ARBITRAGE PRICING MODEL: A CRITICAL EXAMINATION OF ITS EMPIRICAL APPLICABILITY FOR THE LONDON STOCK EXCHANGE

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The capital Asset Pricing Model (C.A.P.M.), developed by Sharpe (1964), Lintner (1965) and Mossin (1966), is an elegant and simple model for pricing risky securities. Unfortunately it has been strongly criticised by Roll (1977) because the market portfolio has not been identified and thus tests of its empirical validity cannot be constructed.

An alternative approach to characterisation of expected returns on risky securities is the Arbitrage Pricing Model (A.P.M.) proposed by Ross (1976, 1977). Its advantage is that several empirical studies have concluded that the A.P.M. can be verified empirically. Gehr (1978), Roll and Ross (1980), Chen (1981), Reinganum (1981) and Hughes (1982) provided some evidence towards this end. These tests are, however, based on a number of assumptions concerning the structure of data whose validity cannot always be guaranted. Unfortunately the studies mentioned previously took it to be the case that these assumptions are met, and no special tests were made to verity them. These tests of the A.P.M. therefore may be characterised as incomplete and so it cannot be inferred that the A.P.M. has been tested in an unambiguous fashion.

The necessary assumptions which ensure an unambiguous test of the A.P.M. using time series data can be summarised as follows :

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$\tilde{e}_{it}$ = the secrity specific disturbance with $E(\tilde{e}_{it}/\tilde{\delta}_{rt}) = 0$. The e's are commonly distributed. Also the security's e's are independent with any other security's e's and each disturbance has finite variance.

Ross (1976, 1977) shows that if no arbitrage profits can be made, then the security return generating model implies the following approximatelly linear risk - return relationship.

$$r_i \approx r_Z + b_{i1}(r_{L1} - r_Z) + b_{i2}(r_{L2} - r_Z) + \ldots + b_{eK}(r_{LK} - r_Z)$$

where

r_{LK} = the expected return on a portfolio with unit sensitivity to the K^{th} factor and zero sensitivity on the remaining K - 1 factors.

r_Z = the expected return on a portfolio that is orthogonal with each portfolio LK, for each K.

3. REVIEW OF THE EMPIRICAL EVIDENCE

A number of studies relating to topics dealing with this work have appeared since its inception.

Gibbons (1981) examined empirically whether the number of factors affecting portfolio returns remains the same across three different portfolio groups. To determine the relevant number of factors required to describe the covariance structure of 41 stock portfolios., and 9 bond portfocios, he utilised the appropriate likelihood ratio technique and concluded that, when one analysed stock and bond portfolios together, additional factors, common to both groups, had an in fluence on returns. These results, however, were not found when he analysed only one group of portfolios.

Kyrzanowski and To (1982) tested (1) the relationship between the number of factors that determine security returns and the sample size in terms of time periods and (2) the relationship between the number of factors that affect security returns and the size of the group being factored. For the first test both Rao and alpha factor analysis were used to determine in each of the six time intervals the relevant number of factors that is related to the security returns. The results showed that, on average, the number of factors associated with security returns remained approximately the same accross various samples of the same size and across various time intervales. For the second test Kryzanowski and to randomly dtew from a security group of size 50 four (overlapping) subgroups containing 10, 20, 30 and 40 securities, respecrively. They also employed Rao and alpha factor analysis to determine for each subgroup the relevant number of factors that accounts for the security intercorrelations.

Both factor analytic methods showed that the number of relevant factors increased with the group size. The results of Gibbons and Kryzanowski and To would be statistically more powerfull if they utilised more groups of portfolios (securities). Consequently given first the importance of the assumption regarding the existence of a security return generating model which remains the same across different security groups and across various time periods, and secondly the lack of statistical power of the previous tests, this paper is concerned with an investigation of such an assumption using time series data from the London Stock Exchange.

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4. SAMPLE AND RESEARCH METHODOLOGY

The empirical verification of the assumption that there exists a security return generating model which remains the same across different security groups and across various time periods requires a large sample, in terms of both the number of securities and the number of time periods. By examining different time periods it was found that the number of securities with continuous monthly data decreased as the length of the time period was increased. Therefore this study's sample was selected to satisfy the following two objectives :

(1) The number of observations per security to be as large is possible.

(2) A reasonable number of securities has to be contained in the sample and each security must be listed on the London Stock Exchange for the entire sample period.

Given these objectives the sample period selected was from November 1, 1956 to December 31, 1981, that is 302 monthly observations for each security, with 200 securities having continuous monthly data during the entire sample period. The second selection objective of the sample may introduce a survival bias in the sense that it has only included firms in existence during the entire sample period, the sample is thus based towards long - lasting firms and the results of the study have to be interpreted with this in mind.

The number of the securities in each sample was initialy divided into several random master groups of equal size. Before deciding the size of the master group consideration was made of the following :

(1) It is necessary to formulate a substantial number of master groups to provide grounds for statistical inference of the tests designed for the study.

(2) In order to increase the statistical power of the test concerning the relationship between the number of factors and the group size it is necessary to generate from each master group a considerable number of subgroups.

(3) The half size of the sample in terms of time periods has to exceed the size of the master groups. If the number of variables is greater than the number of observations then the resulting covariance (correlation) matrix is singular. But the empirical examination of the stated assumption requires non - singular covariance (correlation) matrix.

Having in mind these three requirements it was decided that the size of the master group was to be 40. The 200 company numbers were listed in ascending order and 5 groups each consisting of 40 securities were drawn. 7 subgroups were formed from each master group of the samble containing 5, 10, 15, 20, 25 30 and 35 securities respectively.

The subperiods were chosen to satisfy the following :

(1) To be non - overlapping. This requirement is extremely important fo our analysis. Indeed by utilising non - overlapping subperiods, the possibility of deriving replicable factors having influence on security returns during the rime period covered by a common length of the returns during the time period covered by a common length of the subperiods is minimised.

(2) To have equal lengths so that the performed tests are equally affected by the sample size in terms of time periods.

(3) To contain a number of return observations that is greater than the size of the master group (in order to exclude the possilility of deriving a singular covariance (correlation) matrix.

According to these requirements the rentire period of 302 observations was divided into two different sets subperiods. The first set is made from three non - overlapping subperiods of 100 olservations each (11/1956- 2/1965, 3/1965 - 6/1973 and 7/1973 - 12/1981), while the second is generated from two non - overlapping subperiods of 151 observations each (11/1956 - 5/1969 and 6/1969 - 12/1981).

Among the factor analytic methods the maximum likelihood is usually preferable since more is known about its statistical properties. However, the estimates derived by Rao's factor analysis constitute another, set of maximum likelihood estimates. Furthemore, the algorithm developed by Jorestog (1963) to solve the maximum likelihood estimation equations is extremely sensitive to ill-conditioned correlation matrices (i.e. to correlation matrices wich are usually singular) where the maximum likelihood method may produce invalid results. For these reasons it was decided to employ the factor analytic method of Rao.

One of the main advantages of Rao's factor analysis is that it provides the capability of estimating the relevant number of factors. This can be accomplished by assuming that K, the number of common factors which influence the security returns, is known in advance and then using a chi - square statistic to examine how well the model fits the data.

The test statistic is described by the following equation :

$$C_3 = (T_1 - 1 - \frac{2N+5}{6} - \frac{2}{3}K) \left[\ln (\lambda_{K+1} \ldots \lambda_K) - N - K) \ln (\frac{\lambda_{K+1} + \cdots + \lambda_K}{N - K}) \right] \quad (1)$$

where

C_3 is distributed as a chi - squared variate with ½ $[(N - K)^2 - N - K]$ degrees of freedom.

T_1 = the size of the random samples in terms of time periods.

n = the natural logarithm operator.

$K+1 \ldots, K$ = the last (N - K) characteristic roots of the first order conditions for the maximum value of the squared canonical correlation between the set of hypothesised factors and set of securities.

The null hypothesis that there exist exactly K factors is accepted if :

$$C_3 \langle x^2_{a,\,1/2} \left[(N-K)^2 - N - K \right] \quad (2)$$

where

a = a level of significance.

5. DESCRIPTION OF THE EMPIRICAL RESULTS

Table 1 shows that in 93 out of 100 cases the number of factors increases as the group size increases. By averaging the number of factors across security groups of the same size it canr also be seen that the number of factors increases with the group size. These results reveal that the number of factors which determine the security returns, is not the same across various security groups of different sizes and across various groups of the same size.

In view of the findings in Table 2 (table 3) it can be observed that the number of factors does not remain the same across the subperiods 11/1956 — 2/1965, 3/ 1965 — 6/1973 and (7/1973 — 10/1981 (11/1965 — 5/1969 and 6/1969 — 12/1981).

Another conclusion derived from the results of table 2 (Table 3) is that the number of factors changing across the three (two) subperiods as the group size changes. For example the first group containing 20 subperiods as the group size changes. For example the first group containing 20 securities yields for the subperiod 11/1956 – 2/1965 (11/1956 - 5/1969) 3 (3) factors, whereas the first group containing 40 securities yields for the subperiod 7/1973 - 10/1981 (6/1969 - 12/ 1981) 9 (10) factors.

One possible way to obtain a clear description concerning the relationship between the average number of factors and the group size is to run across - se-

Table 1 The number of factors versus the group size
Period : 11/1956-12/1981, Number of securities : 200

	MASTER GROUP 1		MASTER GROUP 2		MATER GROUP 3	
GROUP SIZE	NUMBER OF FACTORS	CUMMULATIVE PERCENTAGE OF TOTAL VARIANCE ACCOUNTED FOR BY THE COMMON FACTORS	NUMBER OF FACTORS	CUMMULATIVE PERCENTAGE OF TOTAL VARIANCE ACCOUNTED FOR BY THE COMMON FACTORS	NUMBER OF FACTORS	CUMMULATIVE PERCENTAGE OF TOTAL VARIANCE ACCOUNTED FOR BY THE COMMON FACTORS
5	1 (12.5,15)	54.6	1 (6.2,5)	51.4	1 (8.6,5)	47.7
10	2 (42.4,26)	62.2	1 (36.1,35)	48.2	2 (36.1,26)	62.1
15	4 (68,51)	62.4	2 (86.4,76)	51.9	4 (61.9,51)	65.0
20	4 (152.6,116)	61.6	3 (165.6,133)	53.8	4 (149.5,116)	58.6
25	6 (197,165)	67.1	5 (220.7,185)	60.5	4 (234.1,206)	54.2
30	7 (293.1,246)	68.2	7 (288.7,246)	63.1	6 (304.3,270)	60.4
35	8 (395.4,343)	67.4	8 (393.3,343)	63.7	7 (424.4,371)	62.3
40	10 (484.7,425)	65.8	9 (520,456)	63.0	10 (483.7,425)	65.1

Table 1 ... continued

	MASTER GROUP 4		MASTER GROUP 5			
GROUP SIZE	NUMBER OF FACTORS	CUMMULATIVE PERCENTAGE OF TOTAL VARIANCE ACCOUNTED FOR BY THE COMMON FACTORS	NUMBER OF FACTORS	CUMMULATIVE PERCENTAGE OF TOTAL VARIANCE ACCOUNTED FOR BY THE COMMON FACTORS	AVERAGE NUMBER OF FACTORS	AVERAGE OF CUMMULATIVE PERCENTAGE OF TOTAL VARIANCED ACCOUNTED FOR BY THE COMMON FACTORS
5	1[a] (5.5,5)[b]	45.2	2 (19,1)	63.9	1	52.5
10	2 (38,26)	57.7	2 (39.2,26)	52.7	2	56.5
15	3 (82.3,63)	55.9	3 (78.7,63)	59.0	3	60.1
20	5 (122.6,100)	61.6	3 (161,133)	54.0	4	57.9
25	6 (204.2,165)	61.7	4 (249.5,206)	56.9	5	59.1
30	6 (304.3,270)	61.2	4 (367.8,321)	55.3	6	61.6
35	8 (384.4,343)	61.2	5 (493.4,430)	57.6	7	62.4
40	8 (544.6,488)	59.9	7 (586.2,521)	60.5	9	62.8

a The null hypothesis that there exist exactly K factors is accepted at the 1% level of significance.

b The x^2 statistics and the degrees of fredom appear in the parentheses.

Table 2 Number of factors across three nonoverlapping subperiods for the same group of securities
Period : 11/1956-12/1981, Number of securities : 200

	GROUP SIZE 10		GROUP SIZE 20		GROUP SIZE 40	
SUB-PERIOD	NUMBER OF FACTORS	COMMULATIVE PERCENTAGE OF TOTAL VARIANCE ACCOUNTED FOR BY THE COMMON FACTORS	NUMBER OF FACTORS	CUMMULATIVE PERCENTAGE OF TOTAL VARIANCE ACCOUNTED FOR BY THE COMMON FACTORS	NUMBER OF FACTORS	CUMMULATIVE PERCENTAGE OF TOTAL VARIANCE ACCOUNTED FOR BY THE COMMON FACTORS
11/1956-2/1965	1 (22.5,35)	47.8	3 (145.8,133)	51.8	9 (517.1,456)	66.0
	1 (42.6,35)	48.1	4 (132.9,116)	58.4	9 (521.3,456)	66.9
	1 (28.6,45)	44.0	2 (166.4,151)	43.3	7 (591.4,521)	58.5
	2 (39.2,26)	53.8	2 (184.3,151)	46.7	8 (555.1,488)	60.4
	2 (40.3,26)	52.6	2 (190.7,151)	40.5	7 (587.3,521)	56.8
3/1965-6/1973	2 (39.5,26)	57.8	2 (176.0,151)	47.0	10 (483.1,425)	69.9
	1 (50.1,35)	39.3	3 170.8,133)	48.2	10 (490.6,425)	70.1
	2 (35.6,26)	61.2	3 (154.5,133)	51.0	9 (503.4,456)	63.9
	1 (38.1,35)	41.7	3 (159.8,133)	50.9	7 (587.4,521)	59.8
	1 (42.4,35)	40.4	2 (178.4,151)	45.3	6 (626.5,555)	55.2
7/1973-10/1981	2 (30.2,26)	67.9	4 (147.1,116)	69.2	9 (522.3,456)	76.3
	1 (42.4,35)	58.6	2 (181.3,151)	54.3	12 (422.9,366)	78.4
	2 (41.9,35)	70.3	4 (142.1,116)	67.0	10 (475.5,425)	75.9
	2 (25.0,26)	65.4	2 (175.5,151)	56.2	8 (558.9,488)	61.5
	2 (37.6,26)	64.9	3 (159.4,133)	62.4	7 (584.8,521)	60.3

a The null hypothesis that there exist exactly k factors is accepted at the 1% level of significance.

b The x^2 statistics and the degree of freedom appear in the parentheses.

Table 3 Number of factors across two nonoverlapping subperiods for the same group of securities
Period : 11/1956–12/1981, Number of securities : 200

	GROUP SIZE 10		GROUP SIZE 20		GROUP SIZE 40	
SUB-PERIOD	NUMBER OF FACTORS	CUMMULATIVE PERCENTAGE OF TOTAL VARIANCE ACCOUNTED FOR BY THE COMMON FACTORS	NUMBER OF FACTORS	CUMMULATIVE PERCENTAGE OF TOTAL VARIANCE ACCOUNTED FOR BY THE COMMON FACTORS	NUMBER OF FACTORS	CUMMULATIVE PERCENTAGE OF TOTAL VARIANCE ACCOUNTED FOR BY THE COMMON FACTORS
11/1956–5/1969	1[a] (44.5,35)[b]	46.8	3 (150.0,133)	52.2	9 (509.5,456)	63.6
	1 (41.7,35)	37.6	5 (121.4,100)	65.9	7 (570.4,521)	52.5
	2 (38.1,26)	43.7	3 (139.0,133)	48.7	7 (569.1,521)	56.2
	1 (42.6,35)	36.0	4 (144.1,116)	58.6	6 (624.9,555)	53.6
	1 (39.4,35)	35.8	2 (163.9,151)	39.9	6 (647.2,555)	52.00
6-1969–12/1981	2 (33.9,26)	65.4	4 (152.5,116)	65.8	10 (488.2,425)	74.4
	1 (40.6,35)	53.5	4 (147.6,116)	62.3	9 (518.1,456)	72.2
	2 (38.5,26)	67.3	4 (150.8,116)	61.7	11 (450.9,395)	75.9
	1 (49.6,35)	54.2	2 (181.1,151)	52.8	8 (551.1,488)	67.3
	2 (45.5,26)	55.9	3 (168.9,133)	60.2	7 (586.2,521)	67.2

a The null hypothesis that there exist exactly k factors is accepted at the 1% level of significance.

b The χ^2 statistics and the degrees of freedom appear in the parentheses.

Table 4 Cross-sectional regressions of the average number of factors on the group size

	ALPHA COEFFICIENT	BETA COEFFICIENT	ADJUSTED COEFFICIENT OF DETERMINATION
Period : 10/1956-12/1981	-0.25 (-1.03*)[a]	0.22 (22.5)	98.6
Number of securities : 200			
Estimated Regression line:	$Y_u = -0.25 + 0.22X_u$ [b]		

a t-statistics appear in the parentheses. The null hypothesis is that the regression coefficient is equal to zero. Asterisks indicate that the null hypothesis is accepted at the 1% level of significance.

b $u = 1, 2, \ldots\ldots, 8$

y_u = the average number of factors.

X_u = the group size.

ctiona! regression having as dependent variable the average number of factors and as independent variable the group size. The results of the cross - sectional regression are show η in Table 4. These findings reveal positive and significat relationships between the average number of factors and the group size.

6. SOME POSSIBLE EXPLANATIONS OF THE RESULTS

It was found that the number of factors affecting the security returns chan ges with the groups size. One possible explanation of the results is the employ ment of a limited number of security monthly observations for the London Stock Exchange anf hence the utilization of small sample sizes in terms of time pe riods. This may be true since the value obtained by equation (1) approximates a chi - sguare distribution only in the number of observations is large. A small number of observations may imply highly correlated returns which in turn increase the value obtained by equation (1). More factors, therefore, will be required to produce a value that approximates a chi — square di stiibution.

Next a major disadvantage of the sequential procedure is that the critical value of the test criterion is fixed, while the null hypothesis of the number of fa ctors is being tested in sequence and thus different chi— squared values are produced. As a consequence the number of factors will increase with the group size. If this is the case, factors will emerge which represent only statistical arti facts and hence the results produced will be unrealistic, this in turn would indicate the inability of the factor analysis solutions to describe security turns generating models.

The possible explanations stated previously are concerned with the mathematical model's assumptions used to test the relationship between the number of factors and the group size, however, they are not the only explanations of the results.

To test the assumption that the number of factors is the same across different groups a random sample of securities was utilised. This random sample generated a large number of random groups. Therefore some of the groups may contain securities of the same industry, while others may be comprised of securities from different industries. As a result the number of factors changes across different security groups of the same size. Moreover, if one adds new securities belonging to other industries to a group which contains securities of some particular industry, the number of factors will in creae. There are also factors of which account for a large proportion of the variability on some securities, but their influence on other securities is negligible. Since the securities of the groups in this study were chosen randomly, it is possible to find groups of securities whose returns are not highly affected by those factors, whereas for other groups of securities such factors are important in determining the returns. As a result the number of factors changes across various groups of the same size and across various groups of different sizes.

Finally in the real world it is possible that some of the factors found to affect the security returns in one period are unimportant in the following period. Example of such include political crises, oil crises, war scares, etc. In this case the number of factors determining the security returns changes through time.

7, COMPARISON WITH PREVIOUS STUDIES

Kryzanowski and To (1982) used only one mastergroup of securities and four (overlapping) subgroups and concluded that the number of relevant factors is an increasing function of the size of the group being factored.

Initially their conclusions can be critisized for the lack of statistical power. By considering only the group of securities they cannot ensure that the other groups will produce the same results. The empirical findings of the other groups will the present chapter indicate that the number of factors is positively related to the group size, but there are some cases where the number of factors does not increase with the group size.

Also they concluded that the eigenvalue-one criterion of the alpha factor analysis the same results as the statistical test of Rao's factor analysis method. However, the~eigenvalue - one rule of thumb is not a reliable criterion! In this study the eigenvalue-one criterion was violated in 90 cases of out 100. There were few cases where Rao's chi - square test produced relevant factors with eigenvalue greater than one, as well as cases where such a test produced relevant factors with eigenvalues less than one. For the second group of cases, there were factors ; producing significant chi - square values although their corresponding eigenvalues where in the range .85 - .90. In view of these results it can be concluded **that** it seems dangerous to apply the eigenvalue - one criterion to choose the relevant number of factors.

Kryzanowski and Chau concluded additionally that on average, the number of factors does not change substantially across various samples in terms of different period lenghts for the same group of securities. This conclusion, however, does not fall in line with the empirical results presented in this study.

Finally, Hughes (1982) used two groups containing 110 securities and a sample size of 120 observations. Her tests can be criricised because the utilised a large group size relative to the number of observations per security. Hughes stated :

> «The number of factors extracted was increased from five to twelve and the shi - square statistic continued to indicate that many additional factors were needed for adequate factoring», (p. 16).

But, the number of factors increases with the group size and the shi - square test she used requires a large number of observations relative to the size of group. In her case, therefore, the K - factor generation model could probably be tejected for every possible value of K (= the number of factors).

8. THE INDICATIONS OF THE EMPIRICAL RESULTS

The validity of the A.P.M. depends upon a unique security return generating model in the sense that the returns of a large number of securities are affected by a small number of relevant factors and each security return is determined by the same factors. Unfortunately, the theory behind the A.P.M. does not specify the number of the relevant factors which have an impact on security returns, as well as the identity of these factors. Hence the security return generating model of the A.P.M. is an unobservable model and as a consequence, the empirical examination of the A.P.M. is performed by utilising techniques depending only implicity on the underling factors.

Moreover for the A.P.M. 's test there exist computational restrictions with regard to the number of securities than can be handled at one time. Such restrictions necessitates splitting the securities of the sample into different groups and performing factor analytic techniquees separately for each group.

In view of the results reported in this paper it can be inferred that Rao's factor analytic technique produces, for the London Stock Exchange, different return generating models for security groups of different sizes as well as for security groups of the same size. It was explained in Section 5 that such results maybe due either to Rao's factor analyric technique or to the existence of different factors affecting the returns on securities of the randomly chosen groups. In either of these cases the following problems can be seen :

1) The identification of the unique security return generating model of the A. P.M.

2) The absence of an explicit description of the factors produced by factor analysing various security groups.

3) The existence of different security return generating models which emerged by factor analysing various groups of securities of different sizes and various security groups of the same size.

Accordingly it can be asserted that :

(i) It is very difficult to assess which is the appropriate group size that has to be utilised in order to investigate the empirical validity of the A.P.M. By using security groups having a given size it cannot be asserted that the producing security return generating model is the unique model of the A.P.M., since such a model exists if is unobservable.

(it) A basic assumption of the A.P.M. concerning the uniqueness of the security return generating model is violated. Thus the A.P.M. cannot be tested unambiguously using the series data from the London Stock Exchange. As a consequence one may challenge the introduction of the A.P.M. i«to the literature as a testable alternative to the A.P.M.

It is evident that these conclusions do not necessarily imply the invalidity of the A.P.M., they simply show our inability to provide a rigorous statistical methodology to test the model.

The conclusions derived in this section about the empirical tests of the A.P.M. arevery similartothoseofRoll's(1977)concerningthetestability of the C.A.P.M. Roll pointed out that fhe C.A.P.M..may be valid, but it cannot be tested unambiguously since there exists the market portfolio identification problem. Given a mean - standard deviation portfolio, there is not a method of assessing whether it provides a good proxy of the market portfolio. The tests performed by utilising a market proxy and employing the appropriate statistical techniques are not tests of the C. A. P. M., they are simply tests of the mean - standard devis tion efficiency of the chosen market proxy.

Similarly it seems that there also exists an identification problem in th case of the A.P.M., since it does not specify the number and the nature of th underlying factors which influence the security returns. In addition the result of this study indicate that there exists a positive relationship between the numbe of factors and the group size. Therefore given a pre-specified group size, ther is no way to ascertain, whether the security return generating model produced vi factor analysis is the unique generating model of the A.P.M. As a result the test performed by using such a generating model are not necessarily tests of the A.P.M

In the mean-standard deviation each mean-standard deviation efficient portfolio produces a security return-risk linear relationship which while having fora as the C.A.P.M., is not the C.A.P.M. Similar situations are obtained in the A.P.M. since from security groups of different sizes different security returns generatin; models have emerged; each security return generating model may produce a security return-risk linear relationship having the same form as the A.P.M., but such ; relationship may not be the A.P.M.

The previously mentioned conclusions regarding the empirical examination the A.P.M are similar to those of Shanken (1982). Reconsidered two equivalen sets of securities in the sense that the portfolios emerging by combining the securi ties of the second set have an equal rate of return of the securities of the first set According to the AiP.M., such equivalent security sets should yield the same secu rity returns generating model as well as the same security pricing relationship However, Shanken proved theoretically that equivalent security sets yield diffe rent security return generating models and hence different return-risk linear rela tionships.

In the light of his theoretical findings and the identification problem in factor analysis; he argued that the relevant security returns generating model is uno· bservable and his argument is similar to that of Roll's concerning the empirica examination of the C.A.P.M. According to Shanken:

> «Roll argues that empirical investigations of the C.A.P.M. which use proxies for the true market portfolio are really tests of the meanva- riance efficiency of those proxies, not tests of the C.A.P.M. The C-A.P.M implies that a particular portfolio, the market portfolio, is efficient The theory is not testable unless that portfolio is observable and used in tests,

Similarly, it is argued here that factor-analytic empirical investigations of the A.P.M. are not necessarily tests of that theory. In the case of the A.P.M. We are confronted with the task of identifyng the relevant factor structure, rather than the true market portfolio. Whereas we have a reasonably clear notion of what is meant by «the true market portfolio», it is not clear in what sense, if any, a uniquely «relevant factor structure» exists. We noted in Section II that there are, in general, many factor structure corresponding to equivalent sets of securities. The A.P.T. does not appear to provide a criterion for singling out one structure as the «relevant» one», (pp. 1135 - 1136).

The results of Table 2 (3) indicate that the security return generating model cannot be used for forecasting purposes.

Since different return generating models were found across various time periods for different security groups, it can be asserted that there is a violation of the A.P.M.'s assumption about the uniqueness of the security returns generating model across various time periods for the same group of securities of-for different security groups. In view of the identification problem of the security returns generating model of the A.P.M., however, there is no way to ascertain which is the appropriate time length that has to be used in order to examine empirically the validity of the A.P.M. By utilising a given sample period it cannot be asserted that the producing security returns generating model is the unique model of the A.P.M., since if such a model exists it cannot be identified.

The instability of the number of factors through time also shows the violation of a major assumption required to transform the A.P.M. into a testable relationship. Therefore it can be inferred that the A.P.M. cannot be tested unambiguously using time series data for the London Stock Exchange and as a consequence the introduction of the A.P.M. into the literature as a testable alternative to the C.A.P.M., may be questionable.

Finally, it may be stated that a test concerning the intertemporal stationarity of the factor beta coefficients will be useless, since the factors affecting the security returns are not the same across various time periods for the same group of securities.

9. CONCLUSIONS

The present study has utilised time series data from the London Stock Exchange and has concentrated upon the empirical verification of the assumption that there exists a security return generating model which remains the same across different security groups and across various time periods. The findings presented in this work indicate that the number of factor changes as the group size changes. Such results highlight the fact that the methodology used for testing the arbitrage pricing model is not the appropriate one, and previous tests of the arbitrage pricing model are not necessarily tests of the model. The arbitrage pricing model may be true, but the existing statistical methodology does not provide an unambiguous test of the model for the London Stock Exchange.

The number of factors also changes across various time periods for the same group of securities and for different security group. These findings suggest that the security returns generating model of the arbitrage pricing theory cannot be used for making predictions. These results, however, do not constitute evidence against the arbitrage pricing model. The arbitrage pricing model may still hold» but the present state of the statistical methodology cannot be utilised to provide an unambiguous test of the model for the London Stock Exchange.

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