# FORECASTING STOCK BETAS: EVIDENCE FOR THE LONDON STOCK EXCHANGE*

By
*George P. Diacogiannis (Ph.D.)*
Warwick Business School (England)

Abstract

This paper investigates the forecasting ability of beta coefficients for individual securities and portfolios using time series data from the London Stock Exchange. Individual security beta estimates of one period are good predictors of the corresponding betas in the subsequent period, whereas portfolio beta estimates are found to be relatively predictable. The estimated betas can be improved by making use of different adjustment techniques and in the case of portfolios this improvement is greater when the portfolio size is increased. Adjustment methods can also be utilized in order to reduce the forecast errors associated with different risk classes.

## INTRODUCTION

The Capital Asset Pricing Model (CAMP) developed by Sharpe (1964), Lintner (1965) and Mossin (1966) is a one-period linear model which highlights a cross - sectional equilibrium relationship between expected returns and systematic risk (the beta coefficient) for securities or portfolios. Empirical applications of the CAPM require that future beta values be predicted as accurately as possible,

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a requirement which is not essential to the theoretical development of the model[1].

It is common in practice to estimate historical betas by utilizing the market model (initially proposed by Sharpe (1965). The usefulness of an estimated beta for measuring the expected risk of a security or portfolio depends, therefore, upon its predictive ability (unless of cource beta changes in a deterministic fashion).

The primary purpose of this paper is to examine the forecasting ability of the beta coefficient as well as to determine whether beta forecasts can be improved by employing the following three alternative adjustment procedures: first, the method developed by Blume (1975), second the Bayesian technique suggested by Vasicek (1973) and third the procedure used in the Security Risk Evaluation service by Merrill Lynch, Pierce, Fenner and Smith, Inc. (the latter was used in order to compare techniques suggested by the theory of finance with that used by a firm in practice). This investigation is performed for the following cases:

(a) Where security and portfolio betas are used.

(b) Where different security risk classes are utilized.

It is also noted that no previous evidence has been published in the UK on the relationship between the predictive ability of security betas and beta risk classes.

This study is organized as follows: The first section reviews briefly some previous work. The next section describes the data and the research methodology used, while the empirical results are presented in the third section. The fourth and final section, contains a summary of the paper.

## I. Previous studies

Using U.S. monthly data Klemkosky and Martin (1975) produced evidence indicating that beta adjustment techniques are useful for improving security and portfolio beta forecasts. The utilization of a Bayesian method in particular

1. (a) Similarly it can be argued for the Black (1972) version of the CAPM.

(b) Roll (1977) pointed out that the CAPM cannot be tested, because the market portfolio is unobservable; despite this fact, the practical applications of an expected risk - return relationship which is based upon a market proxy requires the beta stationarity.

revealed that portfolio betas estimated in one period are highly predictable using the corresponding betas of the previous period.

In another US study, Eubank and Zumwalt (1979) examined for different estimation - prediction period pairs the impact of beta adjustment procedures on security and portfolio beta forecasts for various risk classes. Their major finding was that beta adjustment techniques can be successful in reducing the forecast errors associated with the highest and lowest risk classes. This was specially notable for individual securities and shorter (ie 12 months) estimation and prediction periods.

The study based on New Zealand weekly data by Emanuel (1980) used the beta adjustment methods (of Blume (1975) and Vasicek (1973)) and concluded that for small portfolios their beta coefficients of one period were good predictors of the corresponding betas in the subsequent period.

The only previous work in this area on British data was carried out by Dimson and Marsh (1983). They investigated the stability of the beta of thin trading securities after using a method designed to avoid thin trading bias. The findings of this study indicated that the stability of individual securities betas was moderate, whereas portfolio betas were very stable (the portfolio beta stability was examined by using the transition matrices method, while the present study utilizes the mean square error technique). Also by employing two adjustment techniques (Blume (1975) and Vasicek (1973)) for the security beta coefficients their results showed improvements in beta forecasts.

## II. The Data and Research Methodology

This study uses firms from the London Stock Exchange for the following reasons:

(a) To compare the results with previous studies conducted in USA.

(b) Previous UK researchers have utilised in their analysis the systematic risk without taking into account the degree of predictability of the coefficient.

The data used in this study was drawn from the London Share Price Database (LSPD)..The Returns File of the LSPD contains monthly log-returns (continuously compounded returns) of a majority of the ordinary shares that have been traded at the London Stock Exchange (LSE) since January 1955. To qualify for inclusion in the sample, a firm has initially to satisfy the following criterion:

(a) To have a complete history of monthly returns from January 1969 through December 1983.

This selection criterion may introduce a survival bias in the sense that it only includes firms in existence during the 15 years sample period; the sample is therefore biased towards long-lasting firms and the results of the present study have to be interpreted with this in mind.

Among those firms which were listed continuously on the LSE during the entire sample period there were firms with infrequently traded shares. Including these firms in the sample will bias the estimates of the variances and covariances which in turn will produce biased estimates of the systematic risk. For this reason it was necessary to consider also the following criterion:

(b) Securities having at least one month with no recorded trade over the entire sample period of 180 monthly obserbations are excluded[2].

The sample used in this study contains the first 200 companies selected from the total number of the firms which satisfied the two criteria. The entire sample period was divided into 3 consecutive subperiods having equal length of 60 months each (1/69 – 12/73, 1/74 – 12/78, and 1/79 – 12/83).

The systematic risk for each security or portfolio in the sample is estimated by employing the market model:

$$\tilde{R}_{it} = a_i + b_i \tilde{R}_{Mt} + \tilde{e}_{it}$$

where

$\tilde{R}_{it}$ = the rate of return for security or portfolio i in month t.

$\tilde{R}_{Mt}$ = the rate of return for the Financial Times – Actuaries All Share Index in month t.

$a_i$ = the regression intercept for the security or portfolio i.

$b_i$ = the regression coefficient for the security or portfolio i, which measures its systematic risk.

2. The LSPD contains for each monthly return an equivalent non – trading indicator. This basically indicates the number of days before the end of the month that the last trade occurred. The number 0 associated with a particular monthly return indicates that such a share was traded the last day of the month. In the sample of this study 91% of the firms had a thin traded indicator of 0 in all months of the sample period.

$\tilde{e}_{it}$ = a stochastic disturbance term which has mean zero, constant variance, and is serially uncorrelated. It is also assumed that the disturbance term and the return on the market index are independent (ie the joint distribution of $R_{it}$ and $R_{Mt}$ is bivariate normal).

To examine the forecasting ability of beta the Mean Square Error (MSE) between estimated and predicted beta will be used (Granger and Newbold (1977), Ch. 8). The MSE is given by the following expression:

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (b_{ie} - b_{ip})^2 \tag{1}$$

where

$N$ = the number of securities or portfolios in the sample.

$b_{ie}$ = the estimated beta for the security or portfolio i.

$b_{ip}$ = the predicted beta for the security or portfolio i.

Equation (1) is equivalent to (Granger and Newbold (1977), p. 287)

$$MSE = (\bar{b}_e - \bar{b}_p)^2 + (1 - \gamma_{ep})^2 s_p^2 + (1 - R_{ep}^2)\ s_e^2 \tag{2}$$

where

$\bar{b}_e$, $\bar{b}_p$ = the means of the estimated and predicted beta values.

$s_e^2$, $s_p^2$ = the cross-sectional variances of the estimated and predicted beta values.

$\gamma_{ep}$ = the regression coefficient (slope), from regressing estimated on predicted beta values.

$R_{ep}^2$ = the coefficient of determination between estimated and predicted beta values.

Equation (2) is comprised of the following three components:

$(\bar{b}_e - \bar{b}_p)^2$ = the bias component.

$(1 - \gamma_{ep})^2\ s_p^2$ = the inefficiency component.

$(1-R^2_{ep})\ s^2_e$ = the random disturbance component.

Bias in a forecast measures the shift of the mean of the estimated betas from the man of the predicted beta. The bias is equal to zero if $\bar{b}_e = \bar{b}_p$. Inefficiency in a forecast is related to prediction errors $(b_{ie} - b_{ip})$, which are due to the shift of the regression slope between estimated and predicted betas from 1. It is equal to zero if the slope from regressing estimated on predicted betas equals 1. The random disturbance term contains those forecast errors which are caused by imperfect covariation between the estimated and predicted values of beta. The random disturbance is equal to zero if estimated and predicted betas are perfectly positively correlated. The MSE takes the value of zero if:

(a) The intercept and the slope from regressing estimated betas on predicted betas equal 0 and 1, respectively, and

(b) The coefficient of determination between estimated and predicted betas equals 1.

In order to forecast betas the following four methods are employed:

(A) The Unadjusted Prediction Method

According to this method the beta coefficients for each security or portfolio are estimated by applying the ordinary least square method over the first and second subperiod of the sample. Then the estimated beta of these subperiods are utilized to predict the beta values for subperiods two and three, respectively.

(B) The Blume Method

This method developed by Blume (1975), initially estimated for each security or portfolio in the sample the beta values of subperiods one and two, $b_{ie,1}$ and $b_{ie,2}$; then the following cross-sectional regression is run:

$$\tilde{b}_{ie,2} = q_1 + q_2\ \tilde{b}_{ie,1} + \tilde{u}_i$$

The estimated regression coefficients $q_1$ and $q_2$ are used to produce the predicted beta for the third subperiod as follows:

$$b_{ip,3} = \hat{q}_1 + \hat{q}_2\ b_{ie,2}$$

(C) The Bayesian Method

This method was recommended by Vasicek (1973) and it forecasts the beta by using the expression:

$$b_{ip,t} = \frac{(\bar{b}_{e,t-1}/s^2_{e,t-1}) + (b_{ie,t-1}/s^2_{ie,t-1})}{(1/s^2_{e,t-1}) + (1/s^2_{ie,t-1})}$$

where

$t$ = 2, 3.

$b_{ie,t-1}$ = the beta coefficient of security or portfolio i estimated by using the market model and the subperiod $t-1$.

$s_{ie,t-1}$ = the standard error of $b_{ie,t-1}$.

$\bar{b}_{e,t-1}$ = the average of the cross-sectional beta estimates in period $t-1$.

$s_{e,t-1}$ = the standard deviation of the cross-sectional beta estimates in period $t-1$.

(D) The Merril Lynch, Piece, Fenner and Smith Method

This is a method used by the brokerage firm of Merrill Lynch, Pierce, Fenner and Smith (MLPFS) which forecasts beta by employing the following equation:

$$b_{ip,3} = 1 + k_{12}\,(b_{ie,2} - 1)$$

where

$k_{12}$ = the slope of the regression between estimated beta over the first and second subperiod.

The unadjusted prediction and the Bayesian techniques require two consecutive subperiods, whereas for the Blume and MLPFS methods three consecutive subperiods are needed. The stationarity of a security or portfolio beta is evaluated by comparing the predicted betas of each method with the estimated betas which have actually occurred over the estimation subperiod (the latter betas are calculated using the market model).

## III. Description and Interpretation of the Empirical Results

### A. Mean Square Error of Adjusted and Unadjusted Betas

The ordinary least squares method was applied to estimate the beta coefficient for the three non-overlapping subperiods from January 1969 to December 1973; January 1974 to December 1978; and January 1979 to December 1983. Exhibit 1 provides the cross-sectional distributions of the beta coefficient, from which it can be seen that the cross-sectional mean betas are close to unity. The cross-sectional standard deviations of betas indicate that the betas of the third subperiod varied slightly more than those of the first and second subperiods.

EXHIBIT 1

Cross-Sectional Distributions for Beta Coefficients

| Subperiod | 1/69-12/73 | 1/74-12/78 | 1/79-12/83 |
|---|---|---|---|
| Mean | 1.0067 | 0.9401 | 0.9554 |
| Standard Deviation | 0.2527 | 0.2634 | 0.2778 |
| Maximum | 1.7750 | 1.6330 | 1.5240 |
| Minimum | 0.3430 | 0.2220 | 0.2050 |

Exhibit 2 shows the total MSE and its components for unadjusted and adjusted security betas obtained by using three consecutive subperiods of equal length (60 monthly observations each).

The following interesting features can be noted from these results:

(a) The past estimates of security betas are not good predictors of the corresponding future betas. The total MSE comprised of substantial inefficiency and random disturbance components (33 and 65 per cent of the total MSE, respectively). The empirical methodology which relies upon equation (1) assumes the use of forecasts and actual beta values. However, the present study utilizes estimated rather than actual values of beta. This explains, at least partially, why the random disturbance component of the MSE is large.

(b) The unadjusted MSE can be reduced when an adjusting technique is employed (on average the unadjusted MSE is reduced by 33 per cent). This

EXHIBIT 2

Mean Square Errors of Unadjusted and Adjusted Beta Coefficients for Securities
(Ordinary Least Square Method)

| 3 Subperiods (60 Observations each) | | | Unadjusted [1] | Bayesian [2] |
|---|---|---|---|---|
| Estimation Subperiod | Prediction Subperiod | MSE Components | | |
| 1/69 – 12/73 | 1/74 – 12/78 | Bias | .00232 | .00183 |
| or | | | | |
| 1/74 – 12/78 | 1/79 – 12/83 | Inefficiency | .03420 | .00252 |
| | | Random Disturbance | .06800 | .06500 |
| | | Total MSE | .10452 | .06935 |
| | | | Blume | MLPFS |
| Estimation Subperiods | Prediction Subperiod | | | |
| 1/69 – 12/73 | 1/79 – 12/83 | Bias | .00124 | .00215 |
| and | | | | |
| 1/74 – 12/78 | | Inefficiency | .00147 | .01112 |
| | | Random Disturbance | .06800 | .06800 |
| | | Total MSE | .07071 | .08112 |

1, 2 Average over the following subperiods: (1/69 – 12/73) and (1/74 – 12/78), (1/74 – 12/78) and (1/79 – 12/83).

reduction comes primarily from the inefficiency component of the MSE (a result which is predictable since the adjustment procedures are based on the existence of inefficiency in the forecast). Indeed the Bayesian, Blume and MLPFS method reduced the inefficiency component by 93, 96, and 98 per cent, respectively. The bias component was very small, less than 3 per cent of total MSE, in all the four methods, indicating that the cross – sectional means of the estimated and predicted betas were close. The largest component of the MSE consisted of random disturbance which remained the same when the Blume and MLPFS methods were used.

Comparing the results obtained by utilizing an adjusted technique, the

Bayesian procedure achieved the largest reduction of the unadjusted MSE, while the MLPFS method produced the smallest.

Taking together (a) and (b) above it can be inferred that beta adjusted techniques provide a better forecast for the systematic risk of individual securities than the unadjusted prediction method. This conclusion is in line with the results of Klemkosky and Martin (1975), Eubank and Zumwalt (1979), and Dimson and Marsh (1983).

In Exhibit 3 the total MSEs and their components for different size portfolios are presented. The portfolios were constructed by listing the 200 securities in alphabetical order and assigning the first N-securities to the first portfolio of size N, the second N-securities to the second portfolio of size N, etc. As in the case of individual securities the total 15-year sample period was divided into three consecutive subperiods of equal length (60 monthly observations each). From Exhibit 3 the following observations can be made:

(a) The grouping of securities into portfolios substantially reduces the MSE of the unadjusted betas, and this reduction increase as more securities are included in the portfolio. These results suggest that the forecasting ability of portfolio betas can be improved as portfolio size increase (this is nearly what would be expected theoretically; generally as portfolio size goes up, beta approaches 1 and hence forecasting is easier). The random disturbances constitute the largest parts of the total MSEs but they decrease in moving from the smaller to the larger portfolio.

(b) The MSE is substantially reduced when beta adjusted techniques are utilized, this reduction increase with the portfolio size and it comes primarily from the random disturbance component of the total MSEs. Such a component decreases as one moves from the smaller to the larger portfolio, indicating a positive and increasing relationship between the correlation coefficient of the estimated and predicted betas and the portfolio size. The inefficiency and the bias terms of the MSEs also decrease as the portfolio size increases. Furthermore a comparison of the Bayesian and the Blume method in addition reveals that the former generally outperforms the latter.

These findings indicate that portfolio beta forecasts can be improved when securities are grouped into portfolios and that the improvement is greater when portfolio size is increased. This evidence is similar to that of Klemkosky and Martin (1975), and Eubank and Zumwalt (1979).

EXHIBIT 3

Mean Square Errors for Unadjusted and Adjusted Beta Coefficients for Portfolios (Ordinary Square Method)

| | | | | Unadjusted[1] | |
|---|---|---|---|---|---|
| MSE Components | Portfolio Size: | 5 | 10 | 15 | 25 |
| Bias | | .00188 | .00170 | .00170 | .00169 |
| Inefficiency | | .00341 | .00180 | .00148 | .00113 |
| Random Disturbance | | .01134 | .01000 | .00743 | .00370 |
| Total MSE | | .01663 | 0.1350 | 0.1061 | .00652 |
| | | | | Bayesian[2] | |
| MSE Components | Portfolio Size: | 5 | 10 | 15 | 25 |
| Bias | | .00288 | .00018 | .00013 | .00001 |
| Inefficiency | | .00160 | .00140 | .00125 | .00066 |
| Random Disturbance | | .01060 | .00513 | .00407 | .00197 |
| Total MSE | | .01440 | .00671 | .00445 | .00258 |
| | | | | Blume | |
| MSE Components | Portfolio Size: | 5 | 10 | 15 | 25 |
| Bias | | .00240 | .00200 | .00186 | .00120 |
| Inefficiency | | .00033 | .00050 | .00039 | .00031 |
| Random Disturbance | | .00715 | .00520 | .00443 | .00122 |
| Total MSE | | .00988 | .00770 | .00668 | .00273 |

| MSE Components | Portfolio Size: | MLPFS 5 | 10 | 15 | 25 |
|---|---|---|---|---|---|
| Bias | | .00032 | .00022 | .00022 | .00011 |
| Inefficiency | | .00080 | .00077 | .00052 | .00101 |
| Random Disturbance | | .01030 | .00700 | .00613 | .00170 |
| Total MSE | | .01146 | .00799 | .00687 | .00282 |

1, 2 Average over the following subperiods: (1/69 – 12/73) and (1/74 – 12/78), (1/74 – 12/78) and (1/79 – 12/83).

The results shown in Exhibit 2 can be affected if the market model's assumptions of autocorrelation and heteroscedasticity are seriously violated. For this reason it was decided to assess empirically such assumptions and to reproduce the results by correcting for possible violations. The magnitude of the autocorrelation and the extent of heteroscedasticity in the market model were small and the results, not presented here, indicated that such violations have little impact on the estimated MSEs.

## B. Mean Square Error for Alternative Risk Classes

In this section the relationships between different security risk classes and the MSEs are examined. In order to estimate the MSEs for alternative risk classes the betas of the first and third subperiod were ordered in accordance with the size of the second period betas; The 200 betas of each subperiod were then divided into quintiles and the MSEs were estimated for the lowest, middle, and highest quintiles. The results of Exhibit 4 give rise to the following observations:

(a) The MSEs of the most risky group are always greater than those related to the least risky group; this suggests that the systematic risk of the so - called aggressive securities fluctuates through time more than that of the so-called defensive securities. The largest (smallest) difference between the MSEs of the lowest and highest quintiles occurred for the unadjusted (MLPFS) method. The MSEs of the middle risk class are lower than those of the other two classes, implying that betas near 1 can be forecasted better than higher or lower betas.

(b) The utilization of the three different adjusted techniques produced significant reductions of the unadjusted betas MSEs's. The largest reduction occurred for the highest and lowest quintiles; for example the Bayesian procedure

EXHIBIT 4

Mean Square Errors of Unadjusted and Adjusted Beta Coefficients for Alternative Risk Classes (Individual Securities)

| | | | Unadjusted[1] | |
|---|---|---|---|---|
| MSE Components | Quintile: | 1 | 3 | 5 |
| Bias | | .00932 | .00464 | .02283 |
| Inefficiency | | .03000 | .02176 | .05921 |
| Random Disturbance | | .06961 | .05763 | .08442 |
| Total MSE | | .10893 | .08403 | .16646 |
| | | | Bayesian[2] | |
| MSE Components | Quintile: | 1 | 3 | 5 |
| Bias | | .00009 | .00000 | .00038 |
| Inefficiency | | .00392 | .00004 | .00782 |
| Random Disturbance | | .05843 | .05002 | .06910 |
| Total MSE | | .05883 | .05006 | .07730 |
| | | | Blume | |
| MSE Components | Quintile: | 1 | 3 | 5 |
| Bias | | .00305 | .00079 | .00316 |
| Inefficiency | | .00556 | .00150 | .00250 |
| Random Disturbance | | .06066 | .04790 | .07744 |
| Total MSE | | .06927 | .05019 | .08310 |
| | | | MLPFS | |
| MSE Components | Quintile: | 1 | 3 | 5 |
| Bias | | .00016 | .00003 | .00022 |
| Inefficiency | | .00042 | .00017 | .00091 |
| Random Disturbance | | .07700 | .05723 | .08322 |
| Total MSE | | .07758 | .05743 | .08435 |

1, 2 Average over the following subperiods: (1/69 – 12/73) and (1/74 – 12/78), (1/74 – 12/78) and (1/79 – 12/83).

reduced the total MSE of the unadjusted betas for the lower quintile by 46 per cent and for the higher by 53 per cent. In each case the random disturbance term represents the largest portion of the MSEs and increase in moving from the lowest to the highest risk class; for example when the Bayesian method is employed the random disturbance term for the lowest, middle, and highest quintile comprised 93.3, 99.9, and 89 per cent, respectively, of the total MSE. A comparison between the random disturbance of the unadjusted betas with those of the adjusted betas reveals that the adjustment techniques did not affect considerably this component of the MSE. The smallest reduction of the random disturbance term occurred for the Bayesian procedure. As it was pointed out earlier the present study uses estimated rather than actual beta values, while the methodology employed assumes actual beta values. This implies that the extreme quintiles probably have the worst estimates of actual betas which in turn partially explains the large estimates of the random disturbance component.

The efficiency component of the unadjusted betas also constitutes a large portion of the MSEs (for the lowest, middle, and highest quintile the efficiency component comprised 27, 26, and 36, respectively, of the total MSE). This component was substantially reduced when any of the three adjustment techniques were used. The adjustment methods are also useful for reducing the bias term of the unadjusted betas; the bias of the middle risk class is smaller than those of the other two classes since the betas which are close to 1 are more predictable (in the case of the Bayesian method the bias term equals to zero, indicating that the means of the estimated and predicted betas are equal). Lastly from Exhibit 4 it can be noticed that the Bayesian approach produces a lower MSE than the Blume technique and that the MLPFS underperforms the other two adjusted methods, a result which was also reported earlier in the paper.

These findings are very similar to the results presented by Eubank and Zumwalt (1979), although they found that Blume's method outperforms the Bayesian procedure.

## IV. Conclusions

In this study the predictive ability of individual security and portfolio betas is examined by utilizing time series data from the LSE. Individual security beta estimates of one period are not good predictors of the corresponding betas in the subsequent period, while portfolio betas estimated in one period are relatively predictable using the corresponding betas of the previous period. The beta forecasts can be generally improved when beta adjustment techniques are used and in the case of portfolios additional improvement can be obtained by increasing the portfolio size. Further evidence confirms that beta adjustment techniques are very effective in reducing the forecast errors associated with higher or lower security betas, but they are less effective for betas near the mean of one.

Finally, by comparing the three different adjustment techniques it was observed that the Bayesian method outperforms the other two procedures, whereas the MLPFS approach underperforms the Bayesian and Blume techniques.

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