)
60
50
40
30
20
10
0
-10
58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87
$\hat{Y}, Y$
P_1X_1
$\hat{P_1}X_1$
P_6X_6
$\hat{P_6}X_6$

FIG. 4

These results were used to estimate $\sum_{i=1}^{6} P_i x_i \hat{y} =$ for each year which was plotted against observed y. This is shown in figures 1, 2, 3, 4. The differences $\hat{y} - y$ were small relative to the range of y and $\hat{y}$ values measured on the y axis and this is the reason that the computer printer draws only line instead of the actual two. Moreover, in order to highlight the results better we plotted observed and estimated $P_i x_i$' s for two commodity groups, Food Drinks and Fats, and Machinery, for all four cases. It is observed that the fit of each equation is not so good as in the case of y and $\hat{y}$, (see the negative Pixi in the beginning of the period 1958 - 1987). This is in accord with the basic LES hypothesis that the system can be applied when consumption is above a certain level.

CONCLUSION

As shown previously the LES is a relatively simple system that allows us to obtain a system of demand relationships which satisfy the theoretical conditions imposed by the standard demand theory.

Moreover, we indicated certain theoretical and estimational problems and limitations. Nevertheless, the LES has given very good results in practice and it is a useful model as well as a positive contribution to applied microeconomics.

These good results are verified in the case of our application. The results show that the LES and the basic hypothesis on which it is, based, is empirically valid in the case of Greek imports and exports (1958 -1987).

That is expenditure on imports and exports can be divided into a certain basic amount on imports and exports and into a remaining part that is reallocated on expenditure on imports and exports according to the estinmated bi' s.

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